

SNS COLLEGE OF TECHNOLOGY

An Autonomous Institution

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A' Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai



Department of Computer Science and Engineering

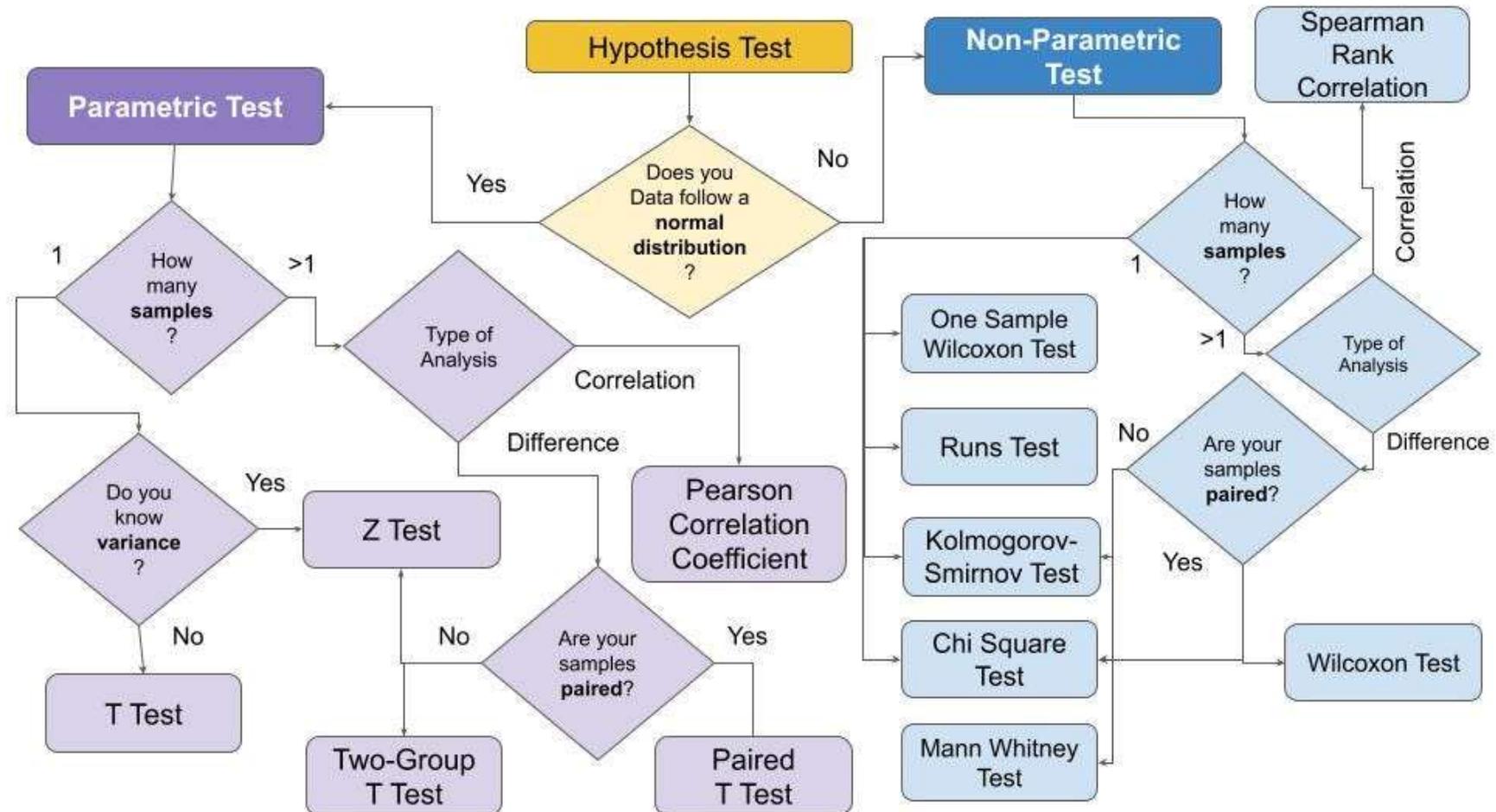
Course Code & Title : 23AD0201 - Data Science Fundamentals

III YEAR / VI SEMESTER - ECE

Unit 3 - INFERENCE STATISTICS

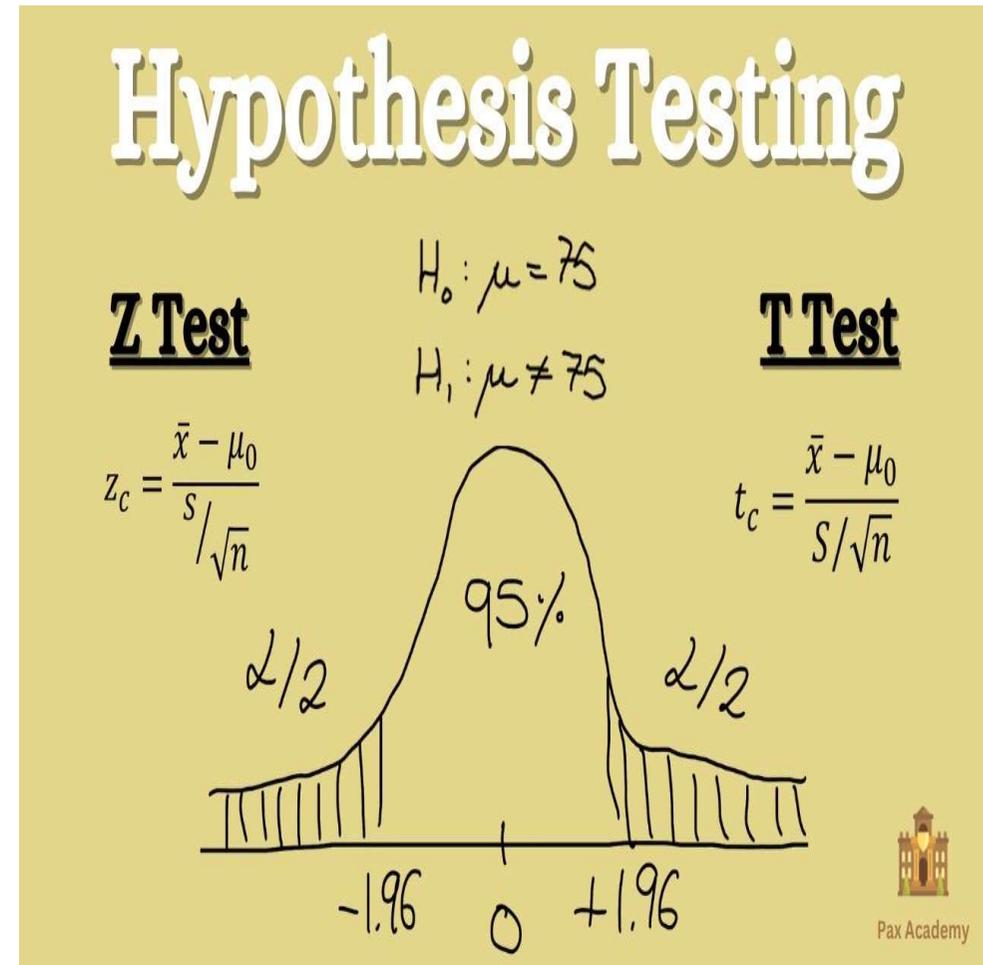
Topic : Hypothesis testing , z-test and z-test procedure

K.KARTHIKEYAN AP /CSE



Hypothesis testing

- Hypothesis testing is a statistical method for making inferences about a population parameter using sample data, typically by comparing a calculated test statistic to a critical value to accept or reject a null hypothesis (H_0)



What is a Z-Test?

A z-test is an inferential tool that determines if the difference between a sample mean and the population mean is statistically significant. It operates under the assumption that the data follows a normal distribution and the population variance is known z-test

Z Test Statistics Formula


$$Z \text{ Test} = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$


Z-Score Formula

$$z = \frac{x - \mu}{\sigma}$$

raw score \rightarrow x μ \leftarrow mean

standard deviation \rightarrow σ



www.inchcalculator.com

where:

x : mean of the sample.

μ : mean of the population.

σ : Standard deviation of the population

Problem -1

Let's understand with the help of example The average family annual income in India is 200k with a standard deviation of 5k and the average family annual income in Delhi is 300k. Then Z-Score for Delhi will be.

$$\begin{aligned} \text{Z-Score} &= \frac{\bar{x} - \mu}{\sigma} \\ &= \frac{300 - 200}{5} \\ &= 20 \end{aligned}$$

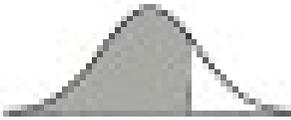
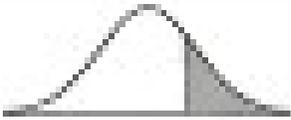
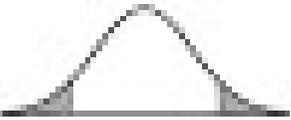
Type of Z-test

There are mainly two types of Z-tests. Let's understand them one by one:

1. One-Sample Z test

A one-sample Z-test is used to determine if the mean of a single sample is significantly different from a known population mean. Use it when:

- The population standard deviation is known.
- The sample size is large (usually $n > 30$).
- The data is approximately normally distributed.

| One-Sample z-test for p | | |
|---|--|--|
| State Hypothesis | | |
| $H_0: p = P$ $H_a: p < P$ | $H_0: p = P$ $H_a: p > P$ | $H_0: p = P$ $H_a: p \neq P$ |
| Find Standardized Test Statistic | | |
| $Z = \frac{\hat{p} - P}{\sqrt{\frac{P(1-P)}{n}}}$ | | Z = standardized test statistic \hat{p} = sample proportion P = H_0 proportion |
| Find the p-value | | |
|  |  |  |
| $H_a: p < P$ | $H_a: p > P$ | $H_a: p \neq P$ |
| On z-table $P(x < z)$ | On z-table $P(x > z)$ | On z-table $2 \times P(x > z)$ |
| Conclusion | | |
| p-value < α | Reject H_0 | |
| p-value > α | Fail to Reject H_0 | |

Suppose a company claims that their new smartphone has an average battery life of 12 hours. A consumer group tests 100 phones and finds an average battery life of 11.8 hours with a known population standard deviation of 0.5 hours.

Step 1: Hypotheses

$$\bullet H_0: \mu = 12 :$$

$$\bullet H_1: \mu \neq 12$$

Step 2: Calculate the Z-Score

We can calculate Z-score using the formula:

$$z = \frac{x - \mu}{\frac{\sigma}{\sqrt{n}}}$$

Where: $\bar{x} = 11.8$, $\mu = 12$, $\sigma = 0.5$ and $n = 100$

After putting the value we get:

$$z = \frac{11.8 - 12}{\frac{0.5}{\sqrt{100}}} = -4$$

Step 3: Decision

Since $|Z| = 4 > 1.96$ (critical value for $\alpha = 0.05$) we reject H_0 indicate significant evidence against the company's claim.

```
import numpy as np
from statsmodels.stats.weightstats import ztest

data = [11.8] * 100
population_mean = 12
population_std_dev = 0.5

z_statistic, p_value = ztest(data, value=population_mean)

print(f'Z-Statistic: {z_statistic:.4f}')
print(f'P-Value: {p_value:.4f}')

alpha = 0.05
if p_value < alpha:
    print("Reject the null hypothesis: The average battery
life is different from 12 hours.")
else:
    print("Fail to reject the null hypothesis: The average
battery life is not significantly different from 12 hours.")
```

Output:

Z-Statistic: -560128131373970.2500

P-Value: 0.0000

*Reject the null hypothesis: The average
battery life is different from 12 hour*

Activity

Activity 1: Hypothesis Testing Concept Activity

Materials Needed

- Board / Projector
- Calculator
- Sample data sheet

Scenario

A college claims that the **average score of students in statistics is 70 marks**. A teacher collects scores from **36 students** and finds the **sample mean is 74** with a **standard deviation of 12**.

Student Task

Students must determine whether the college claim is correct using **hypothesis testing**.

Steps for Students

1.State the Hypotheses

- $H_0 : \mu = 70$ (Average mark is 70)
- $H_1 : \mu \neq 70$ (Average mark is not 70)

2.Identify Given Values

- Sample mean (\bar{x}) = 74
- Population mean (μ) = 70
- Standard deviation (σ) = 12
- Sample size (n) = 36

MCQ

MCQ on Hypothesis Testing and Z-Test

1. Hypothesis testing is used to

- A. Collect data
- B. Analyze population parameters using sample data
- C. Draw graphs
- D. Store data

Answer: B

2. The statement that assumes no difference or no effect is called

- A. Alternative hypothesis
- B. Research hypothesis
- C. Null hypothesis
- D. Statistical hypothesis

Answer: C

MIND MAP

Hypothesis Testing & Z-Test

Understanding Hypothesis Testing



Null Hypothesis (H_0)

$H_0: \mu = 70$
"No Effect"

Alternative Hypothesis (H_1)

$H_1: \mu \neq 70$
"There is an Effect"

— Is there enough evidence? —

What is a Z-Test?

A statistical test to compare sample mean to the population mean.

- Large Sample ($n \geq 30$)
- σ Known

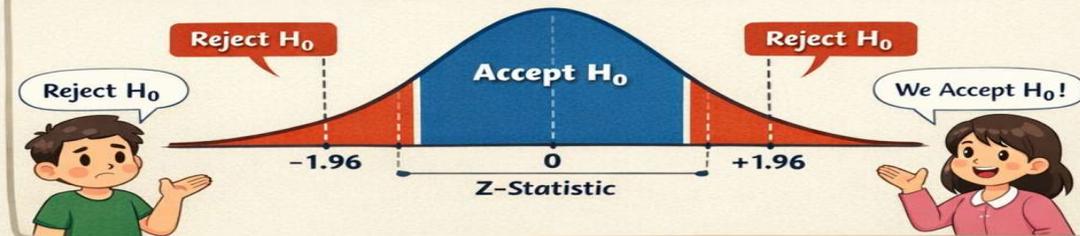
$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

Let's calculate the Z-score!

Steps of Z-Test Procedure

- 1 State Hypotheses
- 2 Set Significance Level (α)
- 3 Calculate Z-Statistic
- 4 Find Critical Value
- 5 Make a Decision

Z-Test Decision



REFERENCE BOOKS

1.Allen B. Downey, “Think Stats: Exploratory Data Analysis in Python”, Green Tea Press, 2014.

2.Sanjeev J. Wagh, Manisha S. Bhende, Anuradha D. Thakare, “Fundamentals of Data Science”, CRC Press, 2022.

3.Chirag Shah, “A Hands-On Introduction to Data Science”, Cambridge University Press, 2020.

4.Vineet Raina, Srinath Krishnamurthy, “Building an Effective Data Science Practice: A Framework to Bootstrap and Manage a Successful Data Science Practice”, A press, 2021.

