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Coimbatore- 49

DEPARTMENT OF COMMERCE WITH INFORMATION TECHNOLOGY

MANAGERIAL ECONOMICS
Cost Function

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Defining the Function

The Mathematical Concept

A cost function expresses the functional relationship between total cost and the quantity of output produced.

$$C = f(Q)$$

It assumes all other factors (technology, input prices) are held constant (ceteris paribus).

Economic Utility

Cost functions are essential for:

Deriving Marginal and Average Cost curves.

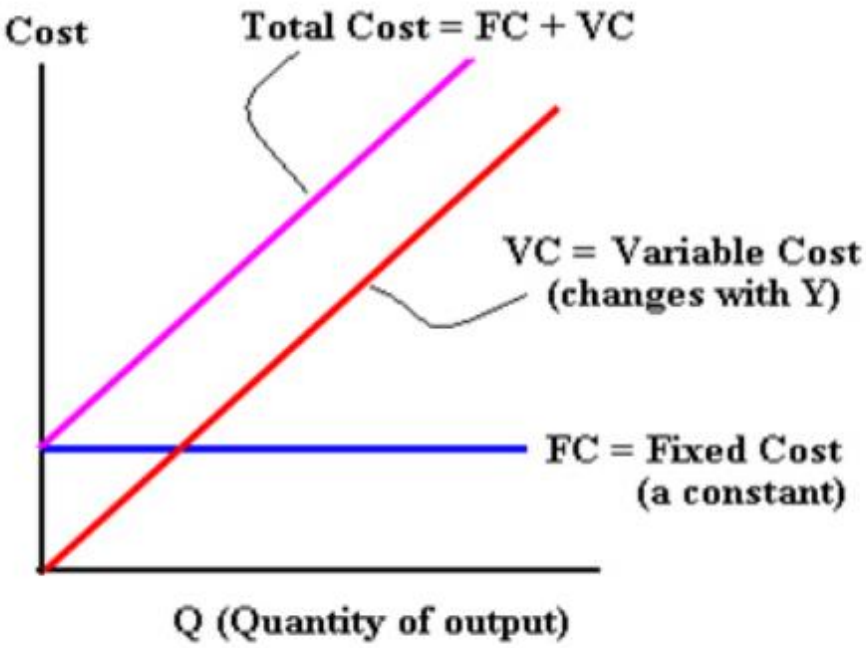
Analyzing efficiency and returns to scale.

Identifying the optimal scale of production.

Short Run Cost Components

Fixed vs. Variable

- **Total Fixed Cost (TFC):** Independent of output (Horizontal line). Includes rent, insurance, depreciation.
- **Total Variable Cost (TVC):** Increases with output (Starts at origin). Includes raw materials, labor.
- **Total Cost (TC):** TC = TFC + TVC . The curve has the same slope as TVC but is shifted up by the amount of TFC.



Linear Cost Function

The Equation

$$TC = a + bQ$$

a : Total Fixed Cost (TFC)
b : Marginal Cost per unit (Constant)

Implications

In a linear model, the Marginal Cost (MC) is constant. Each additional unit costs exactly the same to produce as the last.

This is often a "short-run approximation" but rarely holds true for large variations in output due to the Law of Variable Proportions.

Quadratic Cost Function

The Equation

$$TC = a + bQ + cQ^2$$

- If $c > 0$, costs rise at an increasing rate.

Properties

Marginal Cost (MC) is a linear function of output:

$$MC = \frac{d(TC)}{dQ} = b + 2cQ$$

This implies diminishing returns from the very first unit of variable input.

Cubic Cost Function

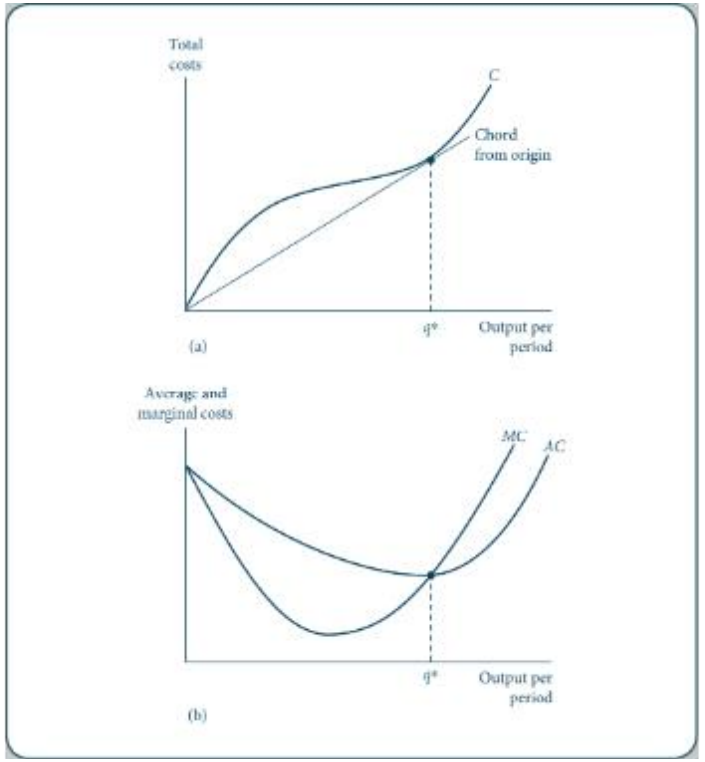
The Theoretical Standard

The cubic form best captures the reality of production (Increasing then Decreasing returns).

$$TC = a + bQ - cQ^2 + dQ^3$$

Shape: The TC curve has an inverse S-shape.

Derivatives: MC and AC curves derived from this function are U-shaped, consistent with economic theory.

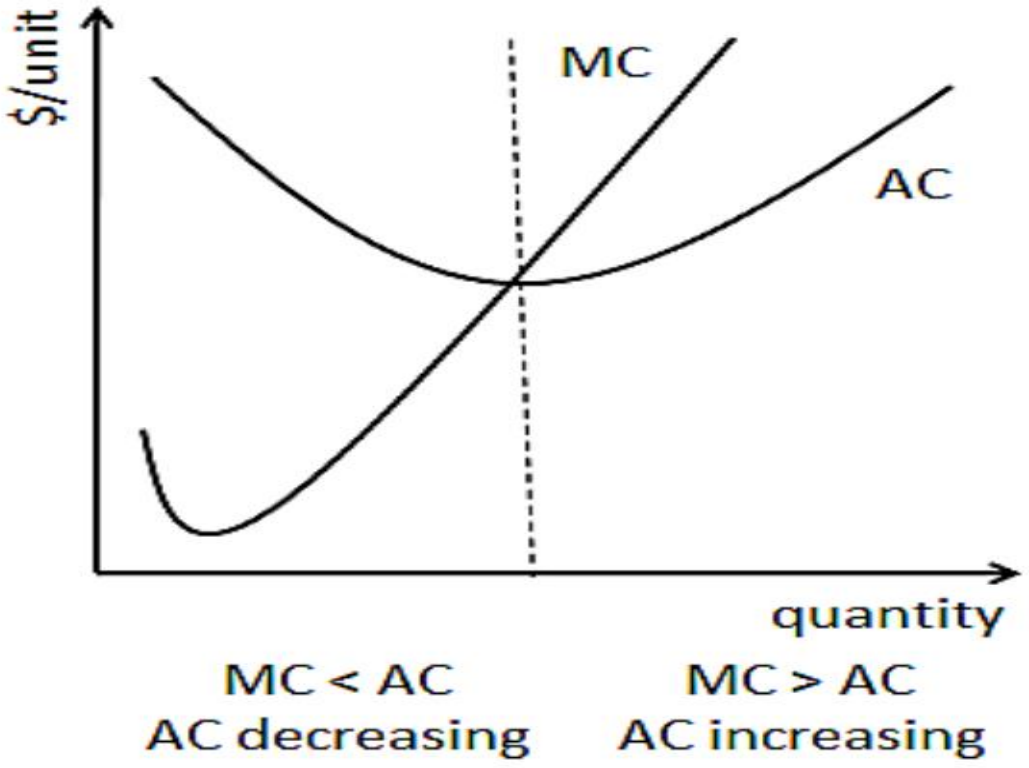


The Geometry of Curves

A fundamental mathematical property of cost functions is the relationship between Marginal and Average values.

The Marginal Cost (MC) curve always intersects the Average Cost (AC) and Average Variable Cost (AVC) curves at their respective minimum points.

When MC < AC , AC falls. When MC > AC , AC rises.



Long Run Implications

No Fixed Costs

In the long run, the constant term a (TFC) becomes zero or variable, as all inputs can be adjusted.

The Long Run Cost Function represents the "lower envelope" of all possible short-run cost curves.

Economies of Scale

The shape of the long-run function is determined by returns to scale.

- AC falling: Economies of Scale.
- AC constant: Constant Returns.
- AC rising: Diseconomies of Scale.

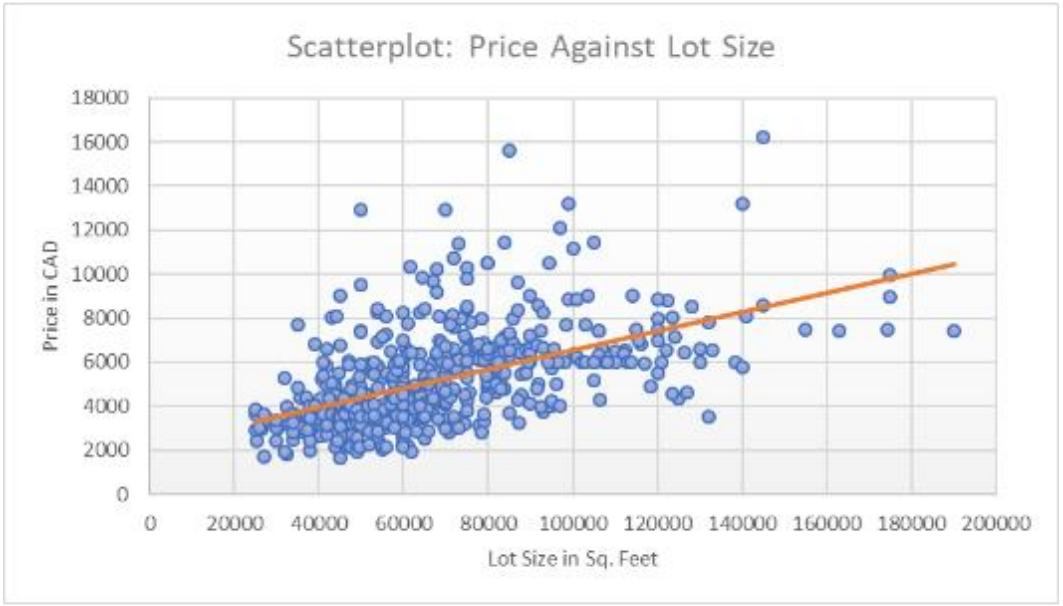
Estimating Cost Functions

Real-world firms estimate their cost functions using regression analysis on historical data (Output vs. Total Cost).

Data Points: Observed combinations of cost and output over time.

Regression Line: The "Line of Best Fit" that minimizes error.

This allows firms to forecast future costs and set budgets effectively.



Application: Profit Max

The ultimate use of the cost function is to find the profit-maximizing level of output.

This occurs where the slope of the Total Cost curve (MC) equals the slope of the Total Revenue curve (MR).

$$MC(Q) = MR(Q)$$