SNS COLLEGE OF TECHNOLOGY

Saravanampatti, Coimbatore – 641 035
Approved by AICTE, Recognized by UGC & Affiliated to Anna University, Chennai

Department of Artificial Intelligence and Data Science

23EET103 – Electric Circuits and Electron Devices

I YEAR /II SEMESTER

UNIT -2 – Impedance and Power

The Anatomy of Electrical Power

From DC sign conventions to AC impedance, phase shifts, and power factor correction.

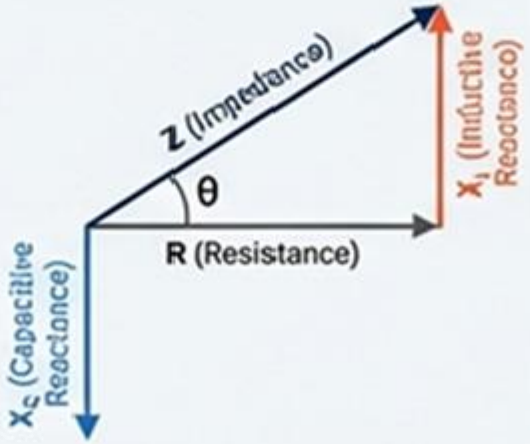
AC Impedance & Phase Angle DIN Pro

$$\boxed{Z = R + j(X_L - X_C)}$$

$$\boxed{|Z| = \sqrt{R^3 + (X_L - X_C)^2}}$$

$$\boxed{\theta = \arctan\left(\frac{X_L - X_C}{R}\right)}$$

where $X_L = 2nfL$ and $X_C = \frac{1}{2\pi fC}$



Power Triangle & Power Factor DIN Pro

$$\boxed{S = P + jQ}$$

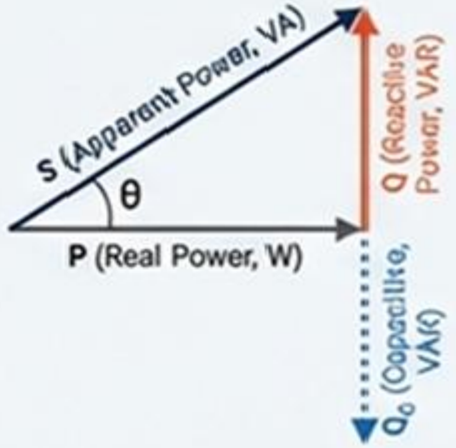
$$\boxed{|S| = \sqrt{P^2 + Q^2}}$$

$$\boxed{PF = \cos(\theta) = \frac{P}{|S|}}$$

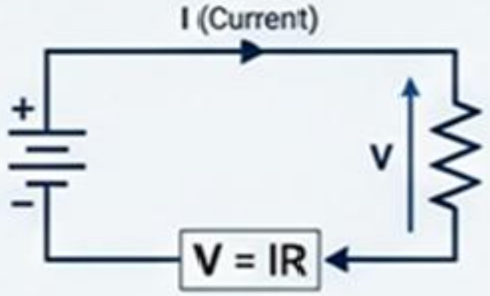
where **P** = $V_{rms} * I_{rms} * \cos(\theta)$ [Real Power, W]

Q = $V_{rms} * I_{rms} * \sin(\theta)$ [Reactive Power, VAR]

S = $V_{rms} * I_{rms}$ [Apparent Power, VA]



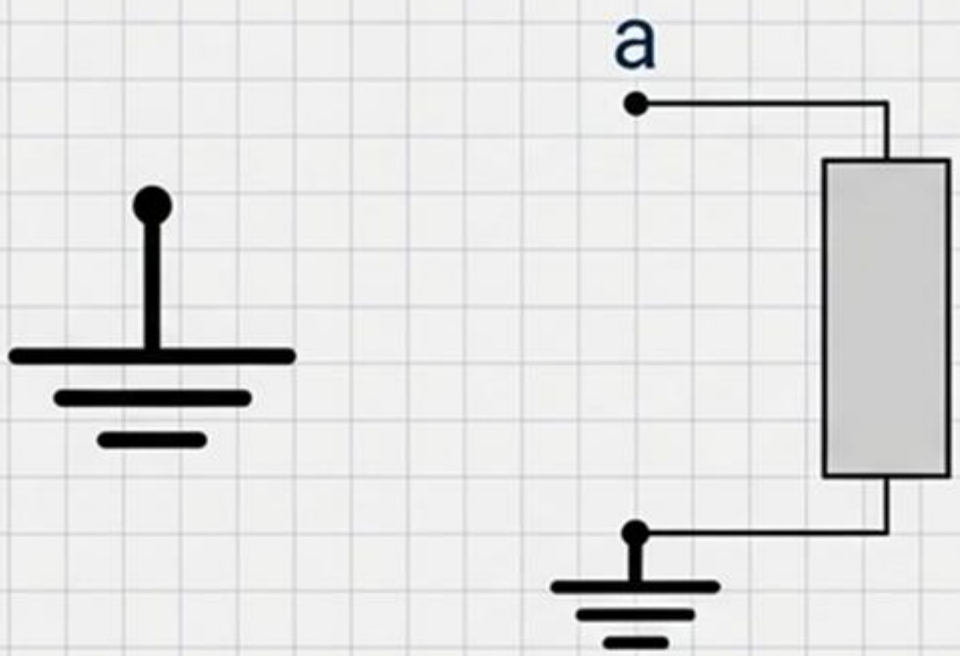
DC Sign Conventions DIN Pro



Passive Sign Convention: Current enters the positive terminal of a passive element.
P = VI > 0 (Absorbing Power).

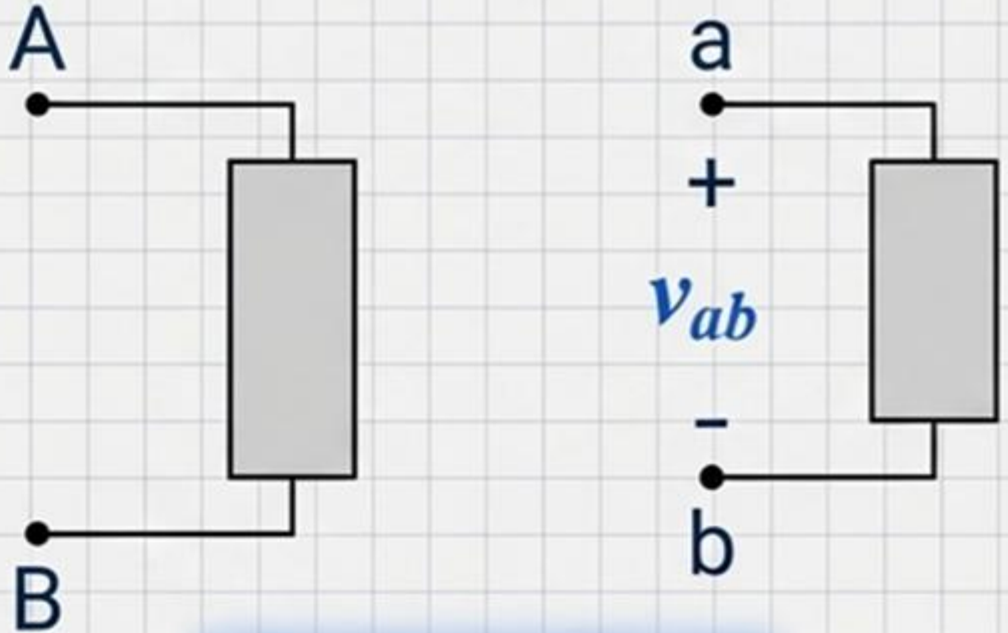
Establishing the Baseline: Ground and Voltage Polarity

Ground Reference



Ground is a reference voltage, typically 0V. Voltages relative to ground are a difference relative to zero, represented with a single subscript (v_a).

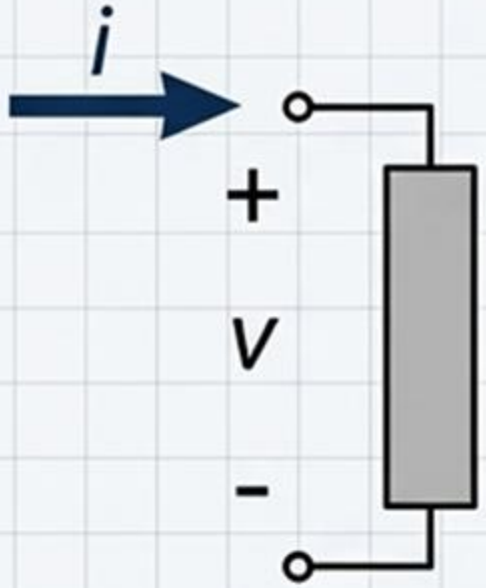
Subscript Polarity



$$\boxed{V_{BA} = -V_{AB}}$$

The first subscript indicates the assumed higher-voltage node; the second is the lower. Reversing the perspective exactly inverts the measurement.

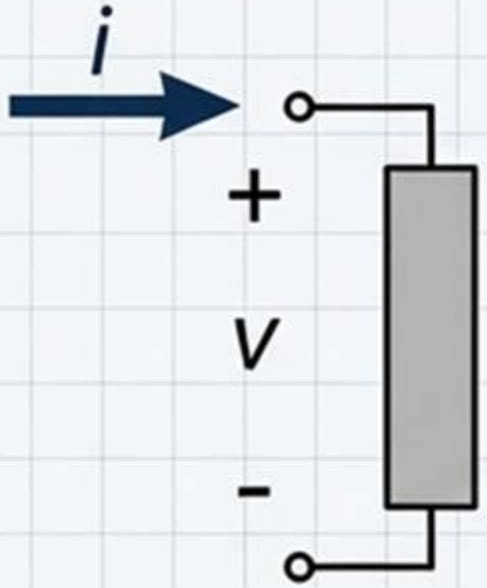
The Passive Sign Convention Matrix



Absorbing

- Consistent with convention.
- Power is positive ($p = +vi$).
- Energy is Dissipated/Absorbed.

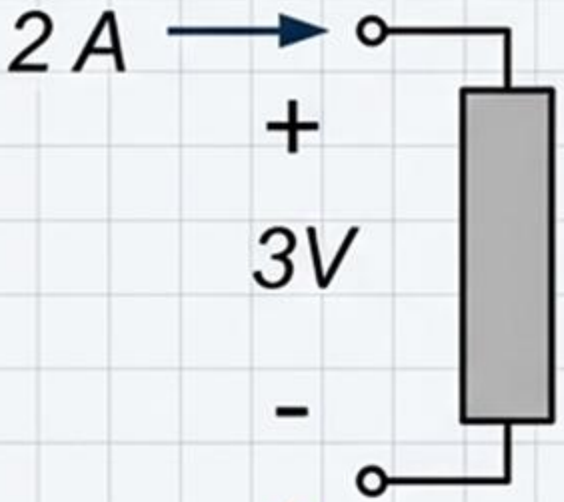
$$\boxed{p = +vi}$$



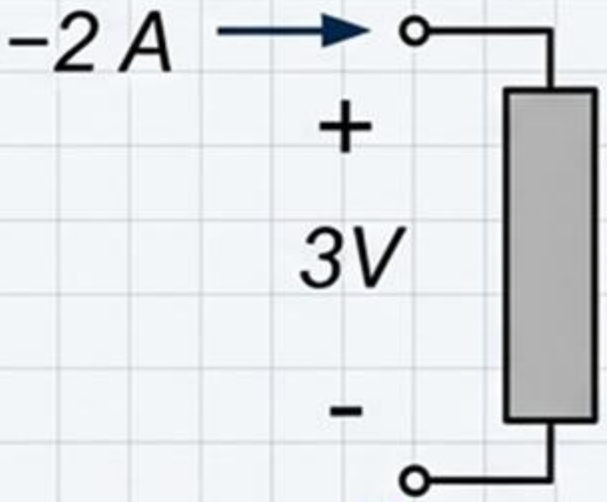
Generating

- Inconsistent with convention.
- Power is negative ($p = -vi$).
- Energy is Supplied/Generated.

$$\boxed{p = -vi}$$



Result: +6W (Power Absorbed)



Result: -6W (Power Generated)

The AC Complication: Defining Impedance (Z)

Imaginary unit
($\sqrt{-1}$)

Impedance
(Total AC opposition in Ohms Ω)

$$Z = R + jX$$

Reactance
(Frequency-dependent magnetic/electric opposition)

Resistance
(Opposes current, dissipates real power)

Polar Coordinate Conversion

Magnitude:

$$|Z| = \sqrt{R^2 + X^2}$$

Phase Angle:

$$\varphi = \arctan\left(\frac{X}{R}\right)$$

Ohm's Law extends to AC circuits: **V = I × Z**. The phase angle (**φ**) dictates exactly how many degrees the voltage leads or lags the current.

The Circuit Element Taxonomy

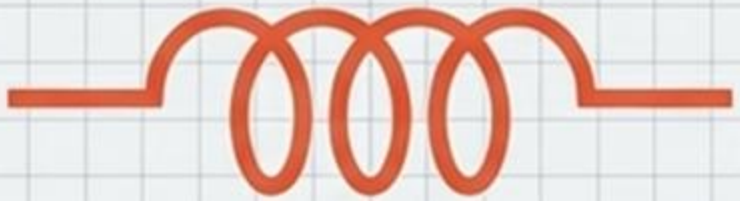
Pure Resistor (**R**)

Impedance: $\boxed{Z = R}$

Reactance: $\boxed{X = 0}$

Phase: 0° (Voltage and Current perfectly in phase)

Pure Inductor (**L**)



Impedance: $\boxed{Z = j\omega L}$

Reactance: $\boxed{X_L = +\omega L}$

Phase: +90° (Voltage leads Current)

Pure Capacitor (**C**)

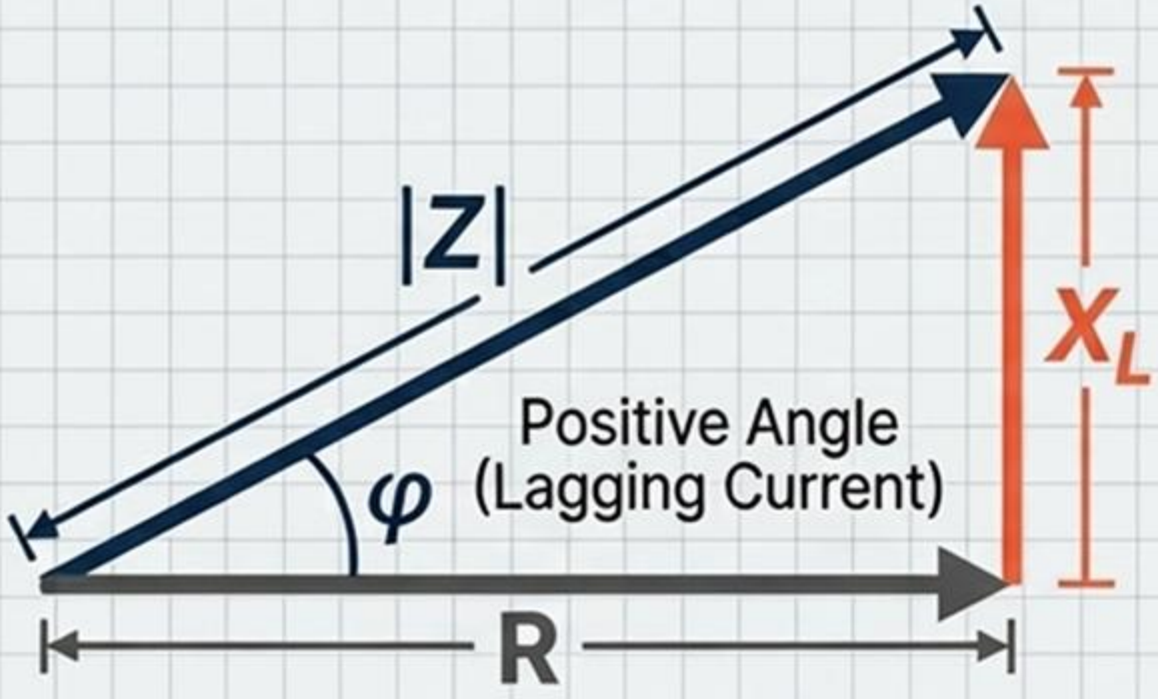
Impedance: $\boxed{Z = -\frac{1}{j\omega C}}$

Reactance: $\boxed{X_C = -\frac{1}{\omega C}}$

Phase: –90° (Current leads Voltage)

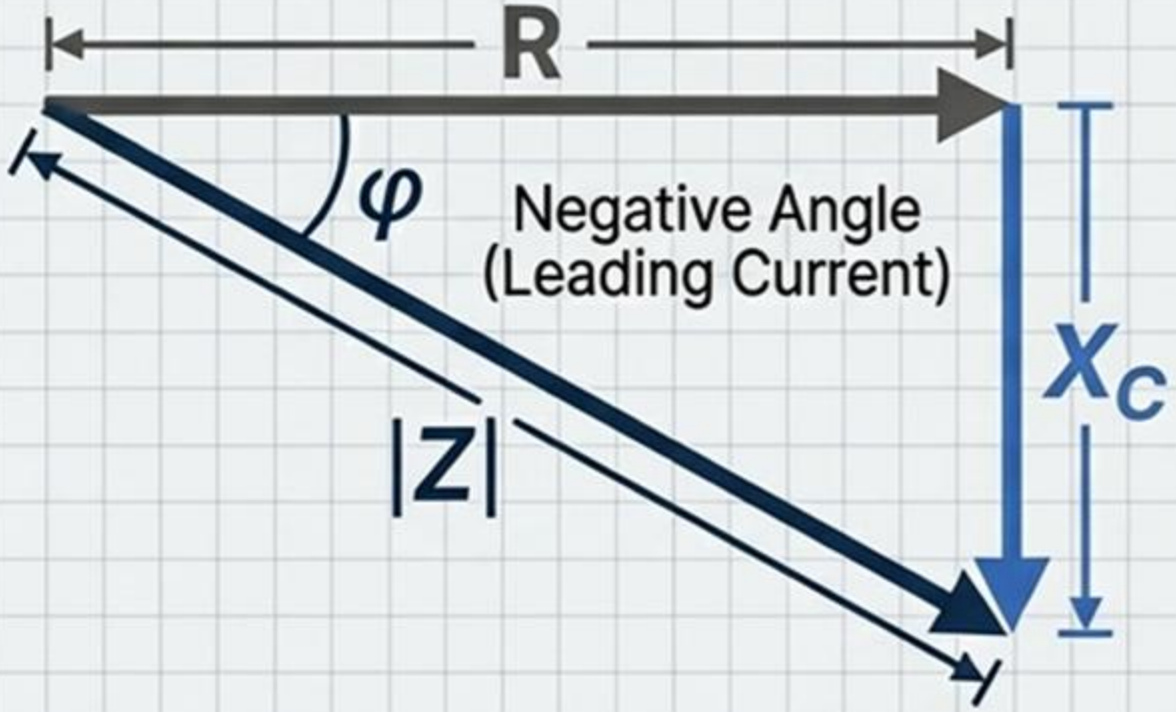
Spatial Math: Visualizing R-L vs. R-C Circuits

The R-L Series Circuit



Current lags voltage by ϕ. Inductive reactance dominates, rotating impedance vector counter-clockwise from the resistive axis.

The R-C Series Circuit



Current leads voltage by ϕ. Capacitive reactance dominates, rotating impedance vector clockwise from the resistive axis.

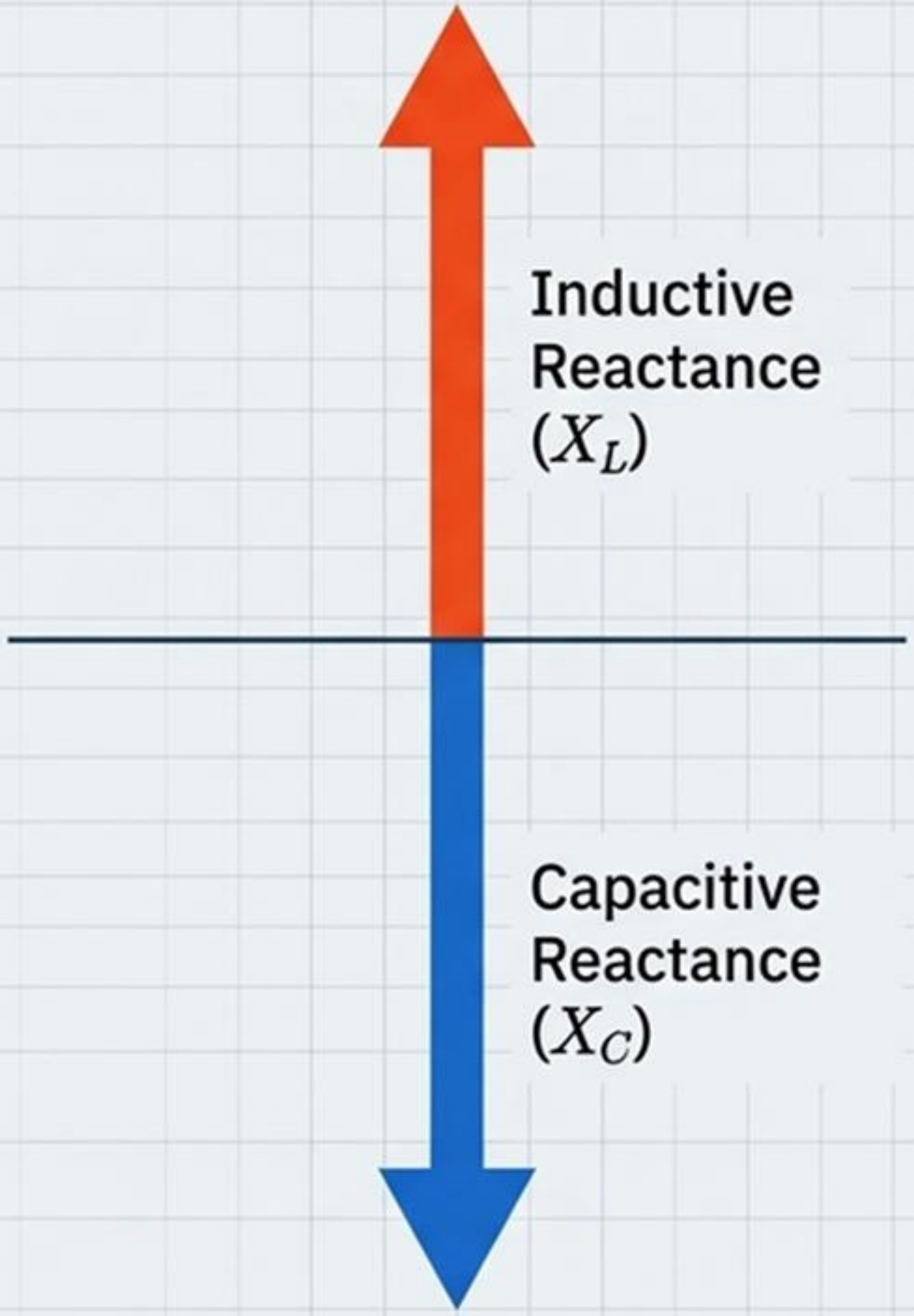
Because inductive and capacitive reactances operate exactly 180° apart on the complex plane, their phase shifts mirror each other perfectly across the resistive axis.

The R-L-C Tug-of-War & Series Resonance

Net Reactance

Equation: $X = X_L - X_C$

- The circuit acts inductively if $X_L > X_C$
- The circuit acts capacitively if $X_L < X_C$



The Resonance Phenomenon

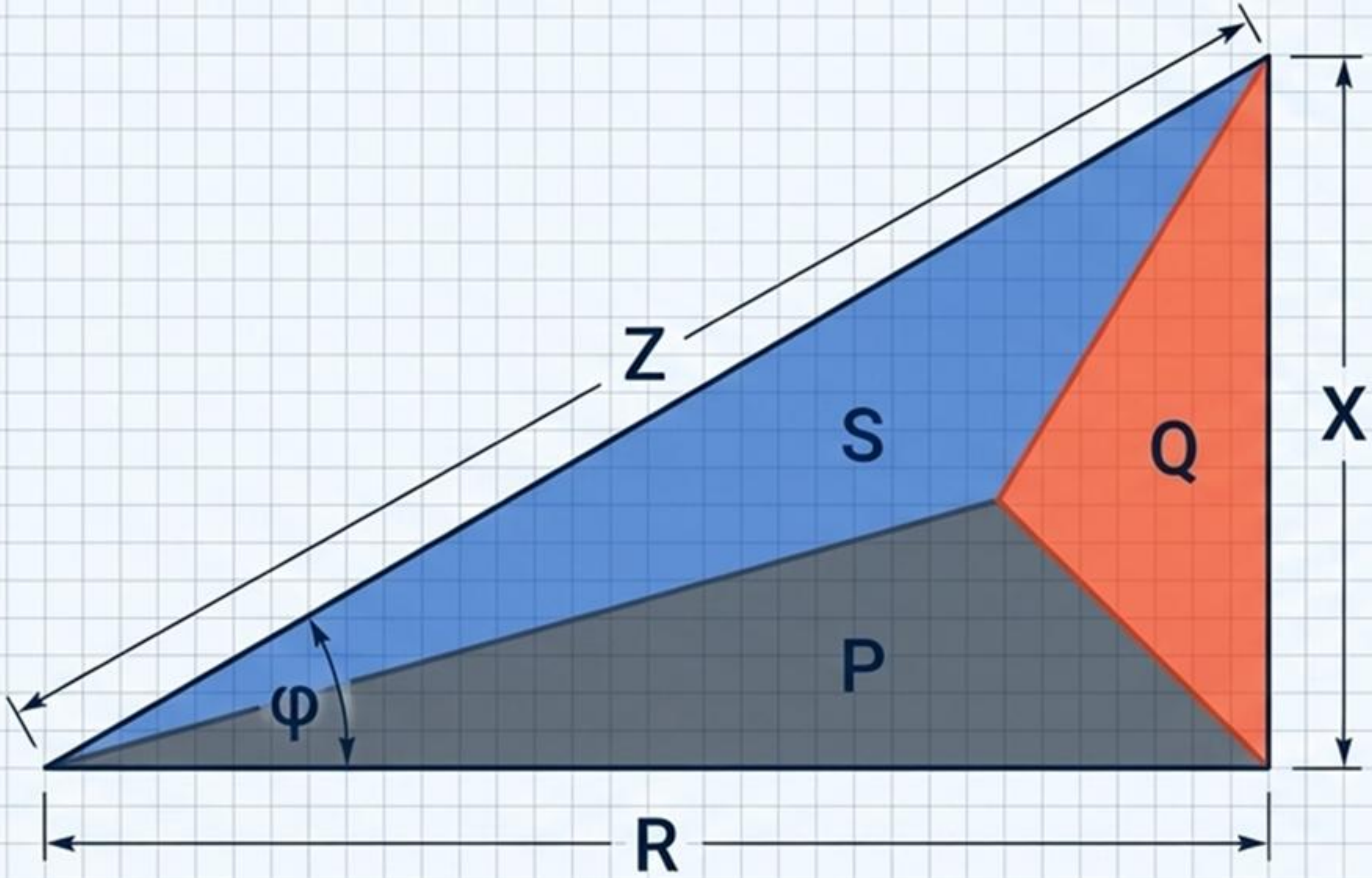
- When X_L perfectly equals X_C, they mathematically cancel out ($X = 0$).
- **Z = R** (Absolute minimum impedance).
- Power Factor = 1.0 (Unity).

Warning: At resonance, reactive voltages V_L and V_C can **individually be much larger than total supply voltage**, despite **perfectly canceling each other out!**

The Dimensionality of Power: The Power Trinity

Real Power (P)	Reactive Power (Q)	Apparent Power (S)
$P = S \times \cos(\varphi)$ $[\text{or } I^2 R]$	$Q = S \times \sin(\varphi)$ $[\text{or } I^2 X]$	$S = V_{\text{rms}} \times I_{\text{rms}}$
Unit: Watts (W)	**Unit: VAR** (Volt-Ampere Reactive)	**Unit: VA** (Volt-Ampere)
Useful work actually performed (heat, light, mechanical energy). Consumed entirely by resistive elements.	Energy oscillating blindly between the source and magnetic/electric fields. Performs zero useful work.	The raw, total power the source must supply to the grid. Geometrically defined as $S = \sqrt{P^2 + Q^2}$.

The Symmetry of AC Circuits



Power Factor = $\cos(\varphi)$

Physical Hardware Ratio:

$$\cos(\varphi) = \frac{R}{|Z|}$$

Energy Output Ratio:

$$\cos(\varphi) = \frac{P}{S}$$

The ratio of physical resistance to total impedance perfectly equals the ratio of useful real power to total apparent power. The geometry of the hardware dictates the geometry of the energy.

Power Factor States & Real-World Loads

Unity PF (PF = 1.0)
Charcoal

$\varphi = 0^\circ$ | **$P = S$, $Q = 0$** (Purely Resistive)

Real-World Load Examples

Heaters, incandescent lamps.

Lagging PF ($0 < PF < 1.0$)
Vermilion Orange

$\varphi > 0^\circ$ | Inductive State ($Q_L > 0$)

Real-World Load Examples
Inter

Motors, transformers, fluorescent lighting.
(The most common industrial state).

Leading PF ($0 < PF < 1.0$)
Cobalt Blue

$\varphi < 0^\circ$ | Capacitive State ($Q_C > 0$)

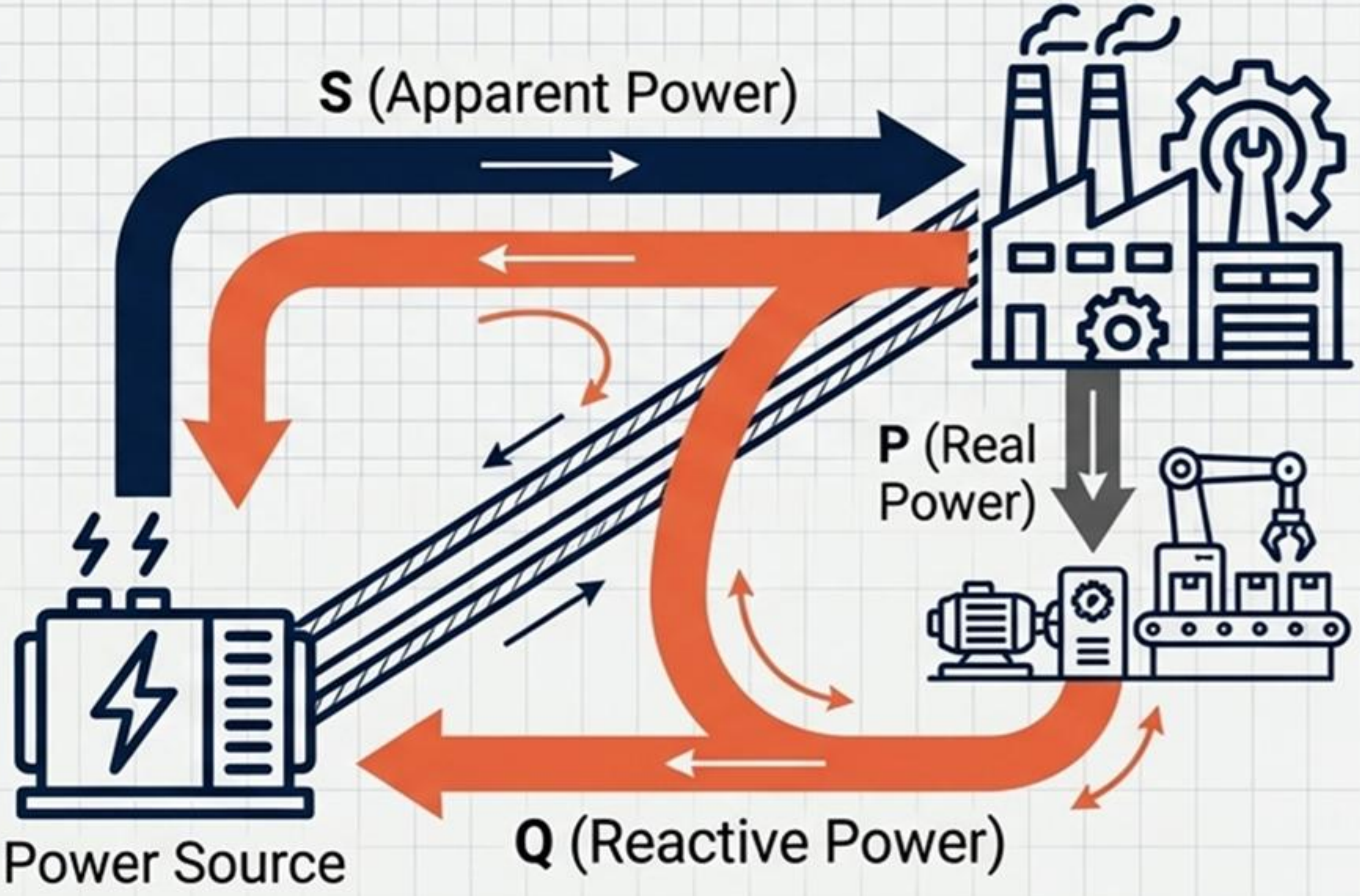
Real-World Load Examples
Inter

Capacitor banks, lightly loaded transmission cables.

The Engineering Problem: The Cost of Poor Power Factor

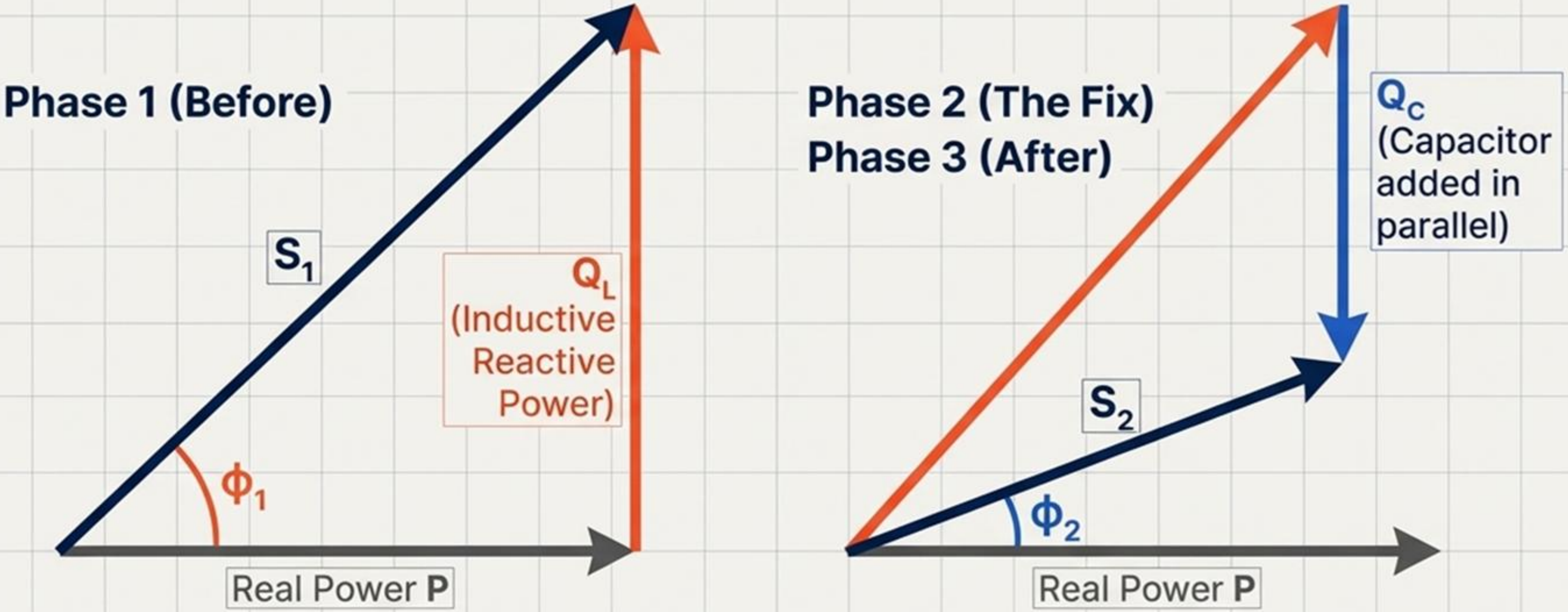
A low PF means drastically higher current (**I**) is drawn to achieve the exact same real work (**P**). This triggers a cascading failure of efficiency:

1. Increased **I²R** conductor heat losses.
2. Necessitates oversized, expensive cables and transformers.
3. Induces severe voltage drops across distribution lines.



Data Reality: At a PF of 0.6, a 10 kW load requires 72.5 Amps. At Unity PF, the exact same 10 kW load requires only 43.5 Amps. The extra 29 Amps merely heat the cables and do zero useful work.

The Correction Mechanism



Crucial Insight: The horizontal base line (Real Power P) remains completely unchanged throughout the process. The net reactive power is simply $Q_{net} = Q_L - Q_C$. By pulling the **S** vector down, we vastly improve efficiency.

Sizing the Solution: Mathematics of Capacitor Correction

Step 1: Find Required Reactive Power (VAR)

$$Q_C = P \times (\tan(\phi_1) - \tan(\phi_2))$$

(Where ϕ_1 is the original angle, and ϕ_2 is the target corrected angle).

Step 2: Size the Physical Capacitor (Farads)

$$C = \frac{Q_C}{2\pi f \times V_{\text{rms}}^2}$$

Application Example

Example: To correct a 10 kW load from **0.6 PF** to **0.9 PF** on a 230V/50Hz supply:

We first calculate **Q_C = 8,490 VAR**. Applying the sizing formula dictates a precise **515 µF** parallel capacitor.

Result: Apparent Power demand plummets from **16.67 kVA** to just **11.11 kVA**, instantly eliminating the utility penalty.

REFERENCES

- https://www.electrical4u.com
- https://www.allaboutcircuits.com
- https://nptel.ac.in/courses/108106172

THANK YOU