

Solution

POLYNOMIALS

Class 10 - Mathematics

1.

(d) $a^2 + 2b$

Explanation:

Given, $P(x) = x^2 - ax - b$

$$\alpha + \beta = a, \alpha\beta = -b$$

$$(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$$

$$a^2 = \alpha^2 + \beta^2 - 2b$$

$$\alpha^2 + \beta^2 = a^2 + 2b$$

2.

(c) $x^2 - 2x - 1$

Explanation:

A quadratic polynomial is always in the form of

$$x^2 - (\text{sum of zeros})x + (\text{product of Zeros})$$

hence the required polynomial is

$$x^2 - (2)x + (-1)$$

$$= x^2 - 2x - 1$$

3.

(c) -36

Explanation:

$p(-4) = 0$ (since -4 is root of $p(x)$)

$$(-4)^2 - 5(-4) + k = 0$$

$$\Rightarrow 16 + 20 + k = 0$$

$$36 + k = 0$$

$$k = -36$$

4.

(c) $\frac{11}{4}$

Explanation:

Here $a = 3, b = 11, c = -4$ Since $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$

$$\alpha + \beta = \frac{-11}{3}, \alpha\beta = \frac{-4}{3}$$

$$\text{So, } \frac{\frac{-11}{3}}{\frac{-4}{3}} = \frac{11}{4}$$

5.

(b) $\frac{2}{3}$

Explanation:

Since α, β are the zeros of $kx^2 - 2x + 3k$, we have

$$\alpha + \beta = -\frac{(-2)}{k} = \frac{2}{k}$$

$$\alpha\beta = \frac{3k}{k} = 3$$

Now, $\alpha + \beta = \alpha\beta$

$$\Rightarrow \frac{2}{k} = 3$$

$$\Rightarrow k = \frac{2}{3}$$

6.

(d) 9

Explanation:

Here $a = 1$, $b = -6$, $c = 8$, $\alpha + \beta = 6$, $\alpha\beta = 8$

$$\begin{aligned} \text{Since } \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} &= \frac{\alpha^3 + \beta^3}{\alpha\beta} \\ &= \frac{(\alpha + \beta)[\alpha^2 + \beta^2 - \alpha\beta]}{\alpha\beta} = \frac{(\alpha + \beta)[\alpha^2 + \beta^2 + 2\alpha\beta - 3\alpha\beta]}{\alpha\beta} \\ &= \frac{(\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha\beta]}{\alpha\beta} \\ &= \frac{6[6^2 - 3 \times 8]}{8} \\ &= 9 \end{aligned}$$

7.

(c) 0

Explanation:

Sum of zeroes of the quadratic equation

$$ax^2 + bx + c = 0 \text{ is } \frac{-b}{a}$$

$$\therefore \text{Sum of zeroes of } x^2 - 1 = x^2 + 0x - 1 = 0 \text{ is } \frac{-0}{1} = 0$$

$$\therefore \alpha + \beta = 0$$

8.

(b) $3, \frac{-3}{2}$

Explanation:

$$p(x) = 2x^2 - 3x - 9$$

$$0 = 2x^2 - 6x + 3x - 9$$

$$= 2x(x - 3) + 3(x - 3)$$

$$0 = (2x + 3)(x - 3)$$

$$x = \frac{-3}{2}, x = 3$$

9.

(c) equal to 0

Explanation:

Given that two of the zeros of the cubic polynomial $ax^3 + bx^2 + cx + d$ are 0,

i.e. $\alpha = 0, \beta = 0$

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\gamma = -\frac{b}{a}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = -\frac{c}{a}$$

$$0 = -\frac{c}{a}$$

$$c = 0$$

10.

(b) ± 3

Explanation:

Let α, β are the zeroes of the given polynomial.

$$\text{Given: } \alpha + \beta = \alpha\beta$$

$$\Rightarrow \frac{-b}{a} = \frac{c}{a}$$

$$\Rightarrow -b = -c$$

$$\Rightarrow -(-27) = 3k^2$$

$$\Rightarrow k^2 = 9$$

$$\Rightarrow k = \pm 3$$

11.

(d) 2

Explanation:

$$\alpha + \beta = \frac{-b}{a}$$

$$\sqrt{2} = \frac{-(-k\sqrt{2})}{2}$$

$$\frac{2\sqrt{2}}{\sqrt{2}} = k$$

$$k = 2$$

12. **(a) 6 and 8**

Explanation:

Sum of the zeroes of the polynomial = $\frac{-b}{a} = \frac{6}{1} = 6$
 And Product of the zeroes of the polynomial = $\frac{c}{a} = \frac{8}{1} = 8$

13.

(b) $-m, m + 3$

Explanation:

Given: equation $x^2 - 3x - m(m + 3) = 0$, where m is a constant

The given equation is the form of $ax^2 + bx + c = 0$

$$\therefore a = 1, b = -3, c = -m(m + 3)$$

We know the roots of the equation can be find out using the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substituting the values of a, b, c , we get

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-m(m+3))}}{2}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{9 + 4m^2 + 12m}}{2}$$

$$\Rightarrow x = \frac{3 \pm (2m+3)}{2}$$

$$\text{or } x = \frac{3 + (2m+3)}{2}, x = \frac{3 - (2m+3)}{2}$$

$\Rightarrow x = m + 3$ and $x = -m$ are the required roots of the equation.

14.

(c) 3, 5

Explanation:

Sum of zeroes of polynomial

$$5x^2 - (3 + k)x + 7 \text{ is } \frac{-[-(3+k)]}{5}, \text{ i.e., } \frac{3+k}{5}$$

$$\text{According to question, } \frac{3+k}{5} = 0 \Rightarrow k = -3$$

Now, $2x^2 - 2(k + 11)x + 30$ becomes $2x^2 - 16x + 30$.

$$\text{i.e., } 2x^2 - 16x + 30 = 0 \text{ or } x^2 - 8x + 15 = 0$$

$$\Rightarrow x = 3, 5$$

Hence, zeroes of polynomial $2x^2 - 16x + 30$ are 3, 5.

15.

(b) -1

Explanation:

Since α and β are the zeros of the quadratic polynomial $f(x) = x^2 + x + 1$

$$\alpha + \beta = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

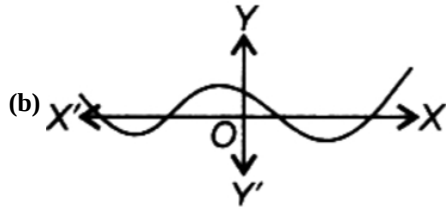
$$= \frac{-1}{1} = -1$$

$$\alpha \times \beta = \frac{\text{constant term}}{\text{coefficient of } x^2} = \frac{1}{1} = 1$$

$$\text{Now, } \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{-1}{1} = -1$$

Thus, the value of $\frac{1}{\alpha} + \frac{1}{\beta}$ is -1.

16.



Explanation:

For more than three distinct real roots the graph must cut x-axis at least four times.

17.

(b) $(x - \sqrt{3})^2$

Explanation:

$$\alpha + \beta = 2\sqrt{3}$$

$$\alpha\beta = 3$$

required quad. poly is

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$x^2 - (2\sqrt{3})x + 3 = 0$$

$$x^2 - 2\sqrt{3}x + 3 = 0$$

$$(x - \sqrt{3})^2 = 0$$

18.

(b) $-\frac{9}{2}$

Explanation:

For $ax^2 + bx + c$, we have $\alpha\beta = \frac{c}{a}$

For $2x^2 + 5x - 9$, we have $\alpha\beta = \frac{-9}{2}$

19.

(b) infinite

Explanation:

infinite

20.

(b) $x^2 + 3x + 2$

Explanation:

According to the question:

$$\alpha + \beta = -3 \text{ and } \alpha\beta = 2$$

The quadratic polynomial whose sum and product of the zeroes are given is given by

$$x^2 - (\alpha + \beta)x + \alpha\beta$$

\Rightarrow Then the quadratic polynomial will be:

$$\Rightarrow x^2 - (-3)x + 2$$

$$\Rightarrow x^2 + 3x + 2$$

Hence, the quadratic polynomial is $x^2 + 3x + 2$

21.

(d) A is false but R is true.

Explanation:

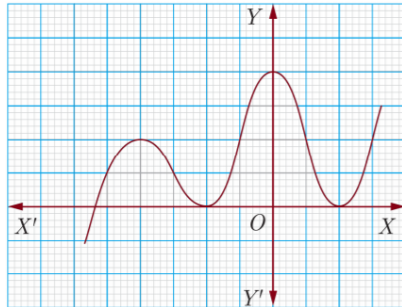
As constant polynomial is only a real number, it has degree as zero, so it has non-zero, so it will never cut x-axis at any point.

22.

(c) A is true but R is false.

Explanation:

As the number of zeroes of polynomial $f(x)$ is the number of points at which $f(x)$ cuts (intersects) then x-axis and number of zero in the given fig. is 3.



So, A is correct but R is not correct.

23. (a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

Explanation:

Quadratic polynomial be $x^2 - \left(-\frac{1}{4}\right)x + \frac{1}{4}$

$$\Rightarrow x^2 + \frac{1}{4}x + \frac{1}{4} \Rightarrow \frac{1}{4}(4x^2 + x + 1)$$

Quadratic polynomial be $4x^2 + x + 1$.

So, both assertion and reason are correct and reason explains assertion.

24.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation:

Reason is true as we know that Sum of zeroes = $-\frac{b}{a}$

Also, we know that Product of zeroes = $\frac{c}{a}$

$$\Rightarrow \frac{5k}{1} = -10 \Rightarrow k = -2$$

So, the Assertion is true. But Reason is not the correct explanation of assertion.

25.

(d) A is false but R is true.

Explanation:

$$x^2 + 7x + 12 = 0$$

$$\Rightarrow x^2 + 4x + 3x + 12 = 0$$

$$\Rightarrow x(x + 4) + 3(x + 4) = 0$$

$$\Rightarrow (x + 4)(x + 3) = 0$$

$$\Rightarrow (x + 4) = 0 \text{ or } (x + 3) = 0$$

$$\Rightarrow x = -4 \text{ or } x = -3$$

Therefore, $x^2 + 7x + 12$ has two real zeroes.

26. Given: $\alpha + \beta = -6$ and $\alpha\beta = -4$,

The quadratic polynomial with α and β as zeros can be written as:

$$x^2 - (\alpha + \beta)x + \alpha\beta$$

$$= x^2 - (-6)x + (-4)$$

$$= x^2 + 6x - 4$$

27. The number of zeroes is 1 as the graph intersects the x-axis at one point only.

28. Compare $f(x) = 5x^2 - 7x + 1$ with $ax^2 + bx + c$ we get,

$$a = 5, b = -7 \text{ and } c = 1$$

Since α and β are the zeroes of $5x^2 - 7x + 1$, we have

$$\alpha + \beta = -\frac{b}{a} = -\frac{(-7)}{5} = \frac{7}{5}$$

$$\alpha\beta = \frac{c}{a} = \frac{1}{5}$$

$$\therefore \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta}$$

$$= \frac{\frac{7}{5}}{\frac{1}{5}}$$

$$= \frac{7}{5} \times \frac{5}{1}$$

$$= 7$$

29. Let $p(t) = t^2 - 15$

For zeroes of $p(t)$,

$$p(t) = 0$$

$$\Rightarrow t^2 - 15 = 0 \Rightarrow (t)^2 - (\sqrt{15})^2 = 0$$

$$\Rightarrow (t - \sqrt{15})(t + \sqrt{15}) = 0. \text{ Using the identity } a^2 - b^2$$

$$= (a - b)(a + b)$$

$$\Rightarrow t - \sqrt{15} = 0 \text{ or } t + \sqrt{15} = 0$$

$$\Rightarrow t = \sqrt{15} \text{ or } t = -\sqrt{15} \Rightarrow t = \sqrt{15}, -\sqrt{15}$$

So, the zeroes of $p(t)$ are $\sqrt{15}$ and $-\sqrt{15}$

We observe that, Sum of its zeroes

$$= (\sqrt{15})(-\sqrt{15}) = 0$$

$$= \frac{0}{1} = \frac{-(\text{Coefficient of } t)}{\text{Coefficient of } t^2}$$

$$\text{Product of its zeroes} = (\sqrt{15}) \times (-\sqrt{15})$$

$$= -15 = \frac{-15}{1} = \frac{\text{Constant term}}{\text{Coefficient of } t^2}$$

Hence relation between zeroes and coefficient is verified.

30. Let the two zeroes of $x^2 - 8x + k$ be $\alpha, \alpha + 2$

$$\therefore 2\alpha + 2 = 8$$

$$\Rightarrow \alpha = 3, \text{ other zero is } 5$$

$$\therefore k = 15$$

31. Here, $h(t) = t^2 - 15 = t^2 - (\sqrt{15})^2$

$$= (t + \sqrt{15})(t - \sqrt{15})$$

$$h(t) = 0, \text{ if } t + \sqrt{15} = 0 \text{ or } t - \sqrt{15} = 0$$

Hence, the zeroes of $h(t)$ are $-\sqrt{15}$ and $\sqrt{15}$

Now in polynomial $t^2 - 15$

$$\text{Sum of the zeroes} = -\sqrt{15} + \sqrt{15} = 0 = -\frac{b}{a}$$

$$\text{Product of zeroes} = -\sqrt{15} \times \sqrt{15} = -15 = \frac{c}{a}$$

Hence, the relationship between zeros and coefficients is verified.

32. Let $\alpha = \frac{3-\sqrt{3}}{5}$ and $\beta = \frac{3+\sqrt{3}}{5}$

$$\text{Given } \alpha + \beta = \frac{3-\sqrt{3}}{5} + \frac{3+\sqrt{3}}{5} = \frac{6}{5}$$

Product of zeroes,

$$\alpha\beta = \left(\frac{3-\sqrt{3}}{5}\right) \times \left(\frac{3+\sqrt{3}}{5}\right) = \frac{3^2 - (\sqrt{3})^2}{5 \times 5} = \frac{6}{25}$$

Polynomial

$$p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$$

$$= x^2 - \frac{6}{5}x + \frac{6}{25}$$

$$p(x) = 25x^2 - 30x + 6$$

33. Since, α and β are the zeroes of the quadratic polynomial $f(x) = 6x^2 + x - 2$.

$$a=6, b=1, c=-2$$

$$\text{sum of zeros} = \alpha + \beta = -\frac{b}{a} = \frac{-1}{6}$$

$$\text{Product of the zeroes} = \alpha\beta = \frac{c}{a} = \frac{-2}{6}$$

Now,

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$= \frac{\left(-\frac{1}{6}\right)^2 - 2\left(-\frac{1}{3}\right)}{-\frac{1}{3}} = \frac{\frac{1}{36} + \frac{2}{3}}{-\frac{1}{3}} = -\frac{\frac{75}{36}}{\frac{1}{3}} = -\frac{75}{108} \times \frac{3}{1} = -\frac{225}{108} = -\frac{25}{12}$$

34. Let $p(x) = x^2 - 5x + 6$. Then,

$$\alpha + \beta = -\frac{-5}{1} = 5$$

$$\alpha\beta = \frac{6}{1} = 6$$

$$\text{Now, } \frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$$

$$= \frac{\alpha + \beta}{\alpha\beta} - 2\alpha\beta$$

$$= \frac{5}{6} - 2 \times 6$$

$$= \frac{5}{6} - 12 = -\frac{67}{6}$$

35. $4x^2 + 17x - 15 = (4x - 3)(x + 5)$

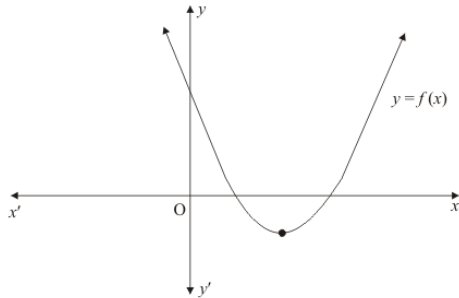
$\therefore x = \frac{3}{4}$ & $x = -5$ are the zeroes of the polynomial.

$$a = 4, b = 17, c = -15$$

$$\text{Sum of zeroes} = \frac{-17}{4} = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$$

$$\text{Product of zeroes} = \frac{-15}{4} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

36. Clearly, $f(x) = ax^2 + bx + c$ represent a parabola opening upwards. Therefore, $a > 0$



Since the parabola cuts the x-axis at two points, this means that the polynomial will have two real solutions.

$$\text{Hence } b^2 - 4ac > 0$$

$$\text{Hence } a > 0 \text{ and } b^2 - 4ac > 0$$

37. Given that,

Quadratic polynomial is $x^2 + 6x + 8$

$$\Rightarrow x^2 + 6x + 8$$

$$\Rightarrow x^2 + 4x + 2x + 8$$

$$\Rightarrow x(x + 4) + 2(x + 4)$$

$$\Rightarrow (x + 2)(x + 4)$$

Zeroes are -2, -4

$$\text{Now, Sum of zeroes} = -2 + (-4) = -6$$

$$\text{Product of zeroes} = (-2) \times (-4) = 8$$

$$\text{Also, Sum of zeroes} = \frac{-b}{a} = \frac{-6}{1} = -6$$

$$\text{Product of zeroes} = \frac{c}{a} = \frac{8}{1} = 8$$

Hence, relationship between zeroes and coefficients verified.

38. Here it is given that

$$f(x) = x^2 - 2$$

$$= (x + \sqrt{2})(x - \sqrt{2})$$

$$f(x) = 0 \text{ if}$$

$$x + \sqrt{2} = 0 \text{ or } x - \sqrt{2} = 0$$

$$\Rightarrow x = -\sqrt{2} \text{ or } x = \sqrt{2}$$

So, the zeros of $f(x)$ are $-\sqrt{2}$ and $\sqrt{2}$.

$$\text{Now in } x^2 - 2 = x^2 - 0 \times x - 2$$

$$\text{Sum of zeros} = -\sqrt{2} + \sqrt{2} = 0 = \frac{0}{2} = -\frac{b}{a}$$

$$\text{Product of zeros} = -\sqrt{2} \times \sqrt{2} = -2 = \frac{-2}{1} = \frac{c}{a}$$

Hence the relationship between zeros and coefficients is verified.

39. The given polynomial is

$$p(x) = 2x^2 + px + 4$$

So, $a = 2$, $b = p$ and $c = 4$

One zero = 2

Let the other zero = m

$$\text{Now, sum of zeroes} = -\frac{p}{2} \dots\dots\dots(i)$$

$$\text{and product of the zeroes} = \frac{4}{2} = 2$$

$$\Rightarrow 2 \times m = 2$$

$$\therefore \text{Other zero} = 1$$

$$\therefore \text{Sum of zeroes} = 2 + 1 = 3 \dots\dots\dots(ii)$$

From (i) and (ii),

$$-\frac{p}{2} = 3$$

$$\Rightarrow p = -6$$

Hence the value of p is -6.

40. By splitting the middle term

$$5t^2 + 12t + 7 = 0$$

$$5t^2 + (5t + 7t) + 7 = 0$$

$$5t^2 + 5t + 7t + 7 = 0$$

$$5t(t + 1) + 7(t + 1) = 0$$

$$(t + 1)(5t + 7) = 0$$

$$(t + 1)(5t + 7) = 0$$

$$\Rightarrow t = -1, -7/5$$

Verification:

Sum of the zeroes = - (coefficient of x) / coefficient of x^2

$$\alpha + \beta = -b/a$$

$$(-1) + (-7/5) = -(12)/5$$

$$= -12/5 = -12/5$$

Product of the zeroes = constant term / coefficient of x^2

$$\alpha\beta = c/a$$

$$(-1)(-7/5) = 7/5$$

$$7/5 = 7/5$$

41. Here, $p(x) = 3x^2 - 2$.

Now $p(x) = 0$

$$\Rightarrow 3x^2 - 2 = 0$$

$$\Rightarrow 3x^2 = 2$$

$$\Rightarrow x^2 = \frac{2}{3}$$

$$\Rightarrow x = \pm\sqrt{\frac{2}{3}}$$

Therefore, zeroes are $\sqrt{\frac{2}{3}}$ and $-\sqrt{\frac{2}{3}}$.

If $p(x) = 3x^2 - 2$, then $a = 3$, $b = 0$ and $c = -2$

$$\text{Now, sum of zeroes} = \sqrt{\frac{2}{3}} + \left(-\sqrt{\frac{2}{3}}\right) = 0 \dots\dots (i)$$

$$\text{Also, } \frac{-b}{a} = \frac{-0}{3} = 0 \dots\dots\dots (ii)$$

From (i) and (ii)

$$\text{Sum of zeroes} = \frac{-b}{a}$$

$$\text{and product of zeroes} = \sqrt{\frac{2}{3}} \times -\sqrt{\frac{2}{3}} = \frac{-2}{3} \dots\dots\dots (iii)$$

$$\text{Also, } \frac{c}{a} = \frac{-2}{3} \dots\dots\dots (iv)$$

From (iii) and (iv)

$$\text{Product of zeroes} = \frac{c}{a}$$

42. $p(x) = x^2 + 3x + 2$

α, β are its zeroes

$$\therefore \alpha + \beta = -3, \alpha\beta = 2$$

Now,

$$(\alpha + 1) + (\beta + 1) = \alpha + \beta + 2 = -3 + 2 = -1$$

$$(\alpha + 1)(\beta + 1) = \alpha\beta + (\alpha + \beta) + 1 = 2 - 3 + 1 = 0$$

$$\therefore \text{Required Polynomial is } k(x^2 + x) \text{ or } x^2 + x$$

43. Consider general quadratic polynomial $p(x) = ax^2 + bx + c, a \neq 0$

$b = 0$ (given)

Let α, β be the zeroes of $p(x)$

$$\therefore \text{Sum of zeroes} = \alpha + \beta = -\frac{b}{a} = \frac{0}{a} = 0$$

$$\Rightarrow \alpha + \beta = 0$$

$$\Rightarrow \alpha = -\beta$$

In other words $\beta = -\alpha$

\therefore The zeroes are $\alpha, -\alpha$.

Hence, the zeroes are equal in magnitude but opposite in sign.

44. Let the given polynomial is $p(x) = x^2 + 7x + 7$

Here, $a = 1, b = 7, c = 7$

$\therefore \alpha, \beta$ are both zeroes of $p(x)$

$$\therefore \alpha + \beta = -\frac{b}{a} = -7 \dots\dots\dots(i)$$

$$\alpha\beta = \frac{c}{a} = 7 \dots\dots\dots(ii)$$

Now,

$$\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta = \frac{\beta + \alpha}{\alpha\beta} - 2\alpha\beta$$

$$= \frac{-7}{7} - 2 \times 7$$

$$= -1 - 14$$

$$= -15$$

Hence the value of $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$ is -15.

45. Let $p(x) = x^2 - 2x - 8$

By the method of splitting the middle term,

$$x^2 - 2x - 8 = x^2 - 4x + 2x - 8$$

$$= x(x - 4) + 2(x - 4) = (x - 4)(x + 2)$$

For zeroes of $p(x)$,

$$p(x) = 0$$

$$\Rightarrow (x - 4)(x + 2) = 0$$

$$\Rightarrow x - 4 = 0 \text{ or } x + 2 = 0$$

$$\Rightarrow x = 4 \text{ or } x = -2$$

$$\Rightarrow x = 4, -2$$

So, the zeroes of $p(x)$ are 4 and -2.

We observe that, Sum of its zeroes

$$= 4 + (-2) = 2$$

$$= \frac{-(-2)}{1} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

Product of its zeroes

$$= 4x(-2) = -8 = \frac{-8}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

Hence, relation between zeroes and coefficients is verified.

46. The given polynomial $p(x) = x^2 + 2\sqrt{2}x - 6$

$$= x^2 + 3\sqrt{2}x - \sqrt{2}x - 6$$

$$= x(x + 3\sqrt{2}) - \sqrt{2}(x + 3\sqrt{2})$$

$$= (x + 3\sqrt{2})(x - \sqrt{2})$$

$$p(x) = 0 \text{ if } x + 3\sqrt{2} = 0 \text{ or } x = \sqrt{2}$$

Zeros of the polynomials are $\sqrt{2}$ and $-3\sqrt{2}$

$$\text{For } p(x) = x^2 + 2\sqrt{2}x - 6$$

$$a = 1, b = 2\sqrt{2}, c = -6$$

$$\text{Sum of the zeroes } \sqrt{2} - 3\sqrt{2} = -2\sqrt{2} = -\frac{2\sqrt{2}}{1} = -\frac{b}{a}$$

$$\text{Product of the zeroes } = \sqrt{2} \times -3\sqrt{2} = \frac{-6}{1} = \frac{c}{a}$$

Hence, the relationship is verified.

$$47. \text{ Let } P(x) = 2x^2 + 3x + \lambda$$

$$\text{Its one zero is } \frac{1}{2} \text{ so } P\left(\frac{1}{2}\right) = 0$$

$$P\left(\frac{1}{2}\right) = 2 \times \left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) + \lambda = 0$$

$$\Rightarrow 2 \times \frac{1}{4} + \frac{3}{2} + \lambda = 0$$

$$\Rightarrow \frac{1}{2} + \frac{3}{2} + \lambda = 0$$

$$\Rightarrow \frac{4}{2} + \lambda = 0$$

$$\Rightarrow 2 + \lambda = 0$$

$$\Rightarrow \lambda = -2$$

Let the other zero be α

$$\text{Then } \alpha + \frac{1}{2} = -\frac{3}{2}$$

$$\Rightarrow \alpha = -\frac{3}{2} - \frac{1}{2} = -\frac{4}{2} = -2$$

48. The given polynomial is

$$p(x) = 6x^2 - 7x - 3$$

Factorize the above quadratic polynomial, we have

$$6x^2 - 7x - 3 = 6x^2 - 9x + 2x - 3$$

$$= 3x(2x - 3) + 1(2x - 3)$$

$$= (3x + 1)(2x - 3)$$

For $p(x) = 0$, either $3x + 1 = 0$ or $2x - 3 = 0$

$$\Rightarrow x = \frac{-1}{3} \text{ or } x = \frac{3}{2}$$

Verification: we have $a = 6, b = -7, c = -3$

$$\text{Sum of zeroes} = \frac{-1}{3} + \frac{3}{2} = \frac{7}{6}$$

$$\text{Also, } \frac{-b}{a} = \frac{-(-7)}{6} = \frac{7}{6}$$

$$\text{Now, product of zeroes} = \left(-\frac{1}{3}\right) \times \frac{3}{2} = \frac{-1}{2}$$

$$\text{Also, } \frac{c}{a} = \frac{-3}{6} = \frac{-1}{2}$$

49. The given quadratic polynomial is $p(x) = 2x^2 - 3x + p$

Since, 3 is a root (zero) of $p(x)$

$$\Rightarrow 2(3)^2 - 3 \times 3 + p = 0$$

$$\Rightarrow 18 - 9 + p = 0$$

$$\Rightarrow 9 + p = 0$$

$$\Rightarrow p = -9$$

$$\text{Now } p(x) = 2x^2 - 3x - 9$$

$$= 2x^2 - 6x + 3x - 9$$

$$= 2x(x - 3) + 3(x - 3)$$

$$= (x - 3)(2x + 3)$$

For roots of polynomial, $p(x) = 0$

$$\Rightarrow (x - 3)(2x + 3) = 0$$

$$\Rightarrow x = 3 \text{ or } x = -\frac{3}{2}$$

Hence the other root is $-\frac{3}{2}$.

50. Here, $\alpha + \beta = \frac{-3}{2\sqrt{5}}$ and $\alpha \cdot \beta = -\frac{1}{2}$ [Given]

$$f(x) = x^2 - (\alpha + \beta)x + \alpha\beta \text{ [Formula]}$$

$$= x^2 - \left(\frac{-3}{2\sqrt{5}}\right)x + \left(-\frac{1}{2}\right)$$

$$\Rightarrow f(x) = x^2 + \frac{3}{2\sqrt{5}}x - \frac{1}{2}$$

$$\Rightarrow f(x) = 2\sqrt{5}x^2 + 3x - \sqrt{5}$$

For zeroes of polynomial $f(x)$, $f(x) = 0$

$$\Rightarrow 2\sqrt{5}x^2 + 3x - \sqrt{5} = 0$$

$$\Rightarrow 2\sqrt{5}x^2 + 5x - 2x - \sqrt{5} = 0$$

$$\Rightarrow \sqrt{5}x(2x + \sqrt{5}) - 1(2x + \sqrt{5}) = 0$$

$$\Rightarrow (2x + \sqrt{5})(\sqrt{5}x - 1) = 0$$

$$\Rightarrow (2x + \sqrt{5}) = 0 \text{ or } \sqrt{5}x - 1 = 0$$

$$\Rightarrow x = \frac{-\sqrt{5}}{2} \text{ or } x = \frac{1}{\sqrt{5}}$$

$$\therefore \alpha = \frac{-\sqrt{5}}{2} \text{ and } \beta = \frac{1}{\sqrt{5}}$$

51. $7y^2 - \frac{11}{3}y - \frac{2}{3}$

$$= \frac{1}{3}(21y^2 - 11y - 2)$$

$$= \frac{1}{3}(21y^2 - 14y + 3y - 2)$$

$$= \frac{1}{3}[7y(3y - 2) + 1(3y - 2)]$$

$$= \frac{1}{3}(3y - 2)(7y + 1)$$

$$\Rightarrow y = \frac{2}{3}, \frac{-1}{7} \text{ are zeroes of the polynomial.}$$

If Given polynomial is $7y^2 - \frac{11}{3}y - \frac{2}{3}$

Then $a = 7$, $b = -\frac{11}{3}$ and $c = -\frac{2}{3}$

$$\text{Sum of zeroes} = \frac{2}{3} + \frac{-1}{7} = \frac{14-3}{21} = \frac{11}{21} \dots\dots (i)$$

$$\text{Also, } \frac{-b}{a} = \frac{-(-\frac{11}{3})}{7} = \frac{11}{21} \dots\dots (ii)$$

From (i) and (ii)

$$\text{Sum of zeroes} = \frac{-b}{a}$$

$$\text{Now, product of zeroes} = \frac{2}{3} \times \frac{-1}{7} = \frac{-2}{21} \dots\dots (iii)$$

$$\text{Also, } \frac{c}{a} = \frac{-\frac{2}{3}}{7} = \frac{-2}{21} \dots\dots (iv)$$

From (iii) and (iv)

$$\text{Product of zeroes} = \frac{c}{a}$$

52. Here, $\alpha + \beta = -2\sqrt{3}$ and $\alpha\beta = -9$

$$f(x) = x^2 - (\alpha + \beta)x + \alpha\beta \text{ [Formula]}$$

$$= x^2 - (-2\sqrt{3})x + (-9)$$

$$\Rightarrow f(x) = x^2 + 2\sqrt{3}x - 9$$

For zeroes of polynomial $f(x)$, $f(x) = 0$

$$\Rightarrow x^2 + 2\sqrt{3}x - 9 = 0$$

$$\Rightarrow x^2 + 3\sqrt{3}x - 1\sqrt{3}x - 9 = 0$$

$$\Rightarrow x(x + 3\sqrt{3}) - \sqrt{3}(x + 3\sqrt{3}) = 0$$

$$\Rightarrow (x + 3\sqrt{3})(x - \sqrt{3}) = 0$$

$$\Rightarrow x + 3\sqrt{3} = 0 \text{ or } (x - \sqrt{3}) = 0$$

$$\Rightarrow x = -3\sqrt{3} \text{ or } x = \sqrt{3}$$

$$\therefore \alpha = -3\sqrt{3} \text{ and } \beta = \sqrt{3}$$

Hence the polynomial is $x^2 + 2\sqrt{3}x - 9$ and its zeros are $-3\sqrt{3}$ and $\sqrt{3}$.

53. Given polynomial is

$$f(x) = x^2 - 2x + 3$$

Compare with $ax^2 + bx + c$, we get

$$a = 1, b = -2 \text{ and } c = 3$$

$$\text{Sum of the zeroes} = \alpha + \beta = -\frac{b}{a} = -\frac{-2}{1} = 2$$

$$\text{Product of the zeroes} = \alpha\beta = \frac{c}{a} = \frac{3}{1} = 3$$

i. Sum of the zeroes of new polynomial = $(\alpha + 2) + (\beta + 2)$

$$= \alpha + \beta + 4$$

$$= 2 + 4 = 6$$

Product of the zeroes of new polynomial = $(\alpha + 2)(\beta + 2)$

$$\begin{aligned}
&= \alpha\beta + 2\alpha + 2\beta + 4 \\
&= \alpha\beta + 2(\alpha + \beta) + 4 \\
&= 3 + 2(2) + 4 \\
&= 11
\end{aligned}$$

So, quadratic polynomial is: $x^2 - (\text{sum of the zeroes})x + (\text{product of the zeroes})$
 $= x^2 - 6x + 11$

Hence, the required quadratic polynomial is $f(x) = (x^2 - 6x + 11)$

ii. Sum of the zeroes of new polynomial $= \frac{\alpha-1}{\alpha+1} + \frac{\beta-1}{\beta+1}$

$$\begin{aligned}
&= \frac{(\alpha-1)(\beta+1) + (\beta-1)(\alpha+1)}{(\alpha+1)(\beta+1)} \\
&= \frac{\alpha\beta + \alpha - \beta - 1 + \alpha\beta + \beta - \alpha - 1}{(\alpha+1)(\beta+1)} \\
&= \frac{\alpha\beta - 1 + \alpha\beta - 1}{\alpha\beta + \alpha + \beta + 1} \\
&= \frac{3-1+3-1}{3+1+2} \\
&= \frac{4}{6} \\
&= \frac{2}{3}
\end{aligned}$$

Product of the zeroes of new polynomial $= \frac{\alpha-1}{\alpha+1} \times \frac{\beta-1}{\beta+1}$

$$\begin{aligned}
&= \frac{(\alpha-1)(\beta-1)}{(\alpha+1)(\beta+1)} \\
&= \frac{\alpha\beta - \alpha - \beta + 1}{\alpha\beta + \alpha + \beta + 1} \\
&= \frac{\alpha\beta - (\alpha + \beta) + 1}{\alpha\beta + (\alpha + \beta) + 1} \\
&= \frac{3 - 2 + 1}{3 + 2 + 1} \\
&= \frac{2}{6} = \frac{1}{3}
\end{aligned}$$

So, the quadratic polynomial is, $x^2 - (\text{sum of the zeroes})x + (\text{product of the zeroes})$
 $= x^2 - \frac{2}{3}x + \frac{1}{3}$

Thus, the required quadratic polynomial is $f(x) = k \left(x^2 - \frac{2}{3}x + \frac{1}{3} \right)$.

54. Sum of the zeroes: $(2 + \beta) = (-1)$

Product of the zeroes : $2\beta = -20$

So, required Quadratic polynomial

$$= [x^2 + (\alpha + \beta)x + 2\beta]$$

$$= [x^2 + (-1)x + (-20)]$$

$$= x^2 - x - 20$$

$$\Rightarrow x^2 - x - 20 = 0 \text{ is the polynomial}$$

55. $x^2 - 6$

Let $p(x) = x^2 - 6$

For zeroes of $p(x)$, $p(x) = 0$

$$\Rightarrow x^2 - 6 = 0 \Rightarrow (x)^2 - (\sqrt{6})^2 = 0$$

$$\Rightarrow (x - \sqrt{6})(x + \sqrt{6}) = 0$$

Using the identity $a^2 - b^2 = (a - b)(a + b)$

$$\Rightarrow x - \sqrt{6} = 0 \text{ or } x + \sqrt{6} = 0$$

$$\Rightarrow x = \sqrt{6} \text{ or } x = -\sqrt{6} \Rightarrow x = \sqrt{6}, -\sqrt{6}$$

So, the zeroes of $x^2 - 6$ are $\sqrt{6}$ and $-\sqrt{6}$

Sum of zeroes

$$= (\sqrt{6}) + (-\sqrt{6}) = 0 = \frac{-0}{1} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

Product of zeroes

$$= (\sqrt{6}) \times (-\sqrt{6}) = -6 = \frac{-6}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

Hence the relation between zeroes and coefficient is verified.

56. Given quadratic polynomial is

$$f(y) = 7y^2 - \frac{11}{3}y - \frac{2}{3}$$

For zeroes of $f(y)$, put $f(y) = 0$

$$\Rightarrow 7y^2 - \frac{11}{3}y - \frac{2}{3} = 0$$

$$\Rightarrow 21y^2 - 11y - 2 = 0$$

$$\Rightarrow 21y^2 - 14y + 3y - 2 = 0 \text{ (by splitting the middle term method)}$$

$$\Rightarrow 7y(3y - 2) + 1(3y - 2) = 0$$

$$\Rightarrow (3y - 2)(7y + 1) = 0$$

Therefore, either $3y - 2 = 0$ or $7y + 1 = 0$

$$\Rightarrow y = \frac{2}{3} \text{ or } y = -\frac{1}{7}$$

Now Verification of the relations between α , β , a , b , and c :

We have $\alpha = \frac{2}{3}$, $\beta = -\frac{1}{7}$, $a = 7$, $b = -\frac{11}{3}$, $c = -\frac{2}{3}$

$$\Rightarrow \alpha + \beta = \frac{-b}{a}$$

$$\Rightarrow \left(\frac{2}{3}\right) - \frac{1}{7} = \frac{+\frac{11}{3}}{7}$$

$$\Rightarrow \frac{14-3}{21} = \frac{11}{3} \times \frac{1}{7}$$

$$\Rightarrow \frac{11}{21} = \frac{11}{21}$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

Hence verified.

Also, we know that $\alpha \cdot \beta = \frac{c}{a}$

$$\Rightarrow \left(\frac{2}{3}\right) \times \left(-\frac{1}{7}\right) = \frac{-\frac{2}{3}}{7}$$

$$\Rightarrow \frac{-2}{21} = \frac{-2}{3} \times \frac{1}{7}$$

$$\Rightarrow \frac{-2}{21} = \frac{-2}{21}$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

Hence verified.

57. Sum of zeroes of a quadratic polynomial is $\frac{-b}{a}$ and the product is $\frac{c}{a}$

$$\text{So } a + b = \frac{-2}{5} \text{ and } ab = \frac{-3}{5}$$

According to question

Sum of zeroes of the polynomial is $\frac{1}{a} + \frac{1}{b}$

$$= \frac{a+b}{ab}$$

$$= \frac{-2}{-3}$$

$$= \frac{2}{3}$$

$$= \frac{2}{3}$$

Product of zeroes of the polynomial is $\frac{1}{ab}$

$$= \frac{1}{ab}$$

$$= \frac{-3}{-5}$$

$$= \frac{3}{5}$$

We know that a quadratic equation is of the form $ax^2 + bx + c$

$$= x^2 - \frac{2}{3}x - \frac{5}{3}$$

58. Here, $f(v) = v^2 + 4\sqrt{3}v - 15$

For zeroes of $f(v)$, put $f(v) = 0$

$$\Rightarrow v^2 + 4\sqrt{3}v - 15 = 0$$

$$\Rightarrow v^2 + 5\sqrt{3}v - 1\sqrt{3}v - 15 = 0 \text{ (By splitting the middle term)}$$

$$\Rightarrow v(v + 5\sqrt{3}) - \sqrt{3}(v + 5\sqrt{3}) = 0$$

$$\Rightarrow (v + 5\sqrt{3})(v - \sqrt{3}) = 0$$

$$\Rightarrow (v + 5\sqrt{3}) = 0 \text{ or } (v - \sqrt{3}) = 0$$

Therefore, either $v = -5\sqrt{3}$ or $v = \sqrt{3}$

Verification of relations between α , β , a , b , c :

we have, $\alpha = -5\sqrt{3}$, $\beta = \sqrt{3}$, $a = 1$, $b = 4\sqrt{3}$, and $c = -15$

$$\alpha + \beta = \frac{-b}{a}$$

$$\Rightarrow -5\sqrt{3} + \sqrt{3} = \frac{-4\sqrt{3}}{1}$$

$$\Rightarrow -4\sqrt{3} = -4\sqrt{3}$$

$\Rightarrow \text{LHS} = \text{RHS}$

Hence, verified.

Also we know that

$$\alpha \cdot \beta = \frac{c}{a}$$

$$\Rightarrow (-5\sqrt{3})(\sqrt{3}) = \frac{-15}{1}$$

$$\Rightarrow -5 \times 3 = -15$$

$$\Rightarrow -15 = -15$$

$\Rightarrow \text{LHS} = \text{RHS}$

Hence, verified.

59. Given, β and $\frac{1}{\beta}$ are zeroes of the polynomial $(\alpha^2 + \alpha)x^2 + 61x + 6\alpha$.

$$\therefore \beta + \frac{1}{\beta} = -\frac{61}{\alpha^2 + \alpha}$$

$$\text{or, } \frac{\beta^2 + 1}{\beta} = \frac{-61}{\alpha^2 + \alpha} \dots\dots(i)$$

$$\text{and } \beta \cdot \frac{1}{\beta} = \frac{6\alpha}{\alpha^2 + \alpha}$$

$$\text{or, } 1 = \frac{6}{\alpha + 1}$$

$$\alpha + 1 = 6$$

$$\alpha = 5$$

Substituting this value of α in (i), we get

$$\frac{\beta^2 + 1}{\beta} = \frac{-61}{5^2 + 5} = -\frac{61}{30}$$

$$30\beta^2 + 30 = -61\beta$$

$$30\beta^2 + 61\beta + 30 = 0$$

$$\text{or, } \frac{-61 \pm \sqrt{(-61)^2 \times 4 \times 30 \times 30}}{2 \times 30}$$
$$= \frac{-61 \pm \sqrt{3721 - 3600}}{60} = \frac{-61 \pm 11}{60}$$

$$\beta = \frac{-5}{6} \text{ or } \frac{-6}{5}$$

$$\text{Hence, } \alpha = 5, \beta = \frac{-5}{6}, \frac{-6}{5}$$

60. Zeroes are -2, -3

factors are $(x + 2), (x + 3)$

$$g(x) = (x + 2)(x + 3) = x^2 + 5x + 6$$

$$\frac{x^4 + 2x^3 - 7x^2 - 8x + 12}{x^2 + 5x + 6} = x^2 - 3x + 2$$

$$x^2 - 3x + 2 = (x - 2)(x - 1)$$

Other zeroes are 2, 1

61. $2x^2 + 3x - 14 = 2x^2 + 7x - 4x - 14$

$$= (x - 2)(2x + 7)$$

$$x = 2, -\frac{7}{2}$$

$$\text{Sum of zeroes} = 2 + \left(-\frac{7}{2}\right) = -\frac{3}{2}$$

$$\text{Product of zeroes} = 2 \times -\frac{7}{2} = -7$$

$$-\frac{b}{a} = -\frac{3}{2}$$

$$\frac{c}{a} = -\frac{14}{2} = -7$$

$$\Rightarrow \text{Hence, sum of zeroes} = -\frac{b}{a}$$

$$\text{Product of zeroes} = \frac{c}{a}$$

62. According to the question, α and β are zeroes of $p(x) = 6x^2 - 5x + k$

$$\text{So, Sum of zeroes} = \alpha + \beta = -\left(\frac{-5}{6}\right) = \frac{5}{6} \dots\dots(i)$$

$$\alpha - \beta = \frac{1}{6} \text{ (Given)} \dots\dots(ii)$$

Adding equations (i) and (ii), we get

$$2\alpha = 1$$

$$\text{or, } \alpha = \frac{1}{2}$$

On putting the value of α in equation (ii), we get

$$\frac{1}{2} - \beta = \frac{1}{6}$$

$$\beta = \frac{1}{2} - \frac{1}{6}$$

$$\beta = \frac{2}{6} = \frac{1}{3}$$

$$\therefore \alpha\beta = \frac{k}{6} = \frac{1}{2} \times \frac{1}{3}$$

Hence, $k = 1$

63. Since α and β are the zeroes of the quadratic polynomial $x^2 + 4x + 3$

So, $\alpha + \beta = -4$

and $\alpha\beta = 3$

Sum of zeroes of new polynomial $= 1 + \frac{\beta}{\alpha} + 1 + \frac{\alpha}{\beta}$

$$= \frac{\alpha\beta + \beta^2 + \alpha\beta + \alpha^2}{\alpha\beta}$$

$$= \frac{\alpha^2 + \beta^2 + 2\alpha\beta}{\alpha\beta}$$

$$= \frac{(\alpha + \beta)^2}{\alpha\beta} = \frac{(-4)^2}{3} = \frac{16}{3}$$

Product of zeroes $= \left(1 + \frac{\beta}{\alpha}\right) \left(1 + \frac{\alpha}{\beta}\right)$

$$= \left(\frac{\alpha + \beta}{\alpha}\right) \left(\frac{\beta + \alpha}{\beta}\right)$$

$$= \frac{(\alpha + \beta)^2}{\alpha\beta}$$

$$= \frac{(-4)^2}{3} = \frac{16}{3}$$

So required polynomial $= x^2 - (\text{Sum of the zeroes})x + \text{Product of the zeroes}$

$$= x^2 - \left(\frac{16}{3}\right)x + \frac{16}{3}$$

$$= \left(x^2 - \frac{16}{3}x + \frac{16}{3}\right)$$

$$= \frac{1}{3}(3x^2 - 16x + 16)$$

64. Since α and β are the zeros of the quadratic polynomial $f(x) = ax^2 + bx + c$

$$\alpha + \beta = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2} = \frac{-b}{a}$$

$$\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{c}{a}$$

We have,

$$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)$$

$$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = a\left(\frac{\alpha^3 + \beta^3}{\alpha\beta}\right) + b\left(\frac{\alpha^2 + \beta^2}{\alpha\beta}\right)$$

$$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = a\left(\frac{(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)}{\alpha\beta}\right) + \left(\frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}\right)$$

By substituting $\alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$ we get,

$$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = a\left(\frac{\left(\frac{-b}{a}\right)^3 - 3 \times \frac{c}{a} \left(\frac{-b}{a}\right)}{\frac{c}{a}}\right) + b\left(\frac{\left(\frac{-b}{a}\right)^2 - 2 \times \frac{c}{a}}{\frac{c}{a}}\right)$$

$$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = a\left(\frac{\frac{-b^3}{a^3} + \frac{3bc}{a^2}}{\frac{c}{a}}\right) + b\left(\frac{\frac{b^2}{a^2} - \frac{2c}{a}}{\frac{c}{a}}\right)$$

$$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = a\left(\frac{-b^3 + 3bca}{a^3} \times \frac{a}{c}\right) + b\left(\frac{b^2 - 2ca}{a^2} \times \frac{a}{c}\right)$$

$$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = a\left(\frac{-b^3 + 3abc}{a^2c}\right) + b\left(\frac{b^2 - 2ca}{ac}\right)$$

$$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = \frac{-b^3 + 3abc}{ac} + \frac{b^3 - 2abc}{ac}$$

$$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = \frac{-b^3 + 3abc + b^3 - 2abc}{ac}$$

$$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = \frac{3abc - 2abc}{ac}$$

$$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = \frac{abc}{ac}$$

$$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = b$$

Hence, the value of $a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)$ is b .

65. Here the given polynomial is

$$f(s) = 2s^2 - (1 + 2\sqrt{2})s + \sqrt{2}$$

$$= s(2s - 1) - \sqrt{2}(2s - 1)$$

$$= (2s - 1)(s - \sqrt{2})$$

Hence $f(s) = 0$ if $2s - 1 = 0$ or $s - \sqrt{2} = 0$

$$s = \frac{1}{2} \text{ or } s = \sqrt{2}$$

Verification of the relation between α , β , a, b and c

$$\alpha = \frac{1}{2}, \beta = \sqrt{2}, a = 2, b = -(1 + 2\sqrt{2}), c = \sqrt{2}$$

$$\alpha + \beta = \frac{-b}{a}$$

$$\Rightarrow \frac{1}{2} + \sqrt{2} = \frac{+(1+2\sqrt{2})}{2}$$

$$\Rightarrow \frac{1}{2} + \sqrt{2} = \frac{1}{2} + \frac{2\sqrt{2}}{2}$$

$$\Rightarrow \frac{1}{2} + \sqrt{2} = \frac{1}{2} + \sqrt{2}$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

$$\text{Now, } \alpha \times \beta = \frac{c}{a}$$

$$\Rightarrow \left(\frac{1}{2}\right)(\sqrt{2}) = \frac{\sqrt{2}}{2}$$

$$\Rightarrow \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

Hence verified.

66. i. 2

ii. 81.2 m

iii. quadratic polynomial

OR

(x - 3) and (x - 2)

67. i. Zeroes of the polynomial are 0 and 5

ii. Maximum height achieved by ball

$$= 25 \times \frac{5}{2} - 5 \times \left(\frac{5}{2}\right)^2$$

$$= \frac{125}{4} \text{ or } 31.25 \text{ m}$$

iii. a. $-5t^2 + 25t = 30$

$$\Rightarrow t^2 - 5t + 6 = 0$$

$$\Rightarrow (t - 2)(t - 3) = 0$$

$$t \neq 3, t = 2$$

OR

b. $-5t^2 + 25t = 20$

$$\Rightarrow t^2 - 5t + 4 = 0$$

$$\Rightarrow (t - 4)(t - 1) = 0$$

$$\Rightarrow t = 4, 1$$

68. i. Parabola

ii. $a > 0$

iii. \therefore The graph cut the x-axis thrice

\therefore No of zeroes = 3

OR

$a < 0$

69. i. Graph of $y = f(x)$ intersects X-axis at two distinct points. So we can say that no of zeros of $y = f(x)$ is 2.

ii. There will not be any zero if graph of $f(x)$ does not intersect x- axis.

iii. $x^2 + (a + 1)x + b$ is the quadratic polynomial.

2 and -3 are the zeros of the quadratic polynomial.

$$\text{Thus, } 2 + (-3) = \frac{-(a+1)}{1}$$

$$\Rightarrow \frac{(a+1)}{1} = 1$$

$$\Rightarrow a + 1 = 1$$

$$\Rightarrow a = 0$$

$$\text{Also, } 2 \times (-3) = b$$

$$\Rightarrow b = -6$$

OR

If -4 is zero of given polynomial then,

$$(-4)^2 - 2(-4) - (7p + 3) = 0$$

$$\Rightarrow 16 + 8 - 7p - 3 = 0$$

$$\Rightarrow 7p = 21$$

$$\Rightarrow p = 3$$

70. i. Point of intersection of graph of polynomial, gives the zeroes of the polynomial.

\therefore zeroes = -4 and 7

ii. Since, zero's are $\alpha = -4$, $\beta = 7$

$$\alpha + \beta = -4 + 7 = 3$$

$$\alpha\beta = -4 \times 7 = -28$$

$$P(x) = x^2 - (\text{Sum of zeroes})x + \text{product of zeroes}$$

$$P(x) = x^2 - 3x + (-28)$$

$$P(x) = x^2 - 3x - 28$$

iii. Product of zeroes = -4×7

$$= -28$$

OR

a is a non-zero real number, b and c are any real numbers c.