Solution

GR 10_REAL NUMBERS_WORKSHEET

Class 10 - Mathematics

1. **(a)** $2^3 \times 53$

   Explanation:

2	424
2	212
2	106
53	53
	1

   424 = $2^3 \times 53$

2. **(a)** co-prime numbers

   Explanation:

   If two numbers do not have a common factor (other than 1), then they are called co-primer numbers. We know that two numbers are coprime if their common factor (greatest common divisor) is 1. e.g. co-prime of 12 are 11, 13.

3.

   (c) an irrational number

   Explanation:

   Let 2 - $\sqrt{3}$ be rational number

   2 - $\sqrt{3}$ = $\frac{p}{q}$ where p and q are composite numbers

   $\sqrt{3}$ = $\frac{p}{q}$ + 2

   $\sqrt{3}$ = $\frac{(p+2q)}{q}$

   since p, q are integers, so $\frac{(p+2q)}{q}$ is rational

   $\therefore \sqrt{3}$ is an irrational number

   it shows our supposition was wrong

   hence 2-$\sqrt{3}$ is an irrational number.

4. **(a)** $(\sqrt{2}-1)^2$

   Explanation:

   $(\sqrt{2}-1)^2$

5.

   (b) 16

   Explanation:

   Let us subtract 5 (the remainder) from each number in order to find their HCF.

   245 - 5 = 240

   1029 - 5 = 1024

   Now, Let us find HCF of 240, 1024

   1024 = 240 $\times$ 4 + 64

   240 = 64 $\times$ 3 + 48

   64 = 48 $\times$ 1 + 16

   48 = 16 $\times$ 3 + 0

   The largest number which divides 245 and 1029 leaving remainder 5 in each case is 16.

6.

   (d) an irrational number

   Explanation:

   If possible let $a\sqrt{b}$ be rational.

   Then $a\sqrt{b} = \frac{p}{q},$ where p and q are non-zero integers, having no common factor other than 1.

Now, $a\sqrt{b} = \frac{p}{q}$
$\Rightarrow \sqrt{b} = \frac{p}{aq}$ ... (i)
But, p and aq are both rational and aq $\neq$ 0
$\because \frac{p}{aq}$ is rational.
Therefore, from eq. (i), it follows that $\sqrt{b}$ is rational.
The contradiction arises by assuming that $a\sqrt{b}$ is rational.
Hence, $a\sqrt{b}$ is irrational.

7. **(a)** p divides b
Explanation:
If p divides b^2, then p also divides b.

8.
(b) 320
Explanation:
Let the two numbers be x and y.
It is given that: x $\times$ y = 1600
HCF = 5
We know, HCF $\times$ LCM = x $\times$ y
$\Rightarrow$ 5 $\times$ LCM = 1600
$\therefore$ LCM = $\frac{1600}{5}$ = 320

9.
(b) 63
Explanation:
HCF = $3^2 \times 7^1$
= 9 $\times$ 7
= 63

10.
(c) pq
Explanation:
Two positive integers are expressed as follows:
a = pq^2
b = p^3q
p and q are prime numbers.
Then, taking the smallest powers of p and q in the values for a and b we get
HCF (a, b) = pq

11.
(c) 120
Explanation:
Least positive integer divisible by 20 and 24 is
LCM of (20, 24).
20 = $2^2 \times 5$
24 = $2^3 \times 3$
$\therefore$ LCM (20, 24) = $2^3 \times 3 \times 5$ = 120
Thus 120 is divisible by 20 and 24.

12.
(b) x^2y^2

Explanation:

$x^2y^5 = y^3(x^2y^2)$

$x^3y^3 = x(x^2y^2)$

Therefore HCF (m, n) is x^2y^2

13. **(a)** does not exist

Explanation:

All numbers except zero have a multiplicative inverse because we cannot multiply any number by it to get 1.

14.

(b) an irrational number

Explanation:

Let $\sqrt{2}$ is a rational number.

$\therefore \sqrt{2} = \frac{p}{q}$, where p and q are some integers and HCF(p, q) = 1 .... (1)

$\Rightarrow \sqrt{2}q = p$

$\Rightarrow (\sqrt{2}q)^2 = p^2$

$\Rightarrow 2q^2 = p^2$

⇒ p^2 is divisible by 2

⇒ p is divisible by 2 .... (2)

Let p = 2m, where m is some integer.

$\therefore \sqrt{2}$ = p

$\Rightarrow \sqrt{2}q = 2m$

$\Rightarrow (\sqrt{2}q)^2 = (2m)^2$

$\Rightarrow 2q^2 = 4m^2$

$\Rightarrow q^2 = 2m^2$

⇒ q^2 is divisible by 2

⇒ q is divisible by 2 .... (3)

From (2) and (3), 2 is a common factor of both p and q, which contradicts (1).

Hence, our assumption is wrong.

Thus, $\sqrt{2}$ is an irrational number.

15.

(c) irrational number

Explanation:

Here, 3 is rational and $2\sqrt{5}$ is irrational.

We know that the sum of a rational and an irrational is an irrational number, therefore, $3 + 2\sqrt{5}$ is irrational.

16. **(a)** 21 litres

Explanation:

We have, 504 = $2^3 \times 3^2 \times 7$ and 735 = $3 \times 5 \times 7^2$.

$\therefore$ H.C.F. (504, 735) = (3×7) = 21

$\therefore$ Capacity of the container = 21 litres.

17. **(a)** 7119

Explanation:

LCM $\times$ HCF = Product of two numbers

56952 $\times$ 113 = 904 $\times$ second number

$\frac{56952 \times 113}{904}$ = second number

Therefore, second number = 7119

18.

(d) q

Explanation:

q is a factor of p. In this case, the highest common factor (HCF) of p and q is q itself because it is the largest number that can evenly divide both p and q. Therefore, if p is a multiple of q, the HCF of p and q is q.

19.

(d) $2^8 \times 3^2$

Explanation:

$2^8 \times 3^2$

20.

(c) 17×500

Explanation:

$850 = 2 \times 5 \times 5 \times 17$

$500 = 2 \times 2 \times 5 \times 5 \times 5$

LCM (850, 500) = $2 \times 2 \times 5 \times 5 \times 5 \times 17 = 17 \times 500$

21. Given that,

At intervals of 6, 12, and 18 minutes, three bells ring.

We know that,

Three bells ring at interval of 6, 12 and 18 minutes

Let us find the L.C.M of 6, 12 and 18.

$6 = 2 \times 3$

$12 = 2 \times 2 \times 3$

$18 = 2 \times 3 \times 3$

LCM(6, 12, 18) = $2 \times 2 \times 3 \times 3 = 36$

Therefore, all the three bells will rang together again at 6 : 36 am.

22. Let $5 - 3\sqrt{2}$ be rational

$\therefore 5 - 3\sqrt{2} = \frac{p}{q}$, p & q are integers, $q \neq 0$, HCF (p, q) = 1

$5 - \frac{p}{q} = 3\sqrt{2}$

$\frac{15q-p}{3q} = \sqrt{2}$

Rational = Irrational

which is a contradiction.

Hence, $5 - 3\sqrt{2}$ is irrational.

23. Let us assume that $2\sqrt{3} - 1$ is a rational. number

Then, there exist positive co-primes a and b such that

$2\sqrt{3} - 1 = \frac{a}{b}$

$2\sqrt{3} = \frac{a}{b} + 1$

$2\sqrt{3} = \frac{\mathrm{a+b}}{\mathrm{b}}$

$\sqrt{3} = \frac{a+b}{2b}$

Here $\frac{\mathrm{a+b}}{\mathrm{b}}$ is a rational number ,so $\sqrt{3}$ is a rational number

This contradicts the fact that $\sqrt{3}$ is an irrational number

Hence $2\sqrt{3} - 1$ is irrational

24. The Highest Common Factor H C F of two or more numbers is the highest number that divides the numbers exactly.

By applying Euclid’s division lemma for 190 and 100

$190 = 100 \times 1 + 90$.

Since remainder ≠ 0, apply division lemma on divisor 100 and remainder 90

$100 = 90 \times 1 + 10$.

Since remainder ≠ 0, apply division lemma on divisor 90 and remainder 10

$90 = 10 \times 9 + 0$.

Here remainder is zero

Hence HCF (190,100)=10

25. 48 = 2 × 2 × 2 × 2 × 3

72 = 2 × 2 × 2 × 3 × 3

108 = 2 × 2 × 3 × 3 × 3

Therefore, L.C.M of 48, 72, 108 is

(2 × 2 × 2 × 3 × 3 × 3)

= 432

So, time when they change again = 432 seconds

But we need to find time after 7 am So, first we convert 432 seconds into minutes.

Time = 432 second

= $\frac{432}{60}$ minutes

∴ Time = 7 minutes 12 seconds

Thus,

Required time = 7am + 7 minutes 12 seconds

= 7 : 07 : 12 am

26. 510 and 92

510= 2 × 3 × 5× 17

92 = 2 × 2 × 23

HCF = 2

LCM = 2 × 2 × 3 × 5 × 17 × 23 = 23460

Product of two numbers 510 and 92 = 510 × 92 = 46920

HCF × LCM = 2 × 23460 = 46920

Hence, product of two numbers = HCF × LCM

27. We know that,

$LCM \times HCF = a \times b$

$\Rightarrow 1449 \times 23 = 161 \times b$

$\Rightarrow b = \frac{1449 \times 23}{161} = 207$

28. Assuming $\frac{2-\sqrt{3}}{5}$ to be a rational number.

$\Rightarrow \frac{2-\sqrt{3}}{5} = \frac{\mathrm{p}}{\mathrm{q}}$, where p and q are integers $\mathrm{q} \neq 0$

$\Rightarrow \sqrt{3} = \frac{2q-5p}{q}$

Here RHS is rational but LHS is irrational.

Therefore our assumption is wrong.

Hence $\frac{2-\sqrt{3}}{5}$ is an irrational number.

29. Let $\mathrm{P(x)} = 2\mathrm{x}^4 - 2\mathrm{y}^4$

$= 2\left[\left(\mathrm{x}^2\right)^2 - \left(\mathrm{y}^2\right)^2\right]$

$= 2\left(x^2 + y^2\right)\left(x^2 - y^2\right)$ Using identity $a^2 - b^2 = (a+b)(a-b)$

$= 2\left(\mathrm{x}^2 + \mathrm{y}^2\right)(\mathrm{x}+\mathrm{y})(\mathrm{x}-\mathrm{y})$

and $Q(x) = 3x^3 + 6x^2y - 3xy^2 - 6y^3$

$= 3x^2(x+2y) - 3y^2(x+2y)$

$= (x+2y)\left(3x^2 - 3y^2\right)$

$= 3(x+2y)\left(x^2 - y^2\right)$

$= 3(x+2y)(x+y)(x-y)$

$\therefore HCF = (x+y)(x-y) = x^2 - y^2$ Using Identity $a^2 - b^2 = (a+b)(a-b)$

30. Since $3 \times 5 \times 7 + 7 = (3 \times 5 + 1) \times 7 = (15+1) \times 7 = 16 \times 7$.

Hence, it is a composite number.

31. We have to find the greatest number which divides 85 and 72 leaving remainder 1 and 2 respectively.

Let assume that x be the greatest number which divides 85 and 72 leaving remainder 1 and 2 respectively.

So, it means

x divides 85 - 1 = 84

and

x divides 72 - 2 = 70

So, from this we concluded that

= x divides 84 and 70

= x = HCF (84, 70)

Now, to find HCF(84, 70), we use method of prime factorization.

Prime factors of 84 = $2 \times 2 \times 3 \times 7$

Prime factors of 70 = $2 \times 5 \times 7$

So,

= HCF (84, 70) = 2×7 = 14

= x = 14

Hence, 14 is the greatest number which divides 85 and 72 leaving remainder 1 and 2 respectively.

32. Let us assume that $2-3\sqrt{5}$ is rational. Then, there exist positive co-primes a and b such that

$2-3\sqrt{5}=\frac{a}{b}$

$3\sqrt{5}=2-\frac{a}{b}$

$3\sqrt{5}=\frac{2\mathrm{b}-\mathrm{a}}{\mathrm{b}}$

$\sqrt{5}=\frac{2\mathrm{b}-\mathrm{a}}{3\mathrm{b}}$

We observe that $\frac{2b-a}{3b}$ is a rational number.

It shows that $\sqrt{5}$ is a rational number.

This contradicts the fact that $\sqrt{5}$ is an irrational number

This contradiction has raised because we assumed that $2-3\sqrt{5}$ is a rational number

Hence, our assumption is wrong ,and $2-3\sqrt{5}$ is an irrational number.

33. It can be observed that Ravi takes lesser time than Sonia for completing 1 round of the circular path. As they are going in the same direction, they will meet again at the same time when Ravi will have completed 1 round of that circular path with respect to Sonia. And the total time taken for completing this 1 round of circular path will be the LCM of time taken by Sonia and Ravi for completing 1 round of circular path respectively i.e., LCM of 18 minutes and 12 minutes.

18 = 2 × 3 × 3 And, 12 = 2 × 2 × 3

LCM of 12 and 18 = 2 × 2 × 3 × 3 = 36

Therefore, Ravi and Sonia will meet together at the starting point after 36 minutes.

34. Let us assume that $5-2\sqrt{3}$ is a rational number.

Then, there must exist positive co primes a and b such that

$\Rightarrow\ 5-2\sqrt{3}=\frac{\mathrm{a}}{\mathrm{b}}$

$\Rightarrow -2\sqrt{3}=\frac{\mathrm{a}}{\mathrm{b}}-5$

$\Rightarrow\ 2\sqrt{3}=5-\frac{\mathrm{a}}{\mathrm{b}}$

$\Rightarrow\ 2\sqrt{3}=\frac{5\mathrm{b}-\mathrm{a}}{\mathrm{a}}$

$\Rightarrow\ \sqrt{3}=\frac{5\mathrm{b}-\mathrm{a}}{2\mathrm{a}}$

The right side $\frac{5\mathrm{b}-\mathrm{a}}{2\mathrm{a}}$ is a rational numbers so $\sqrt{3}$ is a rational number

This contradicts the fact that $\sqrt{3}$ is an irrational number

Hence our assumption is incorrect and $5-2\sqrt{3}$ is an irrational number.

35. Least common factor (LCM) = 78

Greatest divisor is also a HCF = 13

As, we know , HCF $\times$ LCM = a $\times$ b (a and b are two natural no. of which LCM and HCF is given)

Therefore, required product of no. = 13×78 = a $\times$ b =1014 or $2\times3\times13\times13$

Since 13 is common in both no. as its HCF is 13

Required no.= 13×3 = 39 and 13 $\times 2$ = 26

$\therefore$ Pairs of natural numbers are 78 and 13 or 26 and 39.

36. Let us assume that 5 - $\sqrt{3}$ is rational number

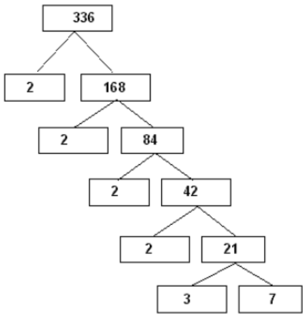
$\therefore$ 5 - $\sqrt{3}$ = $\frac{p}{q}$; q $\neq$ 0 and p, q are integers

$\Rightarrow \sqrt{3}$ = $\frac{5q-p}{q}$

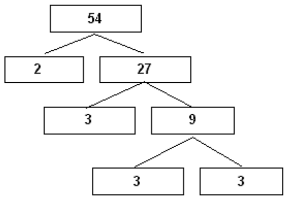
RHS is rational but LHS is irrational

$\therefore$ Our assumption was wrong

$\therefore$ 5 - $\sqrt{3}$ is an irrational number

37. 

So, 336 = 2 × 2 × 2 × 2 × 3 × 7 = 2^4 × 3 × 7



So, 54 = $2 \times 3 \times 3 \times 3 = 2 \times 3^3$

Therefore,

LCM (336, 54) = $2^4 \times 3^3 \times 7 = 3024$

HCM (336, 54) = $2 \times 3 = 6$.

Verification:

LCM × HCF = 3024 × 6 = 1814 and 336 × 54 = 18144

i.e. LCM × HCF = product of two numbers

38. Let us preassume that $3\sqrt{2}$ is a rational number.

In that case, $3\sqrt{2}$ can be writtin as $\frac{p}{q}$, where p and q are co-prime integers and q is not zero.

So, $\frac{p}{q} = \frac{3\sqrt{2}}{1}$

$\Rightarrow \frac{p}{3q} = \frac{\sqrt{2}}{1}$

Since, p is an integer and 3q is also an integer where 3q is not zero.

So, $\frac{p}{3q}$ is a rational number but the equalient number $\sqrt{2}$ should also be a rational number.

But this contradicts the fact that $\sqrt{2}$ is an irrational number.

so, this assumption is wrong and $3\sqrt{2}$ is an irrational number.

39. 35 = 5 × 7

56 = 2^3 × 7

91 = 13 × 7

L.C.M of 35, 56 and 91 = 2^3 × 7 × 5 × 13 = 3640

The smallest number that when divided by 35, 56, 91 leaves a remainder 7 in each case = 3640 + 7 = 3647.

Hence 3647 is the smallest number that, when divided by 35, 56 and 91 leaves a remainder of 7 in each case.

40. Suppose $\sqrt{p}$ be rational $\Rightarrow$ it can be written in the form of $\frac{a}{b}$.

$\sqrt{p} = \frac{a}{b}$ (where a and b are co-prime)

On squaring both sides, we get

$p = \frac{a^2}{b^2}$

a^2 has a factor p.

$pb^2 = a^2$ ....(i)

a also has a factor p.

So $a = pc$

$pb^2 = a^2$

$a^2 = p^2c^2$

Put the value of a^2 in equation (i),

$pb^2 = p^2c^2$

b^2 has a factor p,

$\therefore$ b has a factor p.

$a \ and \ b$ have common factor p.

But we assume that a and b are co-prime

∴ our assumption is wrong.
$\sqrt{p}$ must be an irrational number, (p is a prime number.)
$\sqrt{q}$ is also an irrational number (q is a prime number.)
Sum of two irrational numbers is irrational
$\therefore \sqrt{p}+\sqrt{q}$ is irrational number.

41. Since, the three persons start walking together.
∴ The minimum distance covered by each of them in complete steps = LCM of the measures of their steps
$40=8\times 5=2^3\times 5$
$42=6\times 7=2\times 3\times 7$
$45=9\times 5=3^2\times 5$
Hence LCM (40, 42, 45)
$=2^3\times 3^2\times 5\times 7=8\times 9\times 5\times 7=2520$
∴ The minimum distance each should walk so that each can cover the same distance
= 2520 cm = 25.20 meters.

42. We can prove $7\sqrt{5}$ irrational by contradiction.
Let us suppose that $7\sqrt{5}$ is rational.
It means we have some co-prime integers *a* and *b* (*b*≠ 0)
such that
$7\sqrt{5}=\frac{a}{b}$
$\Rightarrow \sqrt{5}=\frac{a}{7b}$ ....(1)
R.H.S of (1) is rational but we know that $\sqrt{5}$ is irrational.
It is not possible which means our assumption is wrong.
Therefore, $7\sqrt{5}$ cannot be rational.
Hence, it is irrational.

43. **HCF (highest common factor) :** The largest positive integer that divides given two positive integers is called the Highest Common Factor of these positive integers.
We need to find H.C.F of 475 and 495.
By applying Euclid's Division lemma, we have
495 = 475 × 1 + 20.
Since remainder ≠ 0, apply division lemma on 475 and remainder 20
475 = 20 × 23 + 15.
Since remainder ≠ 0, apply division lemma on 20 and remainder 15
20 = 15 × 1 + 5.
Since remainder ≠ 0, apply division lemma on 15 and remainder 5
15 = 5 × 3+ 0.
Therefore, H.C.F. of 475 and 495 = 5

44. (i) The required number of minutes after which they start preparing a new card together = LCM of 10,16 and 20 minutes
Prime factorisation of 10 = 2 × 5
and prime factorisation of 16 = 2 × 2 × 2 × 2
and prime factorisation of 20 = 2 × 2 × 5
Now, LCM(10,16,20) = 2 × 2 × 2 × 2× 5 = 80
Therefore, Number of minutes after which they start preparing a new card together = 80 minutes.
(ii) Recognition and care for elders removes the loneliness due to age related diseases. Moreover they feel happy to help young minds through their experience.

45. $\mathrm{HCF}=\left(\mathrm{x}^2-\mathrm{x}-12\right)=(\mathrm{x}+3)(\mathrm{x}-4)$
$P(x)=\left(x^2-5x+4\right)\left(x^2+5x+a\right)$
$=(x-4)(x-1)\left(x^2+5x+a\right)$
Since, $(x+3)(x-4)$ is the HCF of $P(x)$ and $Q(x)$therefore,
(x+3) and (x-4) are factors of p(x), As (x-4) is already seen
in p(x) and (x+3) is also a factor of p(x).
Thus, by factor theorem, x + 3 = 0 $\Rightarrow x=-3i,e\cdot P(-3)=0$
Hence, $P(-3)=(-7)(-4)(9-15+a)=0$
$\Rightarrow 28(-6+a)=0 \quad \Rightarrow a=6$

Again, $Q(x) = (x^2 + 5x + 6)(x^2 - 5x - 2b)$
$= (x+2)(x+3)(x^2 - 5x - 2b)$
Since, $x - 4$ is a factor of $Q(x)$
$x - 4 = 0 \Rightarrow x = 4$, by factor theorem Q(4) must equal to 0.
$Q(4) = (6)(7)(16 - 20 - 2b) = 0$
$\Rightarrow 42(-4 - 2b) = 0 \Rightarrow 2b = -4 \Rightarrow b = -2$
Hence, a = 6, b = -2

46. The condition of the question is, the number of orange forms taken must be equal to the number of green forms taken.
Let us assume that he takes 10 orange and 10 green forms.
10 green forms can be fit exactly on 2 pages at 5 forms/page. But,10 orange forms can't be fit exactly on any number of pages.
Because, 3 orange forms can be fit exactly on a page. In 10 orange forms, 9 forms can be fit exactly on 3 pages and 1 form will be remaining.
To get the number of forms in orange and green which can be fit exactly on some number of pages, we have to find L.C.M of (3,5). That is 15.
15 orange forms can be fit exactly on 5 pages at 3 forms/page.
15 green forms can be fit exactly on 3 pages at 5 forms/page.
Hence, the smallest number of each form could be printed is 15.

47. We have,
60 = $2^2 \times 3 \times 5$
168 = $2^3 \times 3 \times 7$
330 = $2 \times 3 \times 5 \times 11$
HCF = 6
They can put 6 food items in 1 packet.
so the number of packets required for 60 pieces of pastries = $\frac{60}{6}$ = 10
the number of packets required for 168 pieces of cookies = $\frac{168}{6}$ = 28
the number of packets required for 330 chocolate bars = $\frac{330}{6}$ = 55
Total Packets required = 10+28+55 = 93

48. We will prove $6 + \sqrt{2}$ irrational by contradiction.
Let us suppose that ($6 + \sqrt{2}$) is rational.
It means that we have co-prime integers *a* and *b* (*b* ≠ 0)
Such that
$6 + \sqrt{2} = \frac{a}{b}$
$\Rightarrow \sqrt{2} = \frac{a}{b} - 6$
$\Rightarrow \sqrt{2} = \frac{a - 6b}{b}$ .... (1)
a and b are integers.
It means L.H.S of (1) is rational but we know that $\sqrt{2}$ is irrational. It is not possible.
Therefore, our supposition is wrong. ($6 + \sqrt{2}$) cannot be rational.
Hence, ($6 + \sqrt{2}$) is irrational.

49. The number of participants in each room must be the HCF of 60, 84 and 108.
In order to find the HCF of 60, 84 and 108, we first find the HCF of 60 and 84 by Euclid's division algorithm:

2	60	84	1
	48	60	
	12	24	2
	(HCF)	24	
		0	
		(Remainder)	

Clearly, HCF of 60 and 84 is 12
Now, we find the HCF of 12 and 108

12	108	9

(HCF)	108	
	0	
	(Remainder)	

Clearly, HCF of 12 and 108 is 12. Hence, the HCF of 60,84 and 108 is 12.

Therefore, in each room maximum 12 participants can be seated.

We have,

Total number of participants = 60 + 84 + 108 = 252

$\therefore$ Number of rooms required $= \frac{252}{12} = 21$.

50. We have to find Prime Factors of the following numbers

$48 = 2^4 \times 3$

$72 = 2^3 \times 3^2$

$108 = 2^2 \times 3^3$

so the LCM of 48, 72 and 108is

$LCM = 2^4 \times 3^3$

$LCM = 16 \times 27 = 432$

432 seconds = $\frac{432}{60}$ mins

432 seconds = 7.2 mins

So the time it will change together again is

= 8 : 07 : 12 am

51. Let (4 + 3 √2) be a rational number

Then both (4 + 3 √2) and 4 are rational.

⇒ (4 + 3 √2 – 4) = $3\sqrt{2}$ is rational [Since difference of two rational numbers is rational]

⇒ 3√2 is rational.

on multiplying with $\frac{1}{3}$ we get

⇒ 1/3 (3√2) is rational. (Since product of two rational numbers is rational)

⇒ √2 is rational.

This contradicts the fact that √2 is irrational,

This contradicts because we assumed that (4 + 3√2) is rational.So our assumption is wrong

Hence, (4 + 3√2) is irrational.

52. We do not know whether $\frac{a}{b} < \frac{a+2b}{a+b}$ or, $\frac{a}{b} > \frac{a+2b}{a+b}$.

Therefore, to compare these two numbers, let us compute $\frac{a}{b} - \frac{a+2b}{a+b}$

We have,

$\frac{a}{b} - \frac{a+2b}{a+b} = \frac{a(a+b)-b(a+2b)}{b(a+b)} = \frac{a^2+ab-ab-2b^2}{b(a+b)} = \frac{a^2-2b^2}{b(a+b)}$

$\therefore \quad \frac{a}{b} - \frac{a+2b}{a+b} > 0$

$\Rightarrow \quad \frac{a^2-2b^2}{b(a+b)} > 0$

⇒ $a^2 - 2b^2 > 0$

⇒ $a^2 > 2b^2$

$\Rightarrow \quad a > \sqrt{2}b$

and, $\frac{a}{b} - \frac{a+2b}{a+b} < 0$

$\Rightarrow \quad \frac{a^2-2b^2}{b(a+b)} < 0$

⇒ $a^2 - 2b^2 < 0$

⇒$a^2 < 2b^2$

$\Rightarrow \quad a < \sqrt{2}b$

Thus, $\frac{a}{b} > \frac{a+2b}{a+b}$, if $a > \sqrt{2b}$ and $\frac{a}{b} < \frac{a+2b}{a+b}$, if $a < \sqrt{2}b$.

So, we have the following cases:

CASE I When $a > \sqrt{2}b$

In this case, we have

$\frac{a}{b} > \frac{a+2b}{a+b}$ i.e., $\frac{a+2b}{a+b} < \frac{a}{b}$

We have to prove that

$\frac{a+2b}{a+b} < \sqrt{2} < \frac{a}{b}$

We have,

$a > \sqrt{2}b$

$\Rightarrow$ $a^2 > 2b^2$ [Adding a^2 on both sides]

$\Rightarrow \quad 2a^2 + 2b^2 > \left(a^2 + 2b^2\right) + 2b^2$ [Adding $2b^2$ on both sides]

$\Rightarrow \quad 2\left(a^2 + b^2\right) + 4ab > a^2 + 4b^2 + 4ab$ [Adding 4ab on both sides]

$\Rightarrow \quad 2\left(a^2 + 2ab + b^2\right) > a^2 + 4ab + 4b^2$

$\Rightarrow \quad 2(a+b)^2 > (a+2b)^2$

$\Rightarrow \quad \sqrt{2}(a+b) > a + 2b$

$\Rightarrow \quad \sqrt{2} > \frac{a+2b}{a+b}$ ....(i)

Again,

$a > \sqrt{2}b$

$\Rightarrow \frac{a}{b} > \sqrt{2}$ ....(ii)

From (i) and (ii), we get

$\frac{a+2b}{a+b} < \sqrt{2} < \frac{a}{b}$

CASE II When $a < \sqrt{2}b$

In this case, we have

$\frac{a}{b} < \frac{a+2b}{a+b}$

We have to show that $\frac{a}{b} < \sqrt{2} < \frac{a+2b}{a+b}$

We have,

$a < \sqrt{2}b$

$\Rightarrow \quad a^2 < 2b^2$

$\Rightarrow \quad a^2 + a^2 < a^2 + 2b^2$ [Adding a^2 on both sides]

$\Rightarrow \quad 2a^2 + 2b^2 < a^2 + 2b^2 + 2b^2$ [Adding $2b^2$ on both sides]

$\Rightarrow \quad 2a^2 + 2b^2 < a^2 + 4b^2$

$\Rightarrow \quad 2a^2 + 4ab + 2b^2 < a^2 + 4ab + 4b^2$ [Adding 4ab on both sides]

$\Rightarrow \quad 2(a+b)^2 < (a+2b)^2$

$\Rightarrow \sqrt{2}(a+b) < a + 2b$

$\Rightarrow \quad \sqrt{2} < \frac{a+2b}{a+b}$ . ...(iii)

$\Rightarrow \quad a < \sqrt{2}b \Rightarrow \frac{a}{b} < \sqrt{2}$ ....(iv)

From (iii) and (iv), we get

$\frac{a}{b} < \sqrt{2} < \frac{a+2b}{a+b}$

Hence, $\sqrt{2}$ lies between $\frac{a}{b}$ and $\frac{a+2b}{a+b}$.

53. Suppose $\frac{2\sqrt{3}}{5}$ be a rational number

$\therefore \frac{2\sqrt{3}}{5} = \frac{a}{b}$, a and b are co-prime, b$\neq$ 0

$\Rightarrow \sqrt{3} = \frac{5a}{2b}$

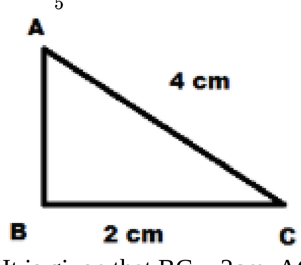
$5a \; and \; 2b$ are integers and $\sqrt{3}$ is irrational.

$\frac{5a}{2b}$ is rational.

$\therefore \sqrt{3} \neq \frac{5a}{2b}$

$\therefore$ Our supposition is wrong

$\Rightarrow \frac{2\sqrt{3}}{5}$ is an irrational number.

54. 

It is given that BC = 2cm, AC = 4 cm.

We have to prove that the area of $\triangle$ABC is irrational.

In $\triangle$ABC we know that

$AC^2 = AB^2 + BC^2$

$(4)^2 = AB^2 + (2)^2$

$AB^2 = 16 - 4$

$= 12$

AB = $\sqrt{12} = 2\sqrt{3}\ cm$

So area of $\triangle ABC = \frac{1}{2} \times BC \times AB$

$= \frac{1}{2} \times 2 \times 2\sqrt{3} = 2\sqrt{3}\ cm^2$ = irrational (As $\sqrt{3}$ is irrational)

So area of $\triangle$ABC is irrational.

55. Let $\sqrt{p} + \sqrt{q}$ is rational number

A rational number can be written in the form of $\frac{a}{b}$

$\sqrt{p} + \sqrt{q} = \frac{a}{b}$

Squaring on both side, we get

$p + q + 2\sqrt{pq} = \left(\frac{a}{b}\right)^2$

$\sqrt{pq} = \frac{1}{2}\left[\left(\frac{a}{b}\right)^2 - p - q\right]$

Now p and q are prime positive number.

So, $\sqrt{p}$ and $\sqrt{q}$ is irrational number .

Also $\sqrt{pq}$ is irrational number .

Since a rational number cannot be equal to an irrational number . Our assumption that $\sqrt{p} + \sqrt{q}$ is rational wrong .

So, $\sqrt{p} + \sqrt{q}$ is an irrational number.

56. It is given that on dividing the polynomial $4x^4 - 5x^3 - 39x^2 - 46x - 2$ by the polynomial g(x), the quotient is $x^2 - 3x - 5$ and the remainder is $- 5x + 8$. We have to find the polynomial g(x).

Now, we know that

Dividend = (Divisor $\times$ Quotient) + Remainder

$4x^4 - 5x^3 - 39x^2 - 46x - 2 = g(x)(x^2 - 3x - 5) + (-5x + 8)$

or, $4x^4 - 5x^3 - 39x^2 - 46x - 2 + 5x - 8 = g(x)(x^2 - 3x - 5)$

or, $4x^4 - 5x^3 - 39x^2 - 41x - 10 = g(x)(x^2 - 3x - 5)$

$g(x) = \frac{4x^4 - 5x^3 - 39x^2 - 41x - 10}{(x^2 - 3x - 5)}$

$$
\begin{array}{r|l}
 & 4x^2 + 7x + 2 \\
x^2 - 3x - 5 & 4x^4 - 5x^3 - 39x^2 - 41x - 10 \\
 & 4x^4 - 12x^3 - 20x^2 \\
 & - \quad + \quad + \\
\hline
 & 7x^3 - 19x^2 - 41x - 10 \\
 & 7x^3 - 21x^2 - 35x \\
 & - \quad + \quad + \\
\hline
 & 2x^2 - 6x - 10 \\
 & 2x^2 - 6x - 10 \\
 & - \quad + \quad + \\
\hline
 & 0
\end{array}
$$

Hence, $g(x) = 4x^2 + 7x + 2$

57. The number of physics books is 192, the number of chemistry books is 240 and the number of mathematics books is 168.

Here, we have to find the HCF of 192, 240 and 168 because the HCF will be the largest number which divides 192, 240 and 168 exactly.

$192 = 2^6 \times 3$

$240 = 2^4 \times 3 \times 5$

$168 = 2^3 \times 3 \times 7$

Now, the HCF of 192, 240 and 168 is $= 2^3 \times 3 = 24$

There must be 24 books in each stack
∴Number of stacks of physics books = $\frac{192}{24} = 8$
And number of stacks of chemistry books =$\frac{240}{24} = 10$
And number of stacks of mathematics books =$\frac{168}{24} = 7$

58. By applying Euclid’s division lemma,
$963 = 657 \times 1 + 306.$
Since remainder ≠ 0, apply division lemma on divisor 657 and remainder 306
$657 = 306 \times 2 + 45.$
Since remainder ≠ 0, apply division lemma on divisor 306 and remainder 45
$306 = 45 \times 6 + 36.$
Since remainder ≠ 0, apply division lemma on divisor 45 and remainder 36
$45 = 36 \times 1 + 9.$
Since remainder ≠ 0, apply division lemma on divisor 36 and remainder 9
$36 = 9 \times 4 + 0.$
Therefore, $H.C.F. = 9.$
HCF of 2 numbers can be expressed as the linear combination of the numbers
$\Rightarrow$ 9 = 45 - 36 $\times$ 1
= $45 - [306 - 45 \times 6] \times 1 = 45 - 306 \times 1 + 45 \times 6$
= $45 \times 7 - 306 \times 1 = [657 - 306 \times 2] \times 7 - 306 \times 1$
= $657 \times 7 - 306 \times 14 - 306 \times 1$
= $657 \times 7 - 306 \times 15$
= $657 \times 7 - [963 - 657 \times 1] \times 15$
= 657 $\times$ 7 - 963 $\times$ 15 + 657 $\times$ 15
= 657 $\times$ 22 - 963 $\times$ 15.
Hence, obtained.

59. On applying the Euclid’s division lemma to find HCF of 152, 272, we get

```
152)272 (1
    152
    ---
    120
```

```
120)152 (1
    120
    ---
     32
```

$272 = 152 \times 1 + 120$
Here the remainder = 0.
Using Euclid’s division lemma to find the HCF of 152 and 120, we get
$152 = 120 \times 1 + 32$
Again the remainder = 0.
Using division lemma to find the HCF of 120 and 32, we get

```
32)120 (3
    96
    --
    24
```

$120 = 32 \times 3 + 24$
Similarly,

```
8)24 (3
  24
  --
   0
```

$32 = 24 \times 1 + 8$

$24 = 8 \times 3 + 0$
HCF of 272 and 152 is 8.
272×8 + 152x = H.C.F. of the numbers
$\Rightarrow 8 = 272 \times 8 + 152x$
$\Rightarrow\ 8 - 272 \times 8 = 152x$
$\Rightarrow 8(1 - 272) = 152x$
$\Rightarrow x = \frac{-2168}{152} = \frac{-271}{19}$

60. Suppose $\sqrt[3]{6}$ be rational number and $\sqrt[3]{6} = \frac{a}{b}$ where $a\ and\ b$ are co-prime and $b \neq 0$
$\Rightarrow (\sqrt[3]{6})^3 = \frac{a^3}{b^3}$
$\Rightarrow 6 = \frac{a^3}{b^3}$
$\Rightarrow 6.b^3 = a^3$
$\Rightarrow a^3$ is divisible by $6 \Rightarrow a$ is divisible by 6.
Let $a = 6c$
$6b^3 = (6c)^3$
$\Rightarrow \quad b^3 = 36c^3$
$\Rightarrow b^3$ is divisible by $6 \Rightarrow b$ is divisible by 6.
$\Rightarrow a\ and\ b$ have a common factor i.e, 6
$\Rightarrow a\ and\ b$ are not co-prime which is a contradiction
$\therefore \sqrt[3]{6}$ is an irratonal.

61. Let us first find the HCF of 210 and 55. Applying Euclid's division lemma on 210 and 55, we get
210 = 55 × 3 + 45 ....(i)
$\because 55 \overline{)210} (3$
165
45
Since, the remainder 45 $\neq$ 0. So, we now apply division lemma on the divisor 55 and the remainder 45 to get
55 = 45 × 1 + 10 ....(i)
$\because 45 \overline{)55} (1$
45
10
We consider the divisor 45 and the remainder 10 and apply division lemma to get
45 = 4 × 10 + 5 ...(iii)
$\because 10 \overline{)45} (4$
40
5
We consider the divisor 10 and the remainder 5 and apply division lemma to get
10 = 5 × 2 + 0 ...(iv)
We observe that the remainder at this stage is zero. So, the last divisor i.e. 5 is the HCF of 210 and 55.
$\therefore$ 5 = 210 × 5 + 55y
$\Rightarrow 55y = 5 - 210 \times 5 = 5 - 1050$
$\Rightarrow 55y = -1045$
$\Rightarrow y = \frac{-1045}{55} = -19$.

62. Let us assume that $4 + \sqrt{3}$ is a rational number equal to a/b where a and b are two integers
$\Rightarrow 4 + \sqrt{3} = \frac{a}{b}$.
$\Rightarrow \sqrt{3} = \frac{a}{b} - 4$
we know that subtraction of two rational number is always a rational number but we get $\sqrt{3}$ as rational number which contradict the fact as $\sqrt{3}$ is an irrational number.
Hence, $4 + \sqrt{3}$ is a irrational number.

63. Maximum number of students that can be seated in one room = HCF of 336, 240, 96
$336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7$
$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$
$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$
$\therefore HCF\ of\ 336, 240, 96 = 2 \times 2 \times 2 \times 2 \times 3 = 48$

Now, number of rooms for participants in Mathematics = $\frac{336}{48}$ = 7

Number of rooms for participants in Physics = $\frac{240}{48}$ = 5

Number of rooms for participants in Biology = $\frac{96}{48}$ = 2

$\therefore$ Total no. of rooms = $7 + 5 + 2 = 14$

64. Fundamental theorem of arithmetic:

"Every composite number can be expressed (factorized) as a product of primes, and this factorization is unique, apart from the order in which the prime factors occur."

2520 = 8 $\times$ 9 $\times$ 5 $\times$ 7 = $2^3 \times 3^2 \times 5 \times 7$

10530 =2 $\times$ 81 $\times$ 5 $\times$13= $2 \times 3^4 \times 5 \times 13$

$\therefore$ LCM = $2^3 \times 3^4 \times 5 \times 7 \times 13$ = 294840

65. LCM of rational number =$\frac{\text{LCM of numerators}}{\text{HCF of denominators}}$

Numbers are $\frac{25}{10}, \frac{5}{10}, \frac{175}{1000}$

Now, 25 = 5$\times$5; 5 = 5$\times$1; 175 = 5$\times$5$\times$7

LCM of (25, 5, 175) = 5$\times$5$\times$7 = 175

Also,

10 = 2$\times$5; 1000 = 2$\times$2$\times$2$\times$5$\times$5$\times$5

HCF of (10,10,1000) = 10

LCM of (2.5, 0.5, 0.175) =$\frac{175}{10} = 17.5$

66. i. The Number of room will be minimum if each room accomodates maximum number of participants. Therefore, the number of participants in each room must be the HCF of 60, 84 and 108. The prime factorisations of 60, 84 and 108 are as under.

 60 = $2^2 \times 3 \times 5$, 84 = $2^2 \times 3 \times 7$ and 108 = $2^2 \times 3^3$

 $\therefore$ HCF of 60, 84 and 108 is $2^2 \times 3$ = 12

 Therefore, in each room 12 participants can be seated.

 ii. The Number of room will be minimum if each room accomodates maximum number of participants. Therefore, the number of participants in each room must be the HCF of 60, 84 and 108. The prime factorisations of 60, 84 and 108 are as under.

 60 = $2^2 \times 3 \times 5$, 84 = $2^2 \times 3 \times 7$ and 108 = $2^2 \times 3^3$

 $\therefore$ HCF of 60, 84 and 108 is $2^2 \times 3$ = 12

 Therefore, in each room 12 participants can be seated.

 $\therefore$ Number of rooms required = $\frac{\text{Total number of participants}}{12}$

 = $\frac{60+84+108}{12}$

 = $\frac{252}{12}$

 = 21

 iii. Prime factorisation of 60 = 2 $\times$ 2 $\times$ 3 $\times$ 5

 Prime factorisation of 84 = 2 $\times$ 2 $\times$ 3 $\times$ 7

 Hence, LCM of 60, 84 = 2 $\times$ 2 $\times$ 3 $\times$ 5 $\times$ 7 = 420

 And HCF of 60, 84 = 2 $\times$ 2 $\times$ 3 = 12

 Now, LCM $\times$ HCF = 420 $\times$ 12 = 5040

 Also, 60 $\times$ 84 = 5040

 i.e., HCF $\times$ LCM = Product of the two numbers

 OR

 Product of numbers = HCF $\times$ LCM

 $\Rightarrow$ 1080 = 30 $\times$ LCM

 $\therefore$ LCM = $\frac{1080}{30}$ = 36

67. i. The number of students in Section A is 32, and the number of students in Section B is 36.

 Step 1: Find the prime factors of each number:

 32 = 2 $\times$ 2 $\times$ 2 $\times$ 2 $\times$ 2

 36 = 2 $\times$ 2 $\times$ 3 $\times$ 3

 Step 2: Identify the common and uncommon prime factors. The common ones are 2 $\times$ 2.

 Step 3: Multiply the common and uncommon prime factors together to get the LCM:

 LCM = 2 $\times$ 2 $\times$ 2 $\times$ 2 $\times$ 2 $\times$ 3 $\times$ 3 = 288

So, the minimum number of books needed to be acquired for the class library is 288, so they can be distributed equally among students of Section A or Section B.

ii. Step 1: Find the prime factors of each number:
$32 = 2 \times 2 \times 2 \times 2$
$36 = 2 \times 2 \times 3 \times 3$
Step 2: Identify the common prime factors and their minimum exponent:
The common prime factors are 2×2.
Step 3: Calculate the HCF by multiplying the common prime factors:
HCF = $2 \times 2 = 4$
So, the HCF of 32 and 36 is 4.

iii. Given number $(7 \times 11 \times 13 \times 15 + 15)$
It can also be written as $15 (7 \times 11 \times 13 + 1)$.
As it is a product of two composite numbers
hence it is a composite number.
OR
Given:
$p = ab^2$
$q = a^2b$
Take the highest power of each prime factor:
LCM = $a^2 \times b^2$
So, the LCM of p and q is a^2b^2.

68. $173250 = 2 \times 5^3 \times 3^2 \times 7 \times 11$
i. 3
ii. a. $173250 = 2 \times 5^3 \times 3^2 \times 7 \times 11$
Number of students in the class = 3 + 2 + 1 + 1 = 7
OR
b. $173250 = 2 \times 5^3 \times 3^2 \times 7 \times 11$
Highest prime number used by students = 11
iii. 5

69. i. 120
ii. (b) 20
iii. Yes, 8 : 13 > 5 :19, thus she had bought more earrings with stones after the price hike
iv. (b) 7
v.
- The demand for jewelry with stones among customers increases.
- People prefer to buy jewelry with stones.

70.
(b) Both A and R are true but R is not the correct explanation of A.
Explanation:
Yes 12 and 17 are coprime numbers and H.C.F. of coprimes is always 1.

71.
(c) A is true but R is false.
Explanation:
HCF (a, b) $\times$ LCM (a, b) = $a \times b$
$\Rightarrow 8 \times \text{LCM} = 280$
$\Rightarrow \text{LCM} = \frac{280}{8} = 35$
A is true but R is false.

72. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation:

Smallest prime is 2 and smallest composite is 4 so H.C.F. of 2 and 4 is 4.

73.

(d) A is false but R is true.

Explanation:

$\frac{3072}{16} = 192 \neq 162$

74.

(d) A is false but R is true.

Explanation:

We know that for any two numbers, Product of the two numbers = HCF $\times$ LCM

HCF $\times$ LCM = 18 $\times$ 169 = 3042 $\neq$ 3072

So, A is false but R is true.