CLASS X (2019-20)
MATHEMATICS STANDARD(041)
SAMPLE PAPER-1

Time : 3 Hours **Maximum Marks : 80**

General Instructions :

(i) All questions are compulsory.

(ii) The questions paper consists of 40 questions divided into four sections A, B, C and D.

(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.

(iv) There is no overall choice. However, an internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted.

Section A

Q.1-Q.10 are multiple choice questions. Select the most appropriate answer from the given options.

1. If p_1 and p_2 are two odd prime numbers such that $p_1 > p_2$, then $p_1^2 - p_2^2$ is [1]

(a) an even number (b) an odd number

(c) an odd prime number (d) a prime number

Ans : (a) an even number

$p_1^2 - p_2^2$ is an even number.

Let us take $p_1 = 5$

and $p_2 = 3$

Then, $p_1^2 - p_2^2 = 25 - 9 = 16$

16 is an even number.

2. The points (7, 2) and $(-1, 0)$ lie on a line [1]

(a) $7y = 3x - 7$ (b) $4y = x + 1$

(c) $y = 7x + 7$ (d) $x = 4y + 1$

Ans : (b) $4y = x + 1$

The point satisfy the line, $4y = x + 1$.

3. If $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$, then the value of k is [1]

(a) 2 (b) -2

(c) $\frac{1}{4}$ (d) $\frac{1}{2}$

Ans : (a) 2

Since, $\frac{1}{2}$ is a root of the quadratic equation

$$x^2 + kx - \frac{5}{4} = 0$$

Then, $$\left(\frac{1}{2}\right)^2 + k\left(\frac{1}{2}\right) - \frac{5}{4} = 0$$

$$\frac{1}{4} + \frac{k}{2} - \frac{5}{4} = 0$$

$$\frac{1 + 2k - 5}{4} = 0$$

$$2k - 4 = 0$$

$$2k = 4$$

$$k = 2$$

4. If the nth term of an A.P. is given by $a_n = 5n - 3$, then the sum of first 10 terms if [1]

(a) 225 (b) 245

(c) 255 (d) 270

Ans : (b) 245

Putting, $n = 1, 10$

we get, $a = 2$

$l = 47$

$$S_{10} = \frac{10}{2}(2 + 47) = 5 \times 49 = 245$$

5. It is given that $\Delta ABC \sim \Delta PQR$ with $\frac{BC}{QR} = \frac{1}{3}$. Then $\frac{\text{ar}(\Delta PRQ)}{\text{ar}(\Delta BCA)}$ is equal to [1]

(a) 9 (b) 3

(c) $\frac{1}{3}$ (d) $\frac{1}{9}$

Ans : (a) 9

Since, $\Delta ABC \sim \Delta PQR$

$$\frac{\text{ar}(\Delta PRQ)}{\text{ar}(\Delta BCA)} = \frac{AR^2}{AC^2}$$

$$= \frac{QR^2}{BC^2} = \frac{9}{1} \qquad \left[\frac{QR}{BC} = \frac{3}{1}\right] = 9$$

6. Ratio in which the line $3x + 4y = 7$ divides the line segment joining the points (1, 2) and $(-2, 1)$ is [1]

(a) 3 : 5 (b) 4 : 6

(c) 4 : 9 (d) None of these

Ans : (c) 4 : 9

$$\frac{3(1) + 4(2) - 7}{3(-2) + 4(1) - 7} = -\frac{4}{-9} = \frac{4}{9}$$

7. $(\cos^4 A - \sin^4 A)$ is equal to [1]

(a) $1 - 2\cos^2 A$ (b) $2\sin^2 A - 1$

(c) $\sin^2 A - \cos^2 A$ (d) $2\cos^2 A - 1$

Ans : (d) $2\cos^2 A - 1$

$$(\cos^4 A - \sin^4 A) = (\cos^2 A)^2 - (\sin^2 A)^2$$

$$= (\cos^2 A - \sin^2 A)(\cos^2 A + \sin^2 A)$$

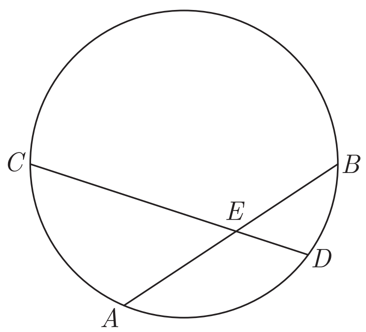
$$= (\cos^2 A - \sin^2 A)(1)$$

$= \cos^2 A - (1 - \cos^2 A) = 2\cos^2 A - 1$

8. Two chords AB and CD of a circle intersect at E such that $AE = 2.4$ cm, $BE = 3.2$ cm and $CE = 1.6$ cm. The length of DE is [1]

(a) 1.6 cm (b) 3.2 cm
(c) 4.8 cm (d) 6.4 cm

Ans : (c) 4.8 cm



Apply the rule, $AE \times EB = CE \times ED$

$2.4 \times 3.2 = 1.6 \times ED$

$ED = 4.8$ cm

9. To divide a line segment AB in the ratio 3 : 4, we draw a ray AX, so that $\angle BAX$ is an acute angle and then mark the points on ray AX at equal distances such that the minimum number of these points is [1]

(a) 3 (b) 4
(c) 7 (d) 10

Ans : (c) 7

Minimum number of these points $= 3 + 4 = 7$

10. If the radius of the sphere is increased by 100%, the volume of the corresponding sphere is increased by [1]

(a) 200% (b) 500%
(c) 700% (d) 800%

Ans : (c) 700%

When the radius is increased by 100%, the corresponding volume becomes 800% and thus increase is 700%.

(Q.11-Q.15) Fill in the blanks.

11. H.C.F. of 6, 72 and 120 is .... [1]

Ans : 6

12. If α and β are the zeroes of the quadratic polynomial $ax^2 + bx + c$, then $\alpha + \beta = - b/$.......... and $\alpha\beta = c/$.......... [1]

Ans : a, a

or

Degree of remainder is always .... than degree of divisor.

Ans : Smaller/less

13. Length of arc of a sector angle 45° of circle of radius 14cm is .... [1]

Ans : $\frac{7}{2}\pi$cm

14. The length of the diagonal of a cube that can be inscribed in a sphere of radius 7.5 cm is .... [1]

Ans : 15 cm

15. A dice is thrown once, the probability of getting a prime number is .... [1]

Ans : 1/2

(Q.16-Q.20) Answer the following

16. Find the positive root of $\sqrt{3x^2 + 6} = 9$. [1]

Ans :

We have $\sqrt{3x^2 + 6} = 9$

Taking square at both side, we get,

$$3x^2 + 6 = 81$$
$$3x^2 = 81 - 6 = 75$$
$$x^2 = \frac{75}{3} = 25$$

Thus $x = \pm 5$

Hence 5 is positive root.

17. The diameter of a wheel is 1.26 m. What the distance covered in 500 revolutions. [1]

Ans :

Distance covered in 1 revolution is equal to circumference of wheel and that is

$$2\pi r = \frac{2\pi d}{2} = \pi d.$$

Distance covered in 500 revolutions

$$= 500 \times \pi \times d$$
$$= 500 \times \pi \times 1.26$$
$$= 500 \times \frac{22}{7} \times 1.26$$
$$= 1980 \text{ m.} = 1.98 \text{ km}$$

18. A rectangular sheet paper 40 cm $\times$ 22 cm is rolled to form a hollow cylinder of height 40 cm. Find the radius of the cylinder. [1]

Ans :

Given,

Height, $h = 40$ cm, circumference $= 22$ cm

$$2\pi r = 22$$
$$r = \frac{22 \times 7}{2 \times 22} = \frac{7}{2} = 3.5 \text{ cm}$$

or

A cylinder, a cone and a hemisphere have same base and same height. Find the ratio of their volumes.

Ans :

Volume of cylinder : Volume of cone : Volume of hemisphere

$$= \pi r^2 h : \frac{1}{3}\pi r^2 h : \frac{2}{3}\pi r^3$$
$$= \pi r^2 h : \frac{1}{3}\pi r^2 h : \frac{2}{3}\pi r^2 \times h \qquad (h = r)$$
$$= 1 : \frac{1}{3} : \frac{2}{3} \text{ or } 3 : 1 : 2$$

19. If the median of a series exceeds the mean by 3, find by what number the mode exceeds its mean? [1]

Ans :

Given, $\text{Median} = \text{Mean} + 3$

$$\text{Mode} = 3\,\text{Median} - 2\,\text{Mean}$$
$$= 3\,(\text{Mean} + 3) - 2\,\text{Mean}$$

$\Rightarrow$ $\text{Mode} = \text{Mean} + 9$

Hence Mode exceeds Mean by 9.

20. 20 tickets, on which numbers 1 to 20 are written, are mixed thoroughly and then a ticket is drawn at random out of them. Find the probability that the number on the drawn ticket is a multiple of 3 or 7. [1]

Ans :

$$\text{Total number of cases} = 20$$
$$n(s) = 20$$
$$A = \text{favourable cases}$$
$$= \{3, 6, 7, 9, 12, 14, 15, 18\}$$

$\therefore$ $n(A) = 8$

$\therefore$ $\text{Required probability} = P(A)$

$$= \frac{n(A)}{n(S)} = \frac{8}{20} = \frac{2}{5}$$

Section B

21. Solve the following pair of linear equations by cross multiplication method: [2]

$$x + 2y = 2$$
$$x - 3y = 7$$

Ans :

We have $x + 2y - 2 = 0$

$$x - 3y - 7 = 0$$

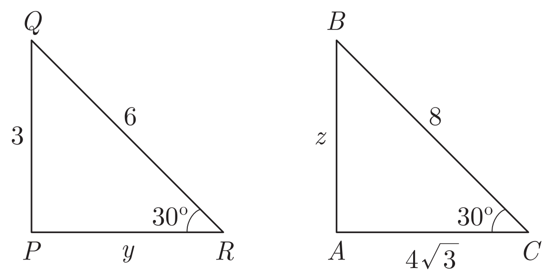
Using the formula

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

we have $\dfrac{x}{-14-6} = \dfrac{y}{-2+7} = \dfrac{1}{-3-2}$

$$\frac{x}{-20} = \frac{y}{5} = \frac{-1}{5}$$
$$\frac{x}{-20} = \frac{-1}{5} \Rightarrow x = 4$$
$$\frac{y}{5} = \frac{-1}{5} \Rightarrow y = -1$$

22. In the given figure, $\Delta\,ABC \sim \Delta\,PQR$. Find the value of $y + z$. [2]



Ans :

In the given figure $\Delta\,ABC \sim \Delta\,PQR$

Thus $\dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{AC}{PR}$

$$\frac{z}{3} = \frac{8}{6} = \frac{4\sqrt{3}}{y}$$
$$\frac{z}{3} = \frac{8}{6} \text{ and } \frac{8}{6} = \frac{4\sqrt{3}}{y}$$
$$z = \frac{8\times 3}{6} \text{ and } y = \frac{4\sqrt{3}\times 6}{8}$$
$$z = 4 \text{ and } y = 3\sqrt{3}$$

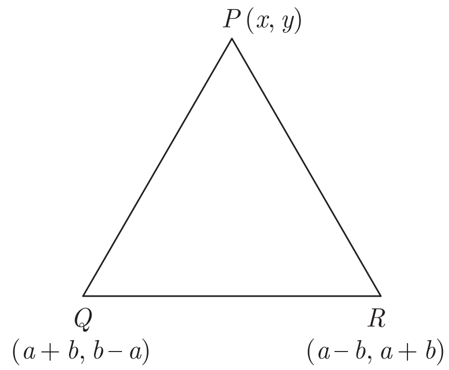
Thus $y + z = 3\sqrt{3} + 4$

23. If the point $P(x, y)$ is equidistant from the points $Q(a+b, b-a)$ and $R(a-b, a+b)$, then prove that $bx = ay$. [2]

Ans :

We have $|PQ| = |PR|$

$$\sqrt{[x-(a+b)]^2 + [y-(b-a)]^2} = \sqrt{[x-(a-b)]^2 + [y-(b+a)]^2}$$



$$[x-(a+b)]^2 + [y-(b-a)]^2 = [x-(a-b)]^2 + [y-(a+b)]^2$$
$$-2x(a+b) - 2y(b-a) = -2x(a-b) - 2y(a+b)$$
$$2x(a+b) + 2y(b-a) = 2x(a-b) + 2y(a+b)$$
$$2x(a+b-a+b) + 2y(b-a-a-b) = 0$$
$$2x(2b) + 2y(-2a) = 0$$
$$xb - ay = 0$$
$$bx = ay$$

Hence Proved

or

Show that the points $A(0,1), B(2,3)$ and $C(3,4)$ are collinear.

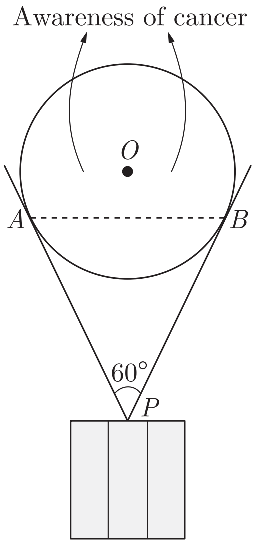
Ans :

If the area of the triangle formed by the points is zero, then points are collinear.

We have $A(0,1), B(2,3)$ and $C(3,4)$

$$\Delta = \frac{1}{2}|0(3-4) + 2(4-1) + 3(1-3)|$$
$$= \frac{1}{2}|0 + (2)(3) + (3)(-2)|$$
$$= \frac{1}{2}|6-6| = 0$$

24. As a part of a campaign, a huge balloon with message of "AWARENESS OF CANCER" was displayed from the terrace of a tall building. It was held by string of length 8 m each, which inclined at an angle of 60° at the point, where it was tied as shown in the figure. [2]



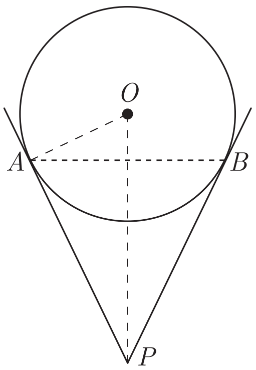
i. What is the length of AB?
ii. If the perpendicular distance from the centre of the circle to the chord AB is 3 cm, then find the radius of the circle.

Ans :

(i) Here, $PA = PB = 8\text{m}$

From the figure it is clear that PA and PB are tangents to the circle.

Now, draw OP which bisects $\angle APB$ and perpendicular to the chord AB.



Thus, we have

$$\angle APC = \angle BPC = 30^\circ$$

and $\angle ACP = \angle BCP = 90^\circ$

In ΔACP,

$$\angle APC + \angle ACP + \angle PAC = 180^\circ$$

After substituting the values, we get

$$30^\circ + 90^\circ + \angle PAC = 180^\circ$$

$$\angle PAC = 180^\circ - 120^\circ = 60^\circ$$

Similarly, $\angle PBC = 60^\circ$

Thus, ΔAPB is an equilateral triangle.

$$AB = AP = BP = 8\,\text{m}$$

(ii) Here, $OC = 3$ m

As, we know that, if a perpendicular drawn from the centre of the circle to the chord, then it bisects the chord.

$$AC = BC = \frac{AB}{2} = \frac{8}{2} = 4$$

In right angled ΔACO

$$OA^2 = AC^2 + OC^2$$

[by Pythagoras theorem]

$$OA = \sqrt{4^2 + 3^2} = 5 \text{ m}$$

Which is the radius of the circle.

25. Find the mean of the data using an empirical formula when it is given that mode is 50.5 and median in 45.5. [2]

Ans :

Given, Mode $= 50.5$

Median $= 45.5$

$$3 \times \text{Median} = \text{Mode} + 2 \text{ Mean}$$

$$\Rightarrow \quad 3 \times 45.5 = 50.5 + 2 \text{ Mean}$$

$$\Rightarrow \quad \text{Mean} = \frac{136.5 - 50.5}{2}$$

Hence, Mean $= 43$

or

A bag contains 6 red and 5 blue balls. Find the probability that the ball drawn is not red.

Ans :

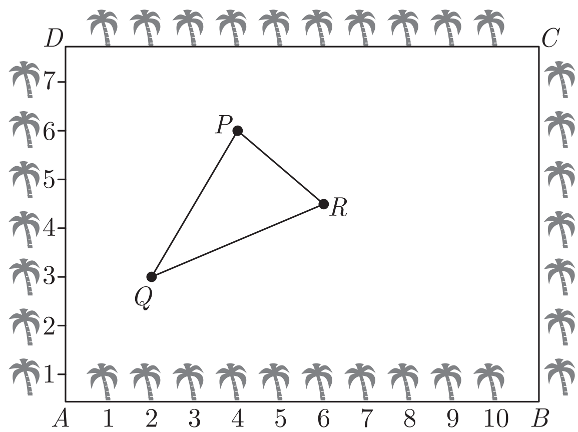
No. of possible outcomes $= 6 + 5 = 11$

No. of favourable outcome $= 5$

$$p \text{ (not red)} = 11 - 6 = 5$$

$$\therefore \quad = \frac{5}{11}$$

26. The Class XII students of a senior secondary school in Kishangarh have been allotted a rectangular plot of land for this gardening activity as shown in figure [2]



Sapling of Neem tree are planted on the boundary at a distance of 1 m from each other. There is a triangular grassy lawn in the plot as shown in above figure.

The students are to sow seeds of flowering plants on the remaining area of the plot.

Then, taking A a origin, find the area of the triangle in this case.

Ans :

When A is taken as origin, AD and AB as coordinate axes. i.e. X and Y-axes, respectively. Hence coordinates of P, Q and R are respectively, (4,6), (3,2) and (6,5).

In this case, AD and AB taken as coordinate axes.

Then, area of ΔPQR

$$= \frac{1}{2}\left| 4(2-5) + 3(5-6) + (6-2) \right|$$

[$\because$ area of triangle]

$$= \frac{1}{2}\left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$$

$$= \frac{1}{2}\left| 4(-3) + 3(-1) + 6 \times 4 \right|$$

$$= \frac{1}{2}\left| -12 - 3 + 24 \right| = \frac{9}{2} \text{ sq. units.}$$

Section C

27. Quadratic polynomial $2x^2 - 3x + 1$ has zeroes as α and β. Now form a quadratic polynomial whose zeroes are 3α and 3β. [3]

Ans :

We have $f(x) = 2x^2 - 3x + 1$

If α and β are the zeroes of $2x^2 - 3x + 1$, then

Sum of zeroes $\alpha + \beta = \frac{-b}{a} = \frac{3}{2}$

Product of zeroes $\alpha\beta = \frac{c}{a} = \frac{1}{2}$

New quadratic polynomial whose zeroes are 3α and 3β is,

$$p(x) = x^2 - (3\alpha + 3\beta)x + 3\alpha \times 3\beta$$
$$= x^2 - 3(\alpha + \beta)x + 9\alpha\beta$$
$$= x^2 - 3\left(\frac{3}{2}\right)x + 9\left(\frac{1}{2}\right)$$
$$= x^2 - \frac{9}{2}x + \frac{9}{2}$$
$$= \frac{1}{2}(2x^2 - 9x + 9)$$

or

If α and β are the zeroes of a quadratic polynomial such that $\alpha + \beta = 24$ and $\alpha - \beta = 8$. Find the quadratic polynomial having α and β as its zeroes.

Ans :

We have $\alpha + \beta = 24$ ...(1)

$\alpha - \beta = 8$ ...(2)

Adding equations (1) and (2) we have

$$2\alpha = 32 \Rightarrow \alpha = 16$$

Subtracting (1) from (2) we have

$$2\beta = 24 \Rightarrow \beta = 12$$

Hence, the quadratic polynomial

$$p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$$
$$= x^2 - (16 + 8)x + (16)(8)$$
$$= x^2 - 24x + 128$$

28. Solve using cross multiplication method: [3]

$$5x + 4y - 4 = 0$$
$$x - 12y - 20 = 0$$

Ans :

We have $5x + 4y - 4 = 0$ ...(1)

$x - 12y - 20 = 0$ ...(2)

By cross-multiplication method,

$$\frac{x}{b_2c_1 - b_1c_2} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{b_1b_2 - a_2b_1}$$

$$\frac{x}{-80 - 48} = \frac{y}{-4 + 100} = \frac{1}{-60 - 4}$$

$$\frac{x}{-128} = \frac{y}{96} = \frac{1}{64}$$

$$\frac{x}{-128} = \frac{1}{-64} \Rightarrow x = 2$$

and $\frac{y}{96} = \frac{1}{-64} \Rightarrow y = \frac{-3}{2}$

Hence, $x = 2$ and $y = \frac{-3}{2}$

29. Find the 20^{th} term of an A.P. whose 3^{rd} term is 7 and the seventh term exceeds three times the 3^{rd} term by 2. Also find its n^{th} term (a_n). [3]

Ans :

Let the first term be a, common difference be d and nth term be a_n.

We have $a_3 = a + 2d = 7$ (1)

$a_7 = 3a_3 + 2$

$a + 6d = 3 \times 7 + 2 = 23$ (2)

Solving (1) and (2) we have

$$4d = 16 \Rightarrow d = 4$$
$$a + 8 = 7 \Rightarrow a = -1$$
$$a_{20} = a + 19d = -1 + 19 \times 4 = 75$$
$$a_1 = a + (n-1)d = -1 + 4n - 4$$
$$= 4n - 5.$$

Hence n^{th} term is $4n - 5$

or

In an A.P. the sum of first n terms is $\frac{3n^2}{2} + \frac{13n}{2}$. Find the 25^{th} term.

Ans :

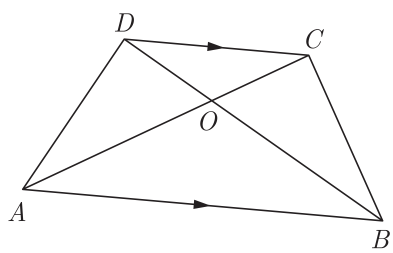
We have $S_n = \frac{3n^2 + 13n}{2}$

$$a_n = S_n - S_{n-1}$$
$$a_{25} = S_{25} - S_{24}$$
$$= \frac{3(25)^2 + 13(25)}{2} - \frac{3(24)^2 + 13(24)}{2}$$
$$= \frac{1}{2}\left\{3(25^2 - 24^2) + 13(25 - 24)\right\}$$
$$= \frac{1}{2}(3 \times 49 + 13) = 80$$

30. In a trapezium $ABCD$, diagonals AC and BD intersect at O and $AB = 3DC$, then find ratio of areas of triangles COD and AOB. [3]

Ans :

As per given condition we have drawn the figure below.



because of AA similarity we have

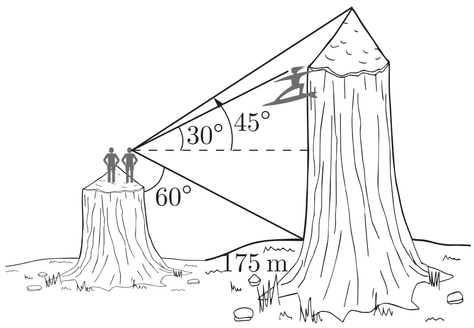
$$\Delta AOB \sim \Delta COD$$

$$\frac{ar(\Delta COD)}{ar(\Delta AOB)} = \frac{CD^2}{AB^2} = \frac{CD^2}{(3CD)^2} = \frac{CD^2}{9CD^2} = \frac{1}{9}$$

$$\text{ratio} = 1:9$$

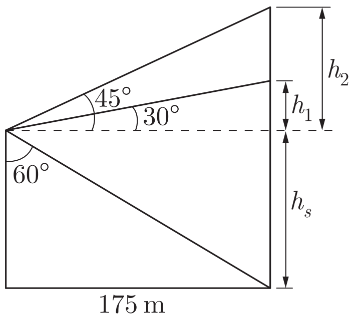
31. A local Outdoors Club has just hiked to the south rim of a large canyon, when they spot a climber attempting to scale the taller northern face. Knowing the distance between the sheer walls of the northern and southern faces of the canyon is approximately 175m, they attempt to compute the distance remaining for the climbers to reach the top of the northern rim. Using a homemade transit, they sight an angle of depression of 60° to the bottom of the north face, and angles of elevation of 30° and 45° to the climbers and top of the northern rim respectively.

(a) How high is the southern rim of the canyon?
(b) How high is the northern rim?
(c) How much farther until the climber reaches the top? [3]



Ans :

Let's first find the distances h_s, h_1 and h_2 in the diagram below, then answer the questions.



$$\tan 60^\circ = \frac{h_s}{175};\ h_s = 175\tan 60^\circ = 175\sqrt{3}\text{ m}$$

$$\tan 30^\circ = \frac{h_1}{175};\ h_1 = 175\tan 30^\circ = \frac{175}{\sqrt{3}}\text{ m}$$

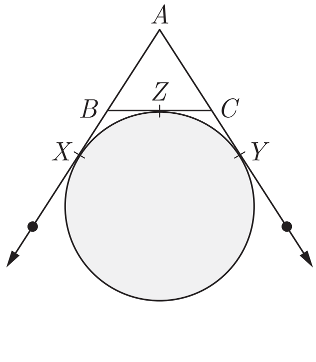
$$\tan 45^\circ = \frac{h_2}{175};\ h_2 = 175\tan 45^\circ = 175\text{ m}$$

(a) $h_s = 175\sqrt{3}$ m is the height of the south rim.
(b) $h_s + h_2 = 175\sqrt{3} + 175 = 175(1+\sqrt{3})$ m is the height of the north rim.
(c) $h_2 - h_1 = 175 - \frac{175}{\sqrt{3}} = 175\left(1 - \frac{1}{\sqrt{3}}\right)$m is how far the climbers have to go to the top.

32. ABC is a triangle. A circle touches sides AB and AC produced and side BC at BC at X, X, Y and Z respectively. Show that $AX = \frac{1}{2}$ perimeter of ΔABC. [3]

Ans :

As per question we draw figure shown below.



Since length of tangents from an external point to a circle are equal,

At A, $AX = AY$ (1)

At B $BX = BZ$ (2)

At C $CY = CZ$ (3)

Perimeter of ΔABC,

$$p = AB + AC + BC$$
$$= (AX - BX) + (AY - CY) + BZ + ZC)$$
$$= AX + AY - BX + BZ + ZC - CY$$
$$= AX + AY = 2AX$$

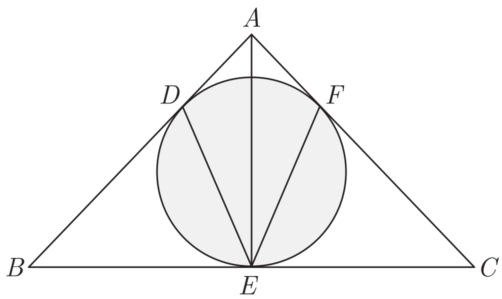
Thus $AX = \frac{1}{2}p$ Hence Proved

or

In $\Delta ABD, AB = AC$. If the interior circle of ΔABC touches the sides AB, BC and CA at D, E and F respectively. Prove that E bisects BC.

Ans :

As per question we draw figure shown below.



Since length of tangents from an external point to a circle are equal,

At A, $AF = AD$ (1)

At B $BE = BD$ (2)

At C $CE = CF$ (3)

Now we have $AB = AC$

$$AD + DB = AF + FC$$
$$BD = FC \quad (AD = AF)$$
$$BE = EC \quad (BD = BE, CE = CF)$$

Thus E bisects BC.

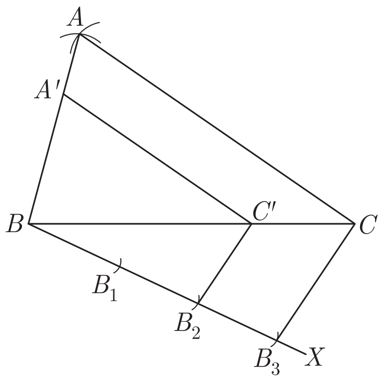
33. Construct a ΔABC in which $AB = 4$ cm, $BC = 5$ cm and $AC = 6$cm. Then construct another triangle whose sides are $\frac{2}{3}$ times the corresponding sides of ΔABC. [3]

Ans :

Steps of construction :

1. Draw a line segment $BC = 5$cm.
2. With B as centre and radius $= AB = 4$cm, draw an arc.
3. With C as centre and radius $= AC = 6$cm, draw another arc, intersecting the arc drawn in step 2 at the point A.
4. Join AB and AC to obtain ΔABC.
5. Below BC, make an acute angle $\angle CBX$.
6. Along BX mark off three points B_1, B_2, B_3 such that $BB_1 = B_1B_2 = B_2B_3$.
7. Join B_3C.
8. From B_2, draw $B_2C' \,||\, B_3C$.
9. From C', draw $C'A' \,||\, CA$, meeting BA at the point A'.

Then $A'BC'$ is the required triangle.

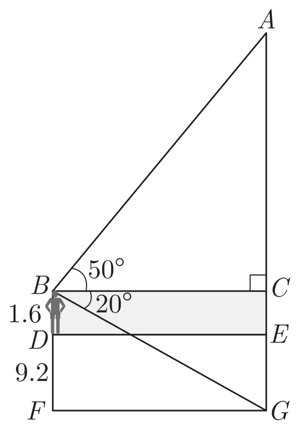


34. Hari, standing on the top of a building, sees the top of a tower at an angle of elevation of 50° and the foot of the tower at an angle of depression of 20°. Hari is 1.6 metre tall and the height of the building on which he is standing is 9.2 mitres. [3]

(a) Draw a rough sketch according to the given information.
(b) How far is the tower from the building?
(c) Calculate the height of the tower.
[$\sin 20^\circ = 0.34$, $\cos 20^\circ = 0.94$, $\tan 20^\circ = 0.36$
$\sin 50^\circ = 0.77$, $\cos 50^\circ = 0.64$, $\tan 50^\circ = 1.19$]

Ans :

(a) Rough sketch



Hari is standing at D. His height BD is 1.6 m.

Height of the building, $DF = 9.2$ m

The angle of elevation of the top of the tower AG is 50°.

The angle of depression of the foot of the tower is 20°.

(b) Distance between the tower and the building,

$$BC = DE = FG$$

In right ΔBCG,

$$\tan 20^\circ = \frac{CG}{BC} \Rightarrow 0.36 = \frac{1.6 + 9.2}{BC}$$

$$BC = \frac{10.8}{0.36} = 30 \text{ m}$$

Hence, distance between the tower and the building $= 30$ m

(c) In right ΔACB

$$AC = BC \tan 50^\circ$$
$$= 30 \times 1.19 = 35.7 \text{ m}$$

Hence, height of the tower

$$= AC + CE + EG$$
$$= 35.7 + 1.6 + 9.2 = 46.5 \text{ m}$$

Section D

35. For any positive integer n, prove that $n^3 - n$ is divisible by 6. [4]

Ans :

We have
$$n^3 - n = n(n^2 - 1)$$
$$= (n-1)n(n+1)$$
$$= (n-1)n(n+1)$$

Thus $n^3 - n$ is product of three consecutive positive integers.

Since, any positive integers a is of the form $3q, 3q+1$ or $3q+2$ for some integer q.

Let $a, a+1, a+2$ be any three consecutive integers.

Case I : $a = 3q$

If $a = 3q$ then,

$$a(a+1)(a+2) = 3q(3q+1)(3q+2)$$

Product of two consecutive integers $(3q+1)$ and $(3q+2)$ is an even integer, say $2r$.

Thus $a(a+1)(a+2) = 3q(2r)$

$= 6qr$, which is divisible by 6.

Case II : $a = 3q+1$

If $a = 3q+1$ then

$$a(a+1)(a+2) = (3q+1)(3q+2)(3q+3)$$
$$= (2r)(3)(q+1)$$
$$= 6r(q+1)$$

which is divisible by 6.

Case III : $a = 3q+2$

If $a = 3q+2$ then

$$a(a+1)(a+2) = (3q+2)(3q+3)(3q+4)$$
$$= 3(3q+2)(q+1)(3q+4)$$

Here $(3q+2)$ and $= 3(3q+2)(q+1)(3q+4)$

$=$ multiple of 6 every q

$= 6r$ (say)

which is divisible by 6. Hence, the product of three consecutive integers is divisible by 6 and $n^3 - n$ is also divisible by 3.

or

Prove that $\sqrt{3}$ is an irrational number. Hence, show that $7+2\sqrt{3}$ is also an irrational number.

Ans :

Assume that $\sqrt{3}$ be a rational number then we have

$$\sqrt{3} = \frac{a}{b}, \qquad (a, b \text{ are co-primes and } b \neq 0)$$

$$a = b\sqrt{3}$$

Squaring both the sides, we have

$$a^2 = 3b^2$$

Thus 3 is a factor of a^2 and in result 3 is also a factor of a.

Let $a = 3c$ where c is some integer, then we have

$$a^2 = 9c^2$$

Substituting $a^2 = 9b^2$ we have

$$3b^2 = 9c^2$$

$$b^2 = 3c^2$$

Thus 3 is a factor of b^2 and in result 3 is also a factor of b.

Thus 3 is a common factor of a and b. But this contradicts the fact that a and b are co-primes. Thus, our assumption that $\sqrt{3}$ is rational number is wrong. Hence $\sqrt{3}$ is irrational.

Let us assume that $7+2\sqrt{3}$ be rational equal to a, then we have

$$7+2\sqrt{3} = \frac{p}{q} \quad q \neq 0 \text{ and } p \text{ and } q \text{ are co-primes}$$

$$2\sqrt{3} = \frac{p}{q} - 7 = \frac{p-7q}{q}$$

or $$\sqrt{3} = \frac{p-7q}{2q}$$

Here $p-7q$ and $2q$ both are integers, hence $\sqrt{3}$ should be a rational number. But this contradicts the fact that $\sqrt{3}$ is an irrational number. Hence our assumption is not correct and $7+2\sqrt{3}$ is irrational.

36. Solve for x : $\left(\frac{2x}{x-5}\right)^2 + \left(\frac{2x}{x-5}\right) - 24 = 0, x \neq 5$ [4]

Ans :

We have $$\left(\frac{2x}{x-5}\right)^2 + 5\left(\frac{2x}{x-5}\right) - 24 = 0$$

Let $\frac{2x}{x-5} = y$ then we have

$$y^2 + 5y - 24 = 0$$

$$(y+8)(y-3) = 0$$

$$y = 3, -8$$

Taking $y = 3$ we have

$$\frac{2x}{x-5} = 3$$

$$2x = 3x - 15$$

$$x = 15$$

Taking $y = -8$ we have

$$\frac{2x}{x-5} = -8$$

$$2x = -8x + 40$$

$$10x = 40$$

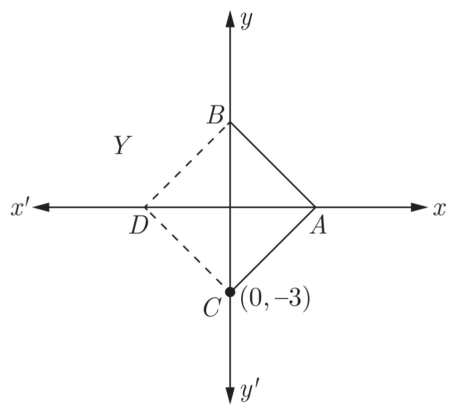
$$x = 4$$

Hence, $x = 15, 4$

37. The base BC of an equilateral triangle ABC lies on y-axis. The co-ordinates of point C are $(0,3)$. The origin is the mid-point of the base. Find the co-ordinates of the point A and B. Also find the co-ordinates of another point D such that $BACD$ is a rhombus. [4]

Ans :

As per question, diagram of rhombus is shown below.



Co-ordinates of point B are $(0,3)$

Thus $BC = 6$ unit

Let the co-ordinates of point A be $(x, 0)$

Now $AB = \sqrt{x^2+9}$

Since $AB = BC$, thus

$$x^2 + 9 = 36$$

$$x^2 = 27 \Rightarrow x = \pm 3\sqrt{3}$$

Co-ordinates of point A is $(3\sqrt{3}, 0)$

Since $ABCD$ is a rhombus

$$AB = AC = CD = DB$$

Thus co-ordinate of point D is $(-3\sqrt{3}, 0)$

or

Prove that the area of a triangle with vertices $(t, t-2), (t+2, t+2)$ and $(t+3)$ is independent of t.

Ans :

Area of the triangle

$$\Delta = \frac{1}{2} | t(t+2-t) + (t+2)(t-t+2) + $$
$$+ (t+3)(t-2-t-2) |$$

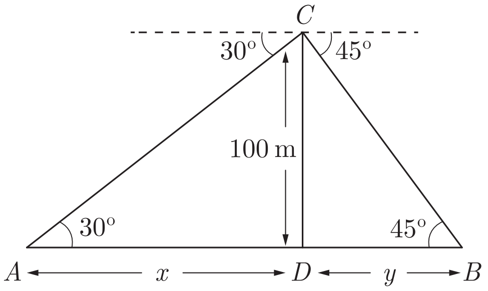
$$= \frac{1}{2}[2t + 2t + 4 - 4t - 12]$$

$= 4$ sq. units. which is independent of t.

38. From the top of tower, 100 m high, a man observes two cars on the opposite sides of the tower with the angles of depression $30°$ & $45°$ respectively. Find the distance between the cars. (Use $\sqrt{3} = 1.73$) [4]

Ans :

Let DC be tower of height 100 m. A and B be two car on the opposite side of tower. As per given in question we have drawn figure below.



In right ΔADC,

$$\tan 30^\circ = \frac{CD}{AD}$$

$$\frac{1}{\sqrt{3}} = \frac{100}{x}$$

$$x = 100\sqrt{3} \qquad ...(1)$$

In right ΔBDC,

$$\tan 45^\circ = \frac{CD}{DB}$$

$$1 = \frac{100}{y}$$

$$\Rightarrow \qquad y = 100 \text{ m}$$

Distance between two cars

$$AB = AD + DB = (100\sqrt{3} + 100)$$

$$= (100 \times 1.73 + 100) \text{ m}$$

$$= (173 + 100) \text{ m}$$

$$= 273 \text{ m}$$

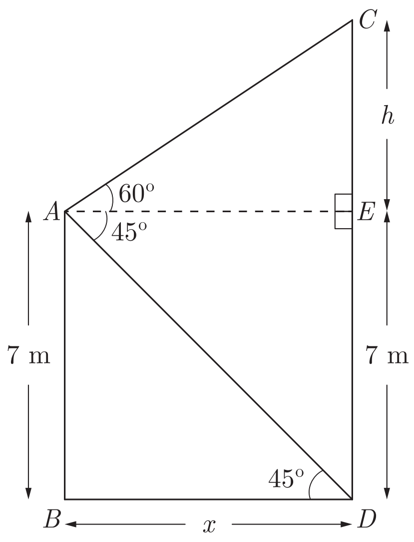
Hence, distance between two cars is 273 m.

or

From the top of a 7 m high building, the angle of elevation of the top of a tower is 60° and the angle of depression of its foot is 45°. Find the height of the tower. (Use $\sqrt{3} = 1.732$)

Ans :

Let AB be the building of height 7 m and CD be the tower of height h. Angle of depressions of top and bottom are given 30° and 60° respectively. As per given in question we have drawn figure below.



Here $\angle CBD = \angle ECB = 45^\circ$ due to alternate angles.

In right ΔABC we have

$$\frac{CD}{BD} = \tan 45^\circ$$

$$\frac{7}{x} = 1$$

$$x = 7$$

In right ΔAEC we have

$$\frac{CE}{AE} = \tan 60^\circ$$

$$\frac{h-7}{x} = \sqrt{3}$$

$$h - 7 = x\sqrt{3}$$

$$h - 7 = 7\sqrt{3}$$

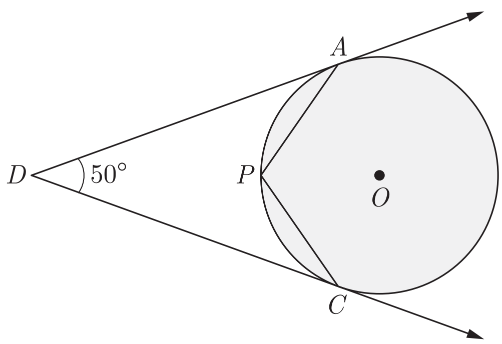
$$h = 7\sqrt{3} + 7$$

$$= 7(\sqrt{3} + 1)$$

$$= 7(1.732 + 1)$$

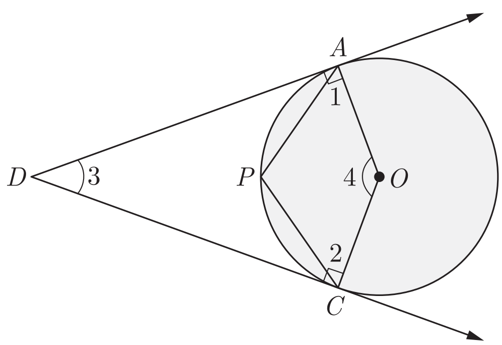
Hence, height of tower = 19.124 m

39. In the given figure, O is the centre of the circle. Determine $\angle APC$, if DA and DC are tangents and $\angle ADC = 50^\circ$. [4]



Ans :

We redraw the given figure by joining A and C to O as shown below.



Since DA and DC are tangents from point D to the circle with centre O, and radius is always perpendicular to tangent, thus

$$\angle DAO = \angle DCO = 90^\circ$$

and

$$\angle ADC + \angle DAO + \angle DCO + \angle AOC = 360^\circ$$

$$50^\circ + 90^\circ + 90^\circ + \angle AOC = 360^\circ$$

$$230^\circ + \angle AOC = 360^\circ$$

$$\angle AOC = 360^\circ - 230^\circ = 130^\circ$$

Now Reflex $\angle AOC = 360^\circ - 130^\circ = 230^\circ$

$$\angle APC = \frac{1}{2} \text{ reflex } \angle AOC$$

(angle subtended at centre...)

$$\angle APC = \frac{1}{2} \times 230^\circ = 115^\circ$$

40. The following distribution gives the weights of 60 students of a class. Find the mean and mode weights of the students. [4]

Weight (in kg)	40-44	44-48	48-52	52-56	56-60	60-64	64-68	68-72
Number of students	4	6	10	14	10	8	6	2

Ans :

C.I.	x_i	f_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
40-44	42	4	-3	-12
44-48	46	6	-2	-12
48-52	50	10	-1	-10
52-56	54	14	0	0
56-60	58	10	1	10
60-64	62	8	2	16
64-68	66	6	3	18
68-72	70	2	4	8
		$\sum f_i = 60$		$\sum f_i u_i = 18$

Let a = Assumed mean = 54

$$\text{Mean, } \overline{x} = a + \frac{\sum f_i u_i}{\sum f_i} \times h$$

$$\text{Mean} = 54 + \frac{18}{60} \times 4 = 55.2$$

Maximum frequency = 14

$\Rightarrow$ Modal class $= 52 - 56,\ l = 52,\ f_1 = 14,$

$f_0 = 10,\ f_2 = 10,\ h = 4$

$$\text{Mode} = 52 + \frac{14 - 10}{28 - 10 - 10} \times 4 = 54$$

Hence, Mean = 55.2 and Mode = 54