CLASS X (2019-20)
MATHEMATICS BASIC(241)
SAMPLE PAPER-3

Time : 3 Hours **Maximum Marks : 80**

General Instructions :

(i) All questions are compulsory.
(ii) The questions paper consists of 40 questions divided into four sections A, B, C and D.
(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.
(iv) There is no overall choice. However, an internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

Section A

Q.1-Q.10 are multiple choice questions. Select the most appropriate answer from the given options.

1. Ratio of volumes of two cylinders with equal height is [1]

(a) $H:h$ (b) $R:r$
(c) $R^2:r^2$ (d) None of these

Ans : (c) $R^2:r^2$

$$\pi R^2h:\pi r^2h = R^2:r^2$$

2. Which of the following statement is false? [1]

(a) All isosceles triangles are similar.
(b) All quadrilateral triangles are similar.
(c) All circles are similar.
(d) None of the above

Ans : (a) All isosceles triangles are similar.

An isosceles triangle is a triangle with two side of equal length hence statement given in option (a) is false.

3. C is the mid-point of PQ, if P is $(4,x)$, C is $(y,-1)$ and Q is $(-2,4)$, then x and y respectively are [1]

(a) -6 and 1 (b) -6 and 2
(c) 6 and -1 (d) 6 and -2

Ans : (a) -6 and 1

Since, $C(y,-1)$ is the mid-point of $P(4,x)$ and $Q(-2,4)$.

We have, $\dfrac{4-2}{2} = y$ ...(1)

and $\dfrac{4+x}{2} = -1$ ...(2)

From equation (1) and (2), we get

$y = 1$

and $x = -6$

4. An equation of the circle with centre at (0, 0) and radius r is [1]

(a) $x^2+y^2=r^2$ (b) $x^2-y^2=r^2$
(c) $x-y=r$ (d) $x^2+r^2=y^2$

Ans : (a) $x^2+y^2=r^2$

Here, $h=k=0$. Therefore, the equation of the circle is $x^2+y^2=r^2$.

5. The ratio of the sides of the triangle to be constructed with the corresponding sides of the given triangle is known as [1]

(a) scale factors (b) length factor
(c) side factor (d) K-factor

Ans : (a) scale factors

The ratio of the sides of the triangle to be constructed with the corresponding sides of the given triangle is known as scale factor.

6. (i) The L.C.M. of x and 18 is 36.
(ii) The H.C.F. of x and 18 is 2.
What is the number x? [1]

(a) 1 (b) 2
(c) 3 (d) 4

Ans : (d) 4

L.C.M. $\times$ H.C.F. = First number $\times$ second number

Hence, required number $= \dfrac{36\times 2}{18} = 4$

7. The linear factors of the quadratic equation $x^2+kx+1=0$ are [1]

(a) $k\geq 2$ (b) $k\leq 2$
(c) $k\geq -2$ (d) $2\leq k\leq -2$

Ans : (d) $2\leq k\leq -2$

We have, $x^2+kx+1 = 0$

On comparing with $ax^2+bx+c = 0$,

we get $a = 1$, $b=k$ and $c=1$

For linear factors, $D \geq 0$

$$b^2-4ac \geq 0$$
$$k^2-4\times 1\times 1 \geq 0$$
$$(k^2-2^2) \geq 0$$
$$(k-2)(k+2) \geq 0$$
$$k \geq 2 \text{ and } k\leq -2$$

8. In a number of two digits, unit's digit is twice the tens digit. If 36 be added to the number, the digits are reversed. The number is [1]

(a) 36 (b) 63

(c) 48 (d) 84

Ans : (c) 48

Let unit's digit : x
tens digit : y

Then, $x = 2y$

Number $= 10y + x$

According to the question.

$$10y + x + 36 = 10x + y$$

$$9x - 9y = 36$$

or $x - y = 4$ ....(1)

Solve, $x = 2y$

$$2y - y = 4$$

$$y = 4$$

Now, from equation,

$$x - 4 = 4 \Rightarrow x = 8$$

$$\text{Number} = 10 \times 4 + 8 = 40 + 8 = 48$$

$$x - y = 4$$

9. An AP starts with a positive fraction and every alternate term is an integer. If the sum of the first 11 terms is 33, then the fourth term is [1]

(a) 2 (b) 3
(c) 5 (d) 6

Ans : (a) 2

Given, $S_{11} = 33$

$$\frac{11}{2}[2a + 10d] = 33 \Rightarrow a + 5d = 3$$

i.e., $a_6 = 3 \Rightarrow a_4 = 2$

[Since, Alternate terms are integers and the given sum is possible]

10. If $\tan 2A = \cot(A - 18^\circ)$, where $2A$ is an acute angle, then the value of A is [1]

(a) 12° (b) 18°
(c) 36° (d) 48°

Ans : (c) 36°

Given, $\tan 2A = \cot(A - 18^\circ)$

$$\cot(90^\circ - 2A) = \cot(A - 18^\circ)$$

$$90^\circ - 2A = A - 18^\circ$$

[since, $(90^\circ - 2A)$ and $(A - 18^\circ)$ both are acute angles]

$$90^\circ + 18^\circ = A + 2A$$

$$3A = 108^\circ$$

$$A = \frac{108^\circ}{3} = 36^\circ$$

(Q.11-Q.15) Fill in the blanks.

11. Area of a circle is .... [1]

Ans : πr^2

12. .... equation is valid for all values of its variables. [1]

Ans : Identity

or

The highest power of a variable in a polynomial is called its ....

Ans : Degree

13. Someone is asked to make a number from 1 to 100. The probability that it is a prime is .... [1]

Ans : $\frac{1}{4}$

14. If p is a prime number and it divides a^2 then it also divides ...., where a is a positive integer. [1]

Ans : a

15. The volume and surface area of a sphere are numerically equal, then the radius of sphere is .... units. [1]

Ans : 3

(Q.16-Q.20) Answer the following

16. Volume and surface area of a solid hemisphere are equal. What is the diameter of hemisphere ? [1]

Ans :

Let radius of sphere be r.

Given, volume of sphere = S.A. of hemisphere

$$\frac{2}{3}\pi r^3 = 3\pi r^2$$

$$r = \frac{9}{2} \text{ units}$$

Diameter $d = \frac{9}{2} \times 2 = 9$ units

or

Find the number of solid sphere of diameter 6 cm can be made by melting a solid metallic cylinder of height 45 cm and diameter 4 cm.

Ans :

Let the number of sphere $= n$

Radius of sphere $= 3$ cm,

Radius of cylinder $= 2$ cm

Volume of spheres = Volume of cylinder

$$n \times \frac{4}{3}\pi r^3 = \pi r_1^2 h$$

$$n \times \frac{4}{3} \times \frac{22}{7} \times (3)^3 = \frac{22}{7} \times (2)^2 \times 45$$

$$36n = 180$$

$$n = \frac{180}{36} = 5$$

Number of solid sphere = 5.

17. Find the value (s) of k if the quadratic equation $3x^2 - k\sqrt{3}x + 4 = 0$ has real roots. [1]

Ans :

If discriminant of quadratic equation is equal to zero, or more than zero, then roots are real.

We have $3x^2 - k\sqrt{3}x + 4 = 0$

Compare with $ax^2 + bx + c = 0$

$$D = b^2 - 4ac$$

For real roots $b^2 - 4ac \geq 0$

$$(-k\sqrt{3})^2 - 4 \times 3 \times 4 \geq 0$$

$$3k^2 - 48 \geq 0$$

$$k^2 - 16 \geq 0$$

$$(k-4)(k+4) \geq 0$$

Thus $k \leq -4$ and $k \geq 4$

18. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find area of minor segment. $(Use\ \pi = 3.14)$ [1]

Ans :

Given,
Radius of circle $r = 10$ cm, central angle $= 90^\circ$
Area of minor segment

$$= \frac{1}{2} \times 10^2 \times \left[\frac{3.14 \times 90}{180} - \sin 90^\circ\right]$$

$$= \frac{1}{2} \times 100 \times [1.57 - 1] = 28.5 \text{ cm}^2$$

19. What is abscissa of the point of intersection of the "Less than type" and of the "More than type" cumulative frequency curve of a grouped data ? [1]

Ans :

The abscissa of the point of intersection of the "Less than type" and "More than type" cumulative frequency curve of a grouped data is median.

20. A dice is thrown once. Find the probability of getting a prime number. [1]

Ans :

Total outcomes $= 6$

Prime numbers $= 2, 3, 5 = 3$

P (prime no.) $= \frac{3}{6} = \frac{1}{2}$

Section B

21. Solve the following system of linear equations by substitution method: [2]

$$2x - y = 2$$

$$x + 3y = 15$$

Ans :

We have $2x - y = 2$ ...(1)

$x + 3y = 15$ ...(2)

From equation (1), we get $y = 2x - 2$ ...(3)

Substituting the value of y in equation (2),

$$x + 6x - 6 = 15$$

or, $7x = 21$

$$x = 3$$

Substituting this value of x in (3), we get

From equation (1), we have

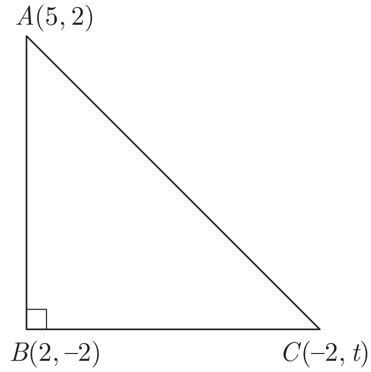
$$y = 2 \times 3 - 2 = 4$$

$$x = 3 \text{ and } y = 4$$

22. If $A(5,2)$, $B(2,-2)$ and $C(-2,t)$ are the vertices of a right angled triangle with $\angle B = 90^o$, then find the value of t. [2]

Ans :

As per question, triangle is shown below.



Now $AB^2 = (2-5)^2 + (-2-2)^2 = 9 + 16 = 25$

$BC^2 = (-2-2)^2 + (t+2)^2 = 16 + (t+2)^2$

$AC^2 = (5+2)^2 + (2-t)^2 = 49 + (2+t^2)$

Since ΔABC is a right angled triangle

$$AC^2 = AB^2 + BC^2$$

$$49 + (2-t)^2 = 25 + 16 + (t+2)^2$$

$$49 + 4 - 4t + t^2 = 41 + t^2 + 4t + 4$$

$$53 - 4t = 45 + 4t$$

$$8t = 8$$

$$t = 1$$

or

For what values of k are the points $(8,1), (3,-2k)$ and $(k,-5)$ collinear?

Ans :

Since points $(8,1), (3,-2k)$ and $(k,-5)$ are collinear, area of triangle formed must be zero.

$$\frac{1}{2}\left[8(-2k+5) + 3(-5,-1) + k(1+2k)\right] = 0$$

$$2k^2 - 15k + 22 = 0$$

$$k = 2, \frac{11}{2}$$

23. Let $\Delta ABC \sim \Delta DEF$. if $ar(\Delta ABC) = 100$ cm^2, $ar(DEF) = 196$ cm^2 and $DE = 7$, then find AB. [2]

Ans :

We have $\Delta ABC \sim \Delta DEF$, thus

$$\frac{ar(\Delta ABC)}{are(\Delta DEF)} = \frac{AB^2}{DE^2}$$

$$\frac{100}{196} = \frac{AB^2}{(7)^2}$$

$$\frac{100}{196} = \frac{AB^2}{49}$$

$$AB^2 = \frac{49 \times 100}{196}$$

$$AB^2 = 25$$

$$AB = 5 \text{ cm}$$

24. Write the relationship connecting three measures of central tendencies. Hence find the median of the give data if mode is 24.5 and mean is 29.75. [2]

Ans :

Given, Modal $= 24.5$

and Mean $= 29.75$

The relationship connecting measures of central tendencies is :

$$3\text{ Median} = \text{Mode} + 2\text{ Mean}$$

$$3\text{ Median} = 24.5 + 2 \times 29.75$$

$$= 24.5 + 59.50$$

$$3\text{ Median} = 84.0$$

$$\therefore \quad \text{Median} = \frac{84}{3} = 28$$

or

A bag contains cards bearing numbers from 11 to 30. A card is taken out from the bag at random. Find the probability that the selected card has multiple of 5 on it.

Ans :

Given, Number of cards = 20

Multiples of 5 from 11 to 30 are 15, 20, 25, 30

Number of favourable outcomes = 4

$$\text{Required probability } = \frac{4}{20} = \frac{1}{5}$$

25. In what ratio does the point $P(-4,6)$ divides the line segment joining the points $A(-6,10)$ and $B(3,-8)$? [2]

Ans :

Let $AP{:}PB = k{:}1$

Now $\frac{3k-6}{k+1} = -4$

$$3k-6 = -4k-4$$

$$7k = 2$$

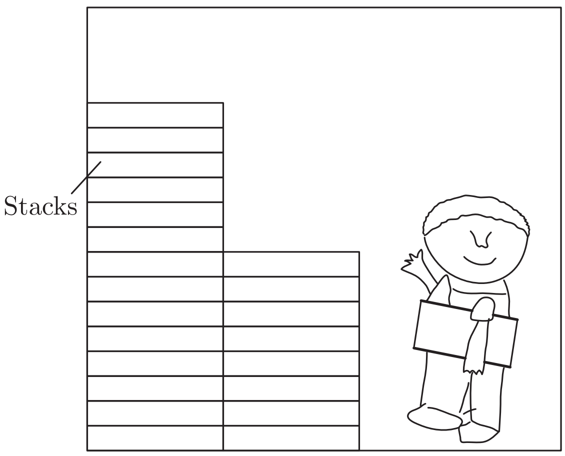
$$k = \frac{7}{2}$$

Hence, $AP{:}PB = 7{:}2$

26. Read the following passage and answer the question that follows.

A book seller has 420 science stream books and 130 Arts stream books. He wants to stack them in such a way that each stack has the same number and they take up the least area of the surface. [2]

Book shop



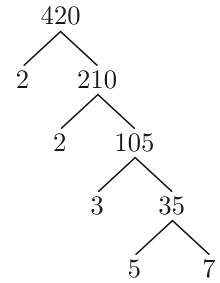
(i) What is the maximum number of books that can be placed in each stack for this purpose?

(ii) Which mathematical concept is used to solve the problems?

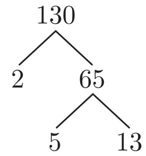
Ans :

(i) Given, number of science books = 420 and number of Arts books = 130

$$420 = 2 \times 2 \times 3 \times 5 \times 7$$



$$130 = 2 \times 5 \times 13$$



Maximum number of books that can be placed in each stack for the given purpose

$$= \text{HCF}(420, 130)$$

$$= 2^1 \times 5^1 = 10$$

(ii) Prime factorisation method.

Section C

27. Solve for x and y : [3]

$$\frac{x}{2} + \frac{2y}{3} = -1$$

$$x - \frac{y}{3} = 3$$

Ans :

We have $\frac{x}{2} + \frac{2y}{3} = -1$

or $3x + 4y = -6$ ...(1)

and $\frac{x}{1} - \frac{y}{3} = 3$

or $3x + y = 9$ ...(2)

Subtracting equation (2) from equation (1), we have

$$5y = -15 \Rightarrow y = -1$$

Substituting $y = -3$ in eq (1), we get

$$3x + 4(-3) = -6$$

$$3x - 12 = -6$$

$$3x = 12 - 6$$

Thus $x = 2$

Hence $x = -2$ and $y = -3$.

28. Find the zeroes of the quadratic polynomial $x^2 - 2\sqrt{2}\,x$ and verify the relationship between the zeroes and the coefficients. [3]

Ans :

We have $x^2 - 2\sqrt{2}\,x = 0$

$$x(x - 2\sqrt{2}) = 0$$

Thus zeroes are 0 and $2\sqrt{2}$.

Sum of zeroes $2\sqrt{2} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

and product of zeroes $0 = \frac{\text{Constan term}}{\text{Coefficient of } x^2}$

Hence verified

or

What should be added to x^3+5x^2+7x+3 so that it is completely divisible by x^2+2x.

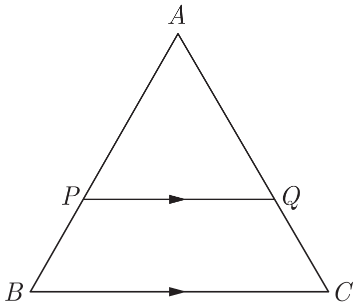
Ans :

$$\begin{array}{r|l} & x+3 \\ x^2+2x & x^3+5x^2+7x+3 \\ & x^3+2x^2 \\ \hline & 3x^2+7x+3 \\ & 3x^2+6x \\ \hline & x+3 \end{array}$$

29. ABC is a triangle, PQ is the line segment intersecting AB in P and AC in Q such that $PQ||BC$ and divides ΔABC into two parts, equal in area, find $BP{:}AB$, [3]

Ans :

As per given condition we have drawn the figure below.



Here, Since $PQ||BC$ and PQ divides ΔABC into two equal parts, thus $\Delta APQ \sim \Delta ABC$

Now $$\frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{AP^2}{AB^2}$$

$$\frac{1}{2} = \frac{AP^2}{AB^2}$$

$$\frac{1}{\sqrt{2}} = \frac{AP}{AB}$$

$$\frac{1}{\sqrt{2}} = \frac{AB-BP}{AB} \quad (AB = AP+BP)$$

$$\frac{1}{\sqrt{2}} = 1-\frac{BP}{AB}$$

$$\frac{BP}{AB} = 1-\frac{1}{\sqrt{2}} = \frac{\sqrt{2}-1}{\sqrt{2}}$$

$$BP:AB = (\sqrt{2}-1):\sqrt{2}$$

30. For what value of n, are the n^{th} terms of two A.Ps 63, 65, 67, ... and 3, 10, 17, ... equal? [3]

Ans :

Let a, d and A, D be the 1^{st} term and common difference of the 2 APs respectively.
n is same

For 1st AP, $a = 63,\ d = 2$

For 2nd AP, $A = 3, D = 7$

Since nth term is same,

$$an = An$$

$$a+(n-1)d = A+(n-1)D$$

$$63+(n-1)2 = 3+(n-1)7$$

$$63+2n-2 = 3+7n-7$$

$$61+2n = 7n-4$$

$$65 = 5n \Rightarrow n = 13$$

When n is 13, the n^{th} terms are equal i.e., $a_{13} = A_{13}$

or

In an A.P., if the 12^{th} term is -13 and the sum of its first four terms is 24, find the sum of its first ten terms.

Ans :

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$a_{12} = a+11d = -13 \qquad ...(1)$$

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

Now $$S_4 = 2[2a+3d] = 24$$

$$2a+3d = 12 \qquad ...(2)$$

Multiplying (1) by 2 and subtracting (2) from it we get

$$(2a+22d)-(2a+3d) = -26-12$$

$$19d = -38$$

$$d = -2$$

Substituting the value of d in (1) we get

$$a+11\times-2 = -13$$

$$a = -13+22$$

$$a = 9$$

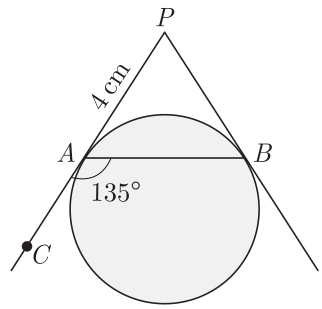
Now, $$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$S_{10} = \frac{10}{2}(2\times 9+9\times -2)$$

$$= 5\times(18-18) = 0$$

Hence, $S_{10} = 0$

31. In the given figure, PA and PB are tangents to a circle from an external point P such that $PA = 4cm$ and $\angle BAC = 135^o$. Find the length of chord AB. [3]



Ans :

Since length of tangents from an external point to a circle are equal,

$$PA = PB = 4 \text{ cm}$$

Here $\angle PAB$ and $\angle BAC$ are supplementary angles,

$$\angle PAB = 180^\circ - 135^\circ = 45^\circ$$

Angle $\angle ABP$ and $= \angle PAB = 45^\circ$ opposite angles of equal sides, thus

$$\angle ABP = \angle PAB = 45^\circ$$

In triangle ΔAPB, we have

$$\angle APB = 180^\circ - \angle ABP - \angle BAP$$

$$= 180^\circ - 45^\circ - 45^\circ = 90^\circ$$

Thus ΔAPB is a isosceles right angled triangle

Now $\quad AB^2 = AP^2 + BP^2 = 2AP^2$

$$= 2 \times 4^2 = 32$$

Hence $\quad AB = \sqrt{32} = 4\sqrt{2}$ cm

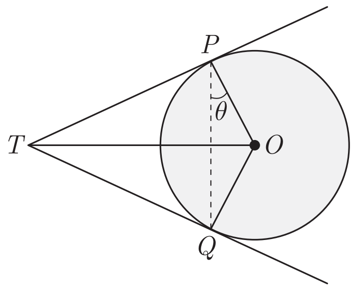
or

Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that

$$\angle PTO = \angle OPQ$$

Ans :

As per question we draw figure shown below.



Let $\angle TPQ$ be θ. the tangent is perpendicular to the end point of radius,

$$\angle TPO = 90^o$$

Now $\quad \angle TPQ = \angle TPO - \theta = (90^o - \theta)$

Since, $TP = TQ$ and opposite angels of equal sides are always equal, we have

$$\angle TQP = (90^o - \theta)$$

Now in ΔTPQ we have

$$\angle TPQ + \angle TQP + \angle PTQ = 180^o$$

$$90^o - \theta + 90^o - \theta + \angle PTQ = 180^o$$

$$\angle PTQ = 180^o - 180^o + 2\theta = 2\theta$$

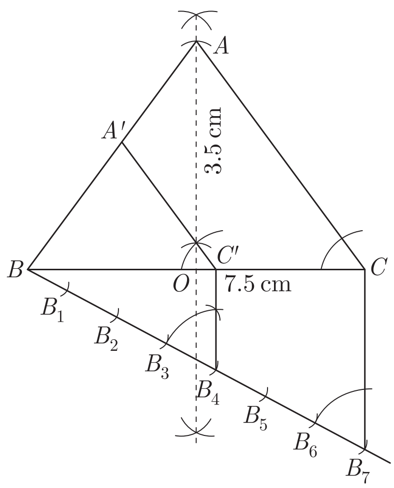
Hence $\angle PTQ = 2\angle OPQ$.

32. Construct an isosceles triangle whose base is 7.5 cm and altitude 3.5 cm then another triangle whose sides are $\frac{4}{7}$ times the corresponding sides of the isosceles triangle. [3]

Ans :

Steps of construction :

1. Draw a line $BC = 7.5$ cm.
2. Draw a perpendicular bisector of BC which intersects the line BC at O.
3. Cut the line $OA = 3.5$ cm.



4. Join A to B and A to C.
5. Draw a ray BX making an acute angle with BC.
6. Locate 7 points at equal distance among $B_1, B_2, ...B_7$ on line segment BX.
7. Join B_7C. Draw a parallel line through B_4 to B_7C intersecting line segment BC at C'.
8. Through C' draw a line parallel to AC intersecting line segment AB at A'.
9. Hence, $\Delta A'BC'$ is a required triangle.

33. Read the following passage and answer the question that follows.

Roja, Renu and Reena are three friends. They decided to sweep a circular park near their homes. They divided the park into three parts by two equal chords AB and AC for convenience. [3]

(i) Prove that the centre of the park lies on the angle bisector of $\angle BAC$.

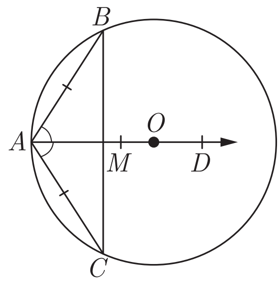
(ii) Which mathematical concept is used in the above problem?

Ans :

(i) Given : A circle $C(O, r)$ and chord $AB =$ chord AC . AD is bisector of $\angle CAB$.

To prove : Centre O lies on the bisector of $\angle BAC$.

Construction: Join BC, meeting bisector AD of $\angle BAC$, at M.



Proof : In triangles BAM and CAM,

$$AB = AC \quad \text{(given)}$$

$$\angle BAM = \angle CAM \quad \text{(given)}$$

and $\quad AM = AM \quad$ (common)

$$\Delta BAM \cong \Delta CAM \quad \text{(SAS)}$$

$$BM = CM$$

and $\quad \angle BMA = \angle CMA$

As $\angle BMA + \angle CMA = 180^\circ \quad$ (linear pair)

$$\angle BMA = \angle CMA = 90^\circ$$

AM is the perpendicular bisector of the chord BC.

AM passes through the centre O.

[Perpendicular bisector of chord of a circle passes through the centre of the circle]

Hence, the centre of the park lies on the angle bisector of $\angle BAC$.

(ii) Congruency of triangles by SAS axiom (Geometry).

34. Determine the values of m and n so that the following system of linear equation have infinite number of solutions : [3]

$$(2m-1)x + 3y - 5 = 0$$

$$3x + (n-1)y - 2 = 0$$

Ans :

We have $\quad (2m-1)x + 3y - 5 = 0 \quad$ ...(1)

Here $a_1 = 2m-1, b_1 = 3, c_1 = -5$

$$3x+(n-1)y-2 = 0 \quad ...(2)$$

Here $a_2 = 3, b_2 = (n-1), c_2 = -2$

For a pair of linear equations to have infinite number of solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2m-1}{3} = \frac{3}{n-1} = \frac{5}{2}$$

or $\quad 2(2m-1) = 15$ and $5(n-1) = 6$

Hence, $\quad m = \frac{17}{4}, n = \frac{11}{5}$

Section D

35. The denominator of a fraction is two more than its numerator. If the sum of the fraction and its reciprocal is $\frac{34}{15}$, find the fraction. [4]

Ans :

Let numerator be x, then denominator will be $x+2$.

and $\quad$ fraction $= \frac{x}{x+2}$

Now $\quad \frac{x}{x+2} + \frac{x+2}{x} = \frac{34}{15}$

$$15(x^2+x^2+4x+4) = 34(x^2+2x)$$

$$30x^2+60x+60 = 34x^2+68x$$

$$4x^2+8x-60 = 0$$

$$x^2+2x-15 = 0$$

$$x^2+5x-3x-15 = 0$$

$$x(x+5)-3(x+5) = 0$$

$$(x+5)(x-3) = 0$$

We reject the $x = -5$. Thus $x = 3$ and fraction $= \frac{3}{5}$

36. Show that the square of any positive integer is of the forms $4m$ or $4m+1$, where m is any integer. [4]

Ans :

Let a be any positive integer, then by Euclid's division algorithm a can be written as

$$a = bq+r$$

Take $b = 4$, then $0 \le r < 4$ because $0 \le r < b$,

Thus $\quad a = 4q, 4q+1, 4q+2, 4q+3$

Case 1 : $a = 4q$

$$a^2 = (4q)^2 = 16q^2 = 4(4q^2)$$

$$= 4m \quad \text{where } m = 4q^2$$

Case 2 : $a = 4q+1$

$$a^2 = (4q+1)^2 = 16q^2+8q+1$$

$$= 4(4q^2+2q)+1$$

$$= 4m+1 \quad \text{where } m = 4q^2+2q$$

Case 3 : $a = 4q+2$

$$a^2 = (4q+2)^2 = 16q^2+16q+4$$

$$= 4(4q^2+4q+1)$$

$$= 4m \quad \text{where } m = 4q^2+4q+1$$

Case 4 : $a^2 = (4q+3)^2 = 16q^2+24q+9$

$$= 16q^2+24q+8+1$$

$$= 4(4q^2+6q+2)+1$$

$$= 4m+1 \quad \text{where } m = 4q^2+6q+2$$

From cases 1, 2, 3 and 4 we conclude that the square of any +ve integer is of the form $4m$ or $4m+1$.

or

Express the HCF/LCM of 48 and 18 as a linear combination.

Ans :

Using Euclid's Division Lemma, we have

$$48 = 18\times2+12 \quad (1)$$

$$18 = 12\times1+6 \quad (2)$$

$$12 = 6\times2+0$$

Thus $\quad \text{HCF}(18,48) = 6$

Now $\quad 6 = 18-12\times1 \quad$ From (2)

$$6 = 18-(48-18\times2) \quad \text{From (1)}$$

$$6 = 18-48\times1+18\times2$$

$$6 = 18\times(2+1)-48\times1 = 18\times3-48\times1$$

$$6 = 18\times3+48\times(-1)$$

Thus $\quad 6 = 18x+48y, \quad$ where $x = 3, y = -1$

Here x and y are not unique.

$$6 = 18\times3+48\times(-1)$$

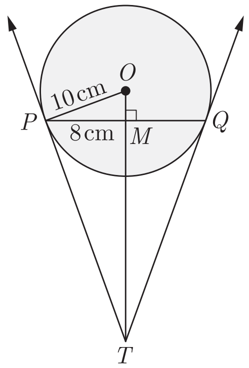
$$= 18\times3+48\times(-1)+18\times48-18\times48$$

$$= 18(3+48)+48(-1-18)$$

$$= 18\times51+48\times(-19)$$

$6 = 18x+48y \quad$ where $x = 51, y = -19$

37. In figure, PQ, is a chord of length 16 cm, of a circle of radius 10 cm. the tangents at P and Q intersect at a point T. Find the length of TP. [4]



Ans :

Here PQ is chord of circle and OM will be perpendicular on it and it bisect PQ. Thus ΔOMP is a right angled triangle.

We have $\quad OP = 10$ cm $\quad$ (Radius)

$\quad PM = 8$ cm $\quad (PQ = 16$ cm$)$

Now in $\Delta OMP, OM = \sqrt{10^2-8^2} = \sqrt{100-64}$

$$= \sqrt{36} = 6 \text{ cm}$$

Now $\quad \angle TPM + \angle MPO = 90^\circ$

Also, $\quad \angle TPM + \angle PTM = 90^\circ$

$$\angle MPO = \angle PTM$$

$$\angle TMP = \angle OMP = 90^\circ$$

$$\Delta TMP \sim \Delta PMO (AA)$$

or, $\quad \frac{TP}{PO} = \frac{MP}{MO}$

$$\frac{TP}{10} = \frac{8}{6}$$

$$TP = \frac{80}{6} = \frac{40}{3}$$

Hence length of TP is $\frac{40}{3}$ cm.

38. Find the values of k so that the area of the triangle with vertices $(k+1,1),(4,-3)$ and $(7,-k)$ is 6 sq. units. [4]

Ans :

We have $(k+1,1),(4,-3)$ and $(7,-k)$
Area of triangle

$$\Delta = \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$$

$$6 = \frac{1}{2}[(k+1)(-3+k)+4(-k-1)+7(1+3)]$$

$$12 = [k^2-2k-3-4k-4+28] = k^2-6k+21$$

$$k^2-6k+9 = 0$$

$$k^2-3k-3k+9 = 0$$

$$k(k-3)-3(k-3) = 0$$

$$(k-3)(k-3) = 0$$

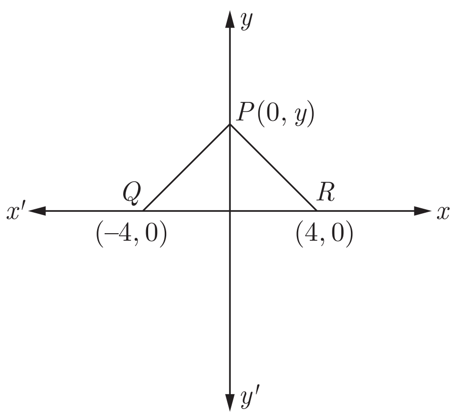
$$k = 3,3$$

or

The base QR of an equilateral triangle PQR lies on x-axis. The co-ordinates of point Q are $(-4,0)$ and the origin is the mid-point of the base. find the co-ordinates of the point P and R.

Ans :

As per question, line diagram is shown below.



Co-ordinates of point R is $(4,0)$

Thus $QR = 8$ units

Let the co-ordinates of point P be $(0,y)$

Since $PQ = QR$

$$(-4-0)^2+(0-y)^2 = 64$$

$$16+y^2 = 64$$

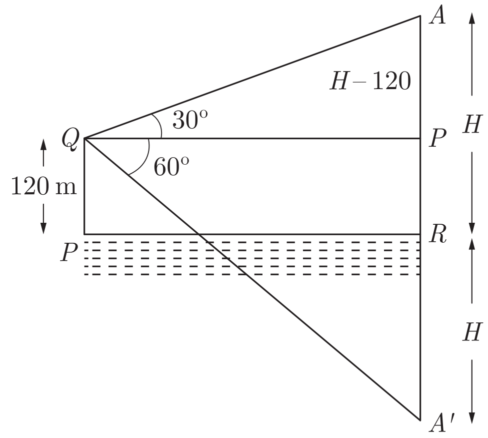
$$y = \pm 4\sqrt{3}$$

Coordinates of P are $(0,4\sqrt{3})$ or $(0,-4\sqrt{3})$

39. The angle of elevation of a cloud from a point 120 m above a lake is 30° and the angle of depression of its reflection in the lake is 60°. Find the height of the cloud. [4]

Ans :

As per given in question we have drawn figure below.



Here, A is cloud and A' is refection of cloud.
In right $\Delta\, AOP$ we have

$$\tan 30^\circ = \frac{H-120}{OP}$$

$$\frac{1}{\sqrt{3}} = \frac{H-120}{OP}$$

$$OP = (H-120)\sqrt{3} \qquad ...(1)$$

In right $\Delta\, OPA'$ we have

$$\tan 60^\circ = \frac{H+120}{OP}$$

$$OP = \frac{H+120}{\sqrt{3}} \qquad ...(2)$$

From (1) and (2), we get

$$\frac{H+120}{\sqrt{3}} = \sqrt{3}(H-120)$$

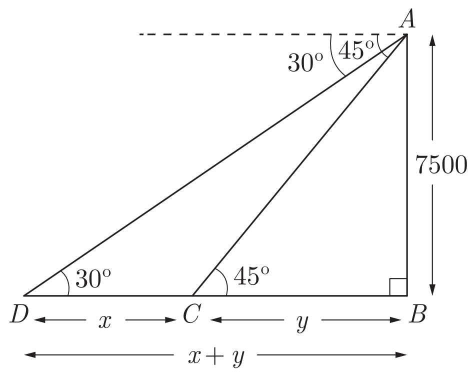
Thus height of cloud is 240 m.

or

The angle of depression of two ships from an aeroplane flying at the height of 7500 m are 30° and 45°. if both the ships are in the same that one ship is exactly behind the other, find the distance between the ships.

Ans :

Let A, C and D be the position of aeroplane and two ship respectively. Aeroplane is flying at 7500 m height from point B. As per given in question we have drawn figure below.



In right $\Delta\, ABC$, we have

$$\frac{AB}{BC} = \tan 45^\circ$$

$$\frac{7500}{y} = y$$

$$y = 7500 \qquad ...(1)$$

In right ΔABD, we have

$$\frac{AB}{BD} = \tan 30^\circ$$

$$\frac{7500}{x+y} = \frac{1}{\sqrt{3}}$$

$$x+y = 7500\sqrt{3} \quad ...(2)$$

Substituting the value of y from (1) in (2) we have

$$x + 7500 = 7500\sqrt{3}$$

$$x = 7500\sqrt{3} - 7500$$

$$= 7500(\sqrt{3}-1)$$

$$= 7500(1.73-1)$$

$$= 7500 \times 0.73$$

$$= 5475 \text{ m}$$

Hence, the distance between two ships is 5475 m.

40. Monthly expenditures on milk in 100 families of a housing society are given in the following frequency distribution : [4]

Monthly expenditure (in Rs.)	0 - 175	175-350	350-525	525-700	700-875	875-1050	1050-1125
Number of families	10	14	15	21	28	7	5

Find the mode and median for the distribution.

Ans :

C.I.	f	$c.f.$
0-175	10	10
157-350	14	24
350-525	15	39
525-700	21	60
700-875	28	88
875-1050	7	95
1050-1225	5	100

$$\text{Median} = \frac{N}{2}\text{th term}$$

$$= \frac{100}{2} = 50\text{th term}$$

$\therefore$ $$\text{Median class} = 525 - 700$$

$$\text{Median} = l + \frac{\frac{N}{2} - c.f.}{f} \times h$$

$$= 525 + \frac{50-39}{21} \times 175$$

$$= 525 + \frac{11}{21} \times 175$$

$$= 525 + 91.6$$

$$= 616.6$$

and $$\text{Modal class} = 700 - 875$$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)$$

$$l = 700,\ f_0 = 21,\ f_1 = 28$$

$$f_2 = 7,\ h = 175$$

$$= 700 + \left(\frac{28-21}{2 \times 28 - 21 - 7}\right) \times 175$$

$$= 700 + \frac{7}{28} \times 175$$

$$= 700 + 43.75$$

$$= 743.75$$