CLASS XII (2019-20)
MATHEMATICS (041)
SAMPLE PAPER-2

Time : 3 Hours **Maximum Marks : 80**

General Instructions :

(i) All questions are compulsory.
(ii) The questions paper consists of 36 questions divided into 4 sections A, B, C and D.
(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
(iv) There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

SECTION-A

DIRECTION : (Q 1-Q 10) are multiple choice type questions. Select the correct option.

Q1. If $f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \,\forall\, x_1, x_2 \varepsilon A$, then what type of a function if $f : A \longrightarrow B$? [1]

(a) One-one (b) Constant
(c) Onto (d) Many one

Q2. $\sin^{-1}\frac{1}{x} = ?$ [1]

(a) $\sec^{-1}x$ (b) $\text{cosec}^{-1}x$
(c) $\tan^{-1}x$ (d) $\sin x$

Q3. $A = \begin{bmatrix} 3 & 6 \\ 5 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 7 & 8 \\ 5 & 6 \end{bmatrix}$, $2A + 3B = ?$ [1]

(a) $\begin{bmatrix} 27 & 24 \\ 22 & 10 \end{bmatrix}$ (b) $\begin{bmatrix} 27 & 36 \\ 25 & 10 \end{bmatrix}$

(c) $\begin{bmatrix} 27 & 36 \\ 25 & 15 \end{bmatrix}$ (d) $\begin{bmatrix} 27 & 36 \\ 35 & 10 \end{bmatrix}$

Q4. If $\lambda \in R$ and $\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ then $\lambda\Delta =$ [1]

(a) $\begin{vmatrix} \lambda a & \lambda b \\ \lambda c & \lambda d \end{vmatrix}$ (b) $\begin{vmatrix} \lambda a & b \\ c & d \end{vmatrix}$

(c) $\begin{vmatrix} \lambda a & b \\ \lambda c & d \end{vmatrix}$ (d) None of these

Q5. $\frac{d}{dx}[\sin^{-1}x] = ?$ [1]

(a) $\frac{1}{\sqrt{1-x^2}}$ (b) $\frac{-1}{\sqrt{1-x^2}}$

(c) $\frac{1}{\sqrt{1+x^2}}$ (d) $\sqrt{1-x^2}$

Q6. The slope of the tangent to the curve, $x = t^2 + 3t - 8,\ y = 2t^2 - 2t - 5$ at the point $(2, -1)$ is- [1]

(a) $\frac{12}{7}$ (b) $\frac{-6}{7}$

(c) $\frac{6}{7}$ (d) $\frac{-12}{7}$

Q7. $\int_0^1 \frac{dx}{1+x^2} = ?$ [1]

(a) $\frac{-\pi}{4}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$ (d) $\frac{-\pi}{2}$

Q8. Area between the $x-$axis and the curve $y = \sin x$, from $x = 0$ to $x = \frac{\pi}{2}$ is [1]

(a) 2 (b) -1

(c) 1 (d) None of these

Q9. Integrating factor of the differential equation $\frac{dy}{dx} + y\sec x = \tan x$ is- [1]

(a) $\sec x + \tan x$ (b) $\sec x - \tan x$

(c) $\sec x$ (d) $\tan x \sec x$

Q10. The position vector of the point (x, y, z) is- [1]

(a) $x\hat{i} - y\hat{j} - z\hat{k}$ (b) $x\hat{i} + y\hat{j} - z\hat{k}$

(c) $x\hat{i} - y\hat{j} + z\hat{k}$ (d) $x\hat{i} + y\hat{j} + z\hat{k}$

Q. 11-15 (Fill in the blanks)

Q11. Let $A = \{1,2,3,4\}$ and R be the equivalence relation on $A \times A$ defined by $(a,b)R(c,d)$ if $a + d = b + c$ the equivalence class $[(1,3)]$ is .... [1]

Q12. If $A = [a_{ij}]$ is a matrix of order 2×2, such that $|A| = -15$ and c_{ij} represents the cofactor of a_{ij}, then $a_{21}c_{21} + a_{22} + c_{22}$ is .... [1]

Q13. A function $f : A \to B$ is said to be .... if every element of B is the image of some element of A under f. [1]

OR

Relation R in a set A is called an .... relation. If each element of A is related to every element of A, i.e. $R = A \times A$.

Q14. A corner point of a feasible region is a point in the region which is the .... of two boundary lines. [1]

Q15. If $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$, then $f(x)$ is an .... function. [1]

OR

$\int e^x (f(x) + f'(x))\,dx$ is equal to ....

DIRECTION : (Q.no. 16-20) Answer the following questions.

Q16. Find the area of the parallelogram determined by the vectors $\hat{i}+2\hat{j}+3\hat{k}$ and $3\hat{i}-2\hat{j}+\hat{k}$. [1]

OR

Find the value of λ, if the vectors $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{c}=3\hat{i}+\lambda\hat{j}+5\hat{k}$ are coplanar.

Q17. If $Y=\tan^{-1}\left(\dfrac{a+x}{1-ax}\right)$, find $\dfrac{dy}{dx}$. [1]

Q18. If A is symmetric, then show that $B'AB$ is symmetric matrix.

Q19. If $\int_0^1(3x^2+2x+K)\,dx=0$, find K. [1]

Q20. If $P(A/B)>P(A)$, the prove that $P(B/A)>P(B)$. [1]

SECTION B

Q21. If $y=\sin^{-1}(\sqrt{x}\sqrt{1-x^2}-x\sqrt{1-x})$, then find $\dfrac{dy}{dx}$. [2]

OR

Find a point on the parabola $y=(x-3)^2$, where the tangent is parallel to the chord joining (3,0) and (4,1).

Q22. If $4\sin^{-1}x+\cos^{-1}x=\pi$, then find the value of x. [2]

Q23. Find the inverse of the matrix $\begin{bmatrix}-3 & 2\\ 5 & -3\end{bmatrix}$.

Hence find the matrix P satisfying the matrix equation $P\begin{bmatrix}-3 & 2\\ 5 & -3\end{bmatrix}=\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}$. [2]

Q24. Prove that [2]

$$\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)=\left(\frac{\pi}{4}+\frac{x}{2}\right), x\,\varepsilon\left(\frac{-\pi}{2},\frac{\pi}{2}\right).$$

OR

Solve for $x:\tan^{-1}\left(\dfrac{1-x}{1+x}\right)=\dfrac{1}{2}\tan^{-1}x, x>0$.

Q25. Prove that if $\dfrac{1}{2}\leq x\leq 1$, then $\cos^{-1}x+\cos^{-1}\left[\dfrac{x}{2}+\dfrac{\sqrt{3-3x^2}}{2}\right]=\dfrac{\pi}{3}$. [2]

Q26. Find the approximate change in the value of $\dfrac{1}{x^2}$, when x changes from $x=2$ to $x=2.002$. [2]

SECTION C

Q27. If $\Delta=\begin{vmatrix}1 & a & a^2\\ a & a^2 & 1\\ a^2 & 1 & a\end{vmatrix}=-4$, then find the value of $\begin{vmatrix}a^3-1 & 0 & a-a^4\\ 0 & a-a^4 & a^3-1\\ a-a^4 & a^3-1 & 0\end{vmatrix}$. [4]

Q28. Find a and b, if the function given by $f(x)=\begin{cases}ax^2+b, & \text{if } x<1\\ 2x+1, & \text{if } x\geq 1\end{cases}$ is differentiable at $x=1$. [4]

OR

Determine the values of a and b such that the following function is continuous at $x = 0$.

$$f(x) = \begin{cases} \dfrac{x+\sin x}{\sin(a+1)x}, & \text{if } -\pi < x < 0 \\ 2, & \text{if } x = 0 \\ \dfrac{2(e^{\sin bx}-1)}{bx}, & \text{if } x > 0 \end{cases}$$

Q29. If $y = \log\left(\sqrt{x} + \dfrac{1}{\sqrt{x}}\right)^2$, then prove that $x(x+1)^2 y_2 + (x+1)^2 y_1 = 2$. [4]

Q30. A person wants to plant some trees in his community park. The local nursery has to perform this task. It charges the cost of planting trees by the formula $C(x) = x^3 - 45x^2 + 600x$, where x is the number of trees and $C(x)$ is the cost of planting x tress in rupees. The local authority has imposed a restriction that it can plant 10 to 20 trees in one community park for a fair distribution. For how many trees should the person place the order so that he has to spend the least amount? How much is the least amount? Use calculus to answer these questions. [4]

Q31. Find the equation (s) of the tangent (s) to the curve $y = (x^3 - 1)(x - 2)$ at the points, where the curve intersects the X-axis. [4]

OR

Find the intervals in which the function:

$$f(x) = -3\log(1+x) + 4\log(2+x) - \frac{4}{2+x}$$ is strictly increasing or strictly decreasing.

Q32. Find $\displaystyle\int \frac{\sec x}{1+\operatorname{cosec} x}\,dx$. [4]

SECTION D

Q33. If the function $f : R \to R$ be defined by $f(x) = 2x - 3$ and $g : R \to R$ by $g(x) = x^3 + 5$, then find fog and show that fog is invertible. Also find $(\text{fog})^{-1}$ and hence find $fog^{-1}(9)$. [6]

OR

A binary operation $*$ is defined on the set R or real numbers by $a * b = \begin{cases} a, & \text{if } b = 0 \\ |a| + b, & \text{if } b \neq 0 \end{cases}$. If at least one of a and b is 0, then prove that $a * b = b * a$.

Check whether $*$ is commutative. Find the identity element for $*$, if it exists.

Q34. Using integration, find the area in the first quadrant bounded by the curve $y = x|x|$, the circle $x^2 + y^2 = 2$ and the Y-axis. [6]

Q35. If $A = \begin{vmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{vmatrix}$, then find A^{-1} and hence solve the following system of equation. [6]

$$\begin{aligned} 3x + 4y + 7z &= 14, \\ 2x - y + 3z &= 4, \\ x + 2y - 3z &= 0 \end{aligned}$$

OR

If $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$, then find the inverse if A using elementary row transformations and hence solve the matrix equation $XA = [1, 0, 1]$.

Q36. Find the distance of point $-2\hat{i} + 3\hat{j} - 4\hat{k}$ from the line $\vec{r} = \hat{i} + 2\hat{j} - \hat{k} + \lambda(\hat{i} + 3\hat{j} - 9\hat{k})$ measured parallel to the plane $x - y + 2z - 3 = 0$. [6]