CLASS XII (2019-20)
MATHEMATICS (041)
SAMPLE PAPER-3

Time : 3 Hours **Maximum Marks : 80**

General Instructions :

(i) All questions are compulsory.
(ii) The questions paper consists of 36 questions divided into 4 sections A, B, C and D.
(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
(iv) There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

SECTION-A

DIRECTION : (Q 1-Q 10) are multiple choice type questions. Select the correct option.

Q1. If $f: R \to R$ such that $f(x) = 3x - 4$ then which of the following is $f^{-1}(x)$? [1]

(a) $\frac{x+4}{3}$ (b) $\frac{1}{3}x - 4$

(c) $3x - 4$ (d) $3x + 5$

Q2. If $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$, then- [1]

(a) $(x = -2, y = 8)$ (b) $(x = 2, y = -8)$

(c) $(x = 3, y = -6)$ (d) $(x = -3, y = 6)$

Q3. The matrix $\begin{bmatrix} 3 & 5 \\ 2 & k \end{bmatrix}$ has no inverse if the value of k is [1]

(a) 0 (b) 5

(c) $\frac{10}{3}$ (d) $\frac{4}{9}$

Q4. $\frac{d}{dx}[\log(\sec x + \tan x)] =$ [1]

(a) $\frac{1}{\sec x + \tan x}$ (b) $\sec x$

(c) $\tan x$ (d) $\sec x + \tan x$

Q5. The slope of the tangent to the curve, $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$ at the point $(2, -1)$ is- [1]

(a) $\frac{12}{7}$ (b) $\frac{-6}{7}$

(c) $\frac{6}{7}$ (d) $\frac{-12}{7}$

Q6. $\int_0^1 \frac{(\tan^{-1}x)^2}{1+x^2}dx =$ [1]

(a) 1 (b) $\frac{\pi^3}{64}$

(c) $\frac{\pi^2}{192}$ (d) None of these

Q7. Solution of the differential equation $ydx - xdy = xydx$ is [1]

(a) $\frac{y^2}{2} - \frac{x^2}{2} = xy + c$ (b) $x = kye^x$

(c) $x = kye^y$ (d) None of these

Q8. If $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 3\hat{i} + 2\hat{j} - \hat{k}$, then the value of $(\vec{a} + 3\vec{b}).(2\vec{a} - \vec{b})$ is- [1]

(a) 15 (b) 18

(c) -18 (d) -15

Q9. The direction ratios of a straight line are 1,3,5. Its direction cosines are [1]

(a) $\frac{1}{\sqrt{35}}, \frac{3}{\sqrt{35}}, \frac{5}{\sqrt{35}}$ (b) $\frac{1}{9}, \frac{1}{3}, \frac{5}{9}$

(c) $\frac{3}{\sqrt{35}}, \frac{5}{\sqrt{35}}, \frac{1}{\sqrt{35}}$ (d) $\frac{5}{\sqrt{35}}, \frac{3}{\sqrt{35}}, \frac{1}{\sqrt{35}}$

Q10. If $P(A) = \frac{3}{8}$, $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$ then $P(A' \cap B') =$ [1]

(a) $\frac{13}{8}$ (b) $\frac{13}{4}$

(c) $\frac{13}{24}$ (d) $\frac{13}{9}$

Q. 11-15 (Fill in the blanks)

Q11. Let $\vec{a}$ and $\vec{b}$ be two given vectors such that $|\vec{a}| = 2$, $|\vec{b}| = 1$ and $\vec{a} \cdot \vec{b} = 1$. The angle between $\vec{a}$ and $\vec{b}$ .... [1]

Q12. If I_3 is the identity matrix of order 3 then the value of $(3I_3)$ will be .... [1]

Q13. The principal value of $\text{cosec}^{-1}(2)$ will be .... [1]

Q14. If $f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \forall x_1, x_2 \in A$, then the function $f : A \to B$ is .... [1]

(a) one-one (b) constant

(c) onto (d) many one

OR

If function $f : N \to N$ be defined by $f(x) = 4x + 3$ then $f^{-1}(x) =$ ....

(a) $4x - 3$ (b) $\frac{4x-3}{2}$

(c) $\frac{x+3}{2}$ (d) $\frac{x-3}{4}$

Q15. The order of the differential equation $\left(\frac{dy}{dx}\right)^2 + y = x$ is .... [1]

(a) 0 (b) 1

(c) 2 (d) 3

OR

The differential equation of family of lines passing through the origin is ....

(a) $x\dfrac{dy}{dx} = y$ (b) $y\dfrac{dy}{dx} = x$

(c) $\dfrac{dy}{dx} = y$ (d) $\dfrac{dy}{dx} = x$

Q16. If A is a matrix of order 2×3 and B is a matrix of order 3×5, then what is the order of matrix $(AB)'$ or $(AB)^T$? [1]

Q17. Find the value of λ, so that the vectors $\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\vec{b} = \hat{i} + \lambda\hat{j} + 3\hat{k}$ are perpendicular to each other. [1]

Q18. Let $f : R \to R$, $f(x) = (x^2 - 3x + 2)$. Find $fof(x)$. [1]

Q19. Prove that the function f given by $f(x) = \log\cos x$ is strictly decreasing. [1]

Q20. Maximise $Z = 3x + 4y$, subject to the constraints $x + y \leq 1$, $x \geq 0$, $y \geq 0$. [1]

SECTION B

Q21. Solve for x $\cos(2\sin^{-1}x) = \frac{1}{9}$, $x > 0$ [2]

OR

Evaluate $\cos\left[\sin^{-1}\frac{1}{4} + \sec^{-1}\frac{4}{3}\right]$

Q22. Find the derivative of $\log\sin x$ w.r.t. x. [2]

Q23. Evaluate $\int(3\,\mathrm{cosec}^2 x - 5x + \sin x)\,dx$. [2]

Q24. If the function $f(x) = \frac{1}{x+2}$, find the points of discontinuity of the composite function $y = f(f(x))$. [2]

OR

If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ and $x \neq y$, prove that $\dfrac{dy}{dx} = -\dfrac{1}{(x+1)^2}$.

Q25. Without expanding, show that [2]

$$\Delta = \begin{vmatrix} \mathrm{cosec}^2\theta & \cot^2\theta & 1 \\ \cot^2\theta & \mathrm{cosec}^2\theta & -1 \\ 42 & 40 & 2 \end{vmatrix} = 0$$

Q26. Show that $\Delta = \begin{vmatrix} x & p & q \\ p & x & q \\ q & q & x \end{vmatrix} = (x-p)(x^2 + px - 2q^2)$ [2]

SECTION C

Q27. Let $f : R \to R$ defined by $f(x) = \frac{2x-1}{3}$, $x \in R$, where x is the number of students in a class and $f(x)$ is money collected by the class for girl child welfare, Show that f is invertible. [4]

Q28. Solve the differential equation $\dfrac{dy}{dx} + \dfrac{y}{x} = x^2$. [4]

OR

Solve $x^2\dfrac{dy}{dx}-xy=1+\cos\left(\dfrac{y}{x}\right)$, $x\neq 0$ and $x=1$, $y=\dfrac{\pi}{2}$.

Q29. Find the values of x which satisfy the equation: [4]

$$\sin^{-1}x+\sin^{-1}(1-x)=\cos^{-1}x.$$

Q30. Find the equation of the plane passing through the points $(2,1,-1)$ and $(-1,3,4)$ and perpendicular to the plane $x-2y+4z=10$. [4]

Q31. Find the unit vector in the direction of the sum of the vectors $\vec{a}=2\hat{i}-\hat{j}+2\hat{k}$ and $\vec{b}=-\hat{i}+\hat{j}+3\hat{k}$. [4]

OR

If $\vec{a}$, $\vec{b}$ and $\vec{c}$ determine the vertices of a triangle, show that $\dfrac{1}{2}[\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b}]$ gives the vector area of me triangle. Hence, deduce the condition that the three points $\vec{a}$, $\vec{b}$ and $\vec{c}$ are collinear. Also, find the unit vector normal to the plane of the triangle.

Q32. Find the vector equation of a line passing through a point with position vector $2\hat{i}-\hat{j}+\hat{k}$, and parallel to the line joining the points $-\hat{i}+4\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+2\hat{k}$. Also, find the Cartesian equivalent of this equation. [4]

SECTION D

Q33. Show that every square matrix can be uniquely expressed as the sum of a symmetric matrix and a skew-symmetric matrix. [6]

OR

If $A=\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & -2\\ a & 2 & b\end{bmatrix}$ is a matrix satisfying $AA^T=9I_3$, then find the values of a and b.

Q34. A manufacturer produces two types of steel trunks. He has two machines A and B. The first type of trunk requires 3h on machine A and 3h on machine B. The second type of trunk requires 3h on machines A and 2h on machine B. Both machines are run daily for 18h and 15h, respectively. There is a profit of ₹30 on first type of trunk and ₹25 on the second type of trunk. How many trunks of each type should be produced and sold to make maximum profit? [6]

Q35. Find the area of the smaller region bounded by the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ and the straight line $\dfrac{x}{a}+\dfrac{y}{b}=1$. [6]

OR

Evaluate $\displaystyle\int_a^b x\,dx$ using integration as limit of sum. [6]

Q36. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere. [6]