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S UBI ECT: MATHEMATICS (041)

BLULE PRINTI: CLASS XII

| Chapter | $\begin{gathered} \hline \text { MCQ } \\ (1 \mathrm{mark}) \end{gathered}$ | $\begin{gathered} \text { FIB } \\ (1 \text { mark }) \\ \hline \end{gathered}$ | $\begin{array}{\|c\|} \hline \text { VSA } \\ \text { (1 mark) } \\ \hline \end{array}$ | $\begin{array}{\|c} \hline \mathbf{S A} \\ \text { (2 marks) } \end{array}$ | $\begin{array}{\|c\|} \hline \text { LA - I } \\ (4 \text { marks }) \\ \hline \end{array}$ | $\begin{array}{\|c\|} \hline \text { LA- II } \\ (6 \text { marks }) \\ \hline \end{array}$ | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Relations and Functions | -- | 1(1) | -- | 2(1)* | 4(1) | -- | 5(2) |
| Inverse Trigonometric Functions | 1(1) | -- | -- |  | -- | -- | 3(2) |
| Matrices | 2(2) | 1(1) | -- | -- | -- | -- | 3(3) |
| Determinants | -- | -- | 1(1) | -- | -- | 6(1)* | 7(2) |
| Continuity \& Differentiability | -- | 1(1) | -- | -- | 4(1)* | -- | 5(2) |
| Applications of Derivatives | -- | 1(1)* | -- | 2(1) | -- | 6(1)* | 9(3) |
| Integrals | 1(1) | -- | $\begin{gathered} 2(2) \\ \mathbf{1 ( 1 ) *} \end{gathered}$ | -- | 4(1) | -- | 8(5) |
| Applications of the Integrals | -- | -- | -- | -- | -- | 6(1) | 6(1) |
| Differential Equations | -- | -- | 1(1) | 2(1) | 4(1) | -- | 7(3) |
| Vector Algebra | 1(1) | 1(1)* | -- | 2(1)* | -- | -- | 4(3) |
| Three-Dimensional Geometry | 3(3) | -- | -- | 2(1) | -- | 6(1) | 11(5) |
| Linear Programming | -- | -- | -- | -- | 4(1) | -- | 4(1) |
| Probability | 2(2) | -- | -- | 2(1) | 4(1)* | -- | 8(4) |
| Total | 10(10) | 5(5) | 5(5) | 12(6) | 24(6) | 24(4) | 80(36) |

## Note: * - Internal Choice Questions

$\mathcal{S U B I} \mathcal{E C T}: \mathcal{M A T H E \mathcal { M A } \mathcal { T } I C S}$
CLASS : XII

## General Instruction:

(i) All the questions are compulsory.
(ii) The question paper consists of $\mathbf{3 6}$ questions divided into 4 sections A, B, C, and D.
(iii) Section A comprises of $\mathbf{2 0}$ questions of $\mathbf{1}$ mark each. Section $\mathbf{B}$ comprises of $\mathbf{6}$ questions of $\mathbf{2}$ marks each. Section C comprises of $\mathbf{6}$ questions of $\mathbf{4}$ marks each. Section D comprises of $\mathbf{4}$ questions of $\mathbf{6}$ marks each.
(iv) There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

## SECTION - A

Questions 1 to 20 carry 1 mark each.

1. The value of $\cos ^{-1}\left(\cos \frac{3 \pi}{2}\right)$ is
(a) $\frac{\pi}{2}$
(b) $\frac{3 \pi}{2}$
(c) $\frac{5 \pi}{2}$
(d) $\frac{7 \pi}{2}$
2. If $A$ and $B$ are two matrices of the order $3 \times m$ and $3 \times n$, respectively, and $m=n$, then the order of matrix $(5 A-2 B)$ is
(a) $\mathrm{m} \times 3$
3 (b) $3 \times 3$
(c) $\mathrm{m} \times \mathrm{n}$ (
(d) $3 \times n$
3. If $\mathrm{A}=\left[\begin{array}{ll}0 & 1 \\ 1 & 0\end{array}\right]$, then $\mathrm{A}^{2}$ is equal to
(a) $\left[\begin{array}{ll}0 & 1 \\ 1 & 0\end{array}\right]$
(b) $\left[\begin{array}{ll}1 & 0 \\ 1 & 0\end{array}\right]$
(c) $\left[\begin{array}{ll}0 & 1 \\ 0 & 1\end{array}\right]$
(d) $\left[\begin{array}{ll}1 & 0 \\ 0 & 1\end{array}\right]$
4. If $|\vec{a}|=10,|\vec{b}|=2$ and $\vec{a} \cdot \vec{b}=12$, then value of $|\vec{a} \times \vec{b}|$ is
(a) 5
(b) 10
(c) 14
(d) 16
5. If $\int \frac{3 e^{x}-5 e^{-x}}{4 e^{x}+5 e^{-x}} d x=\mathrm{ax}+\mathrm{b} \log \left|4 \mathrm{e}^{\mathrm{x}}+5 \mathrm{e}^{-\mathrm{x}}\right|+\mathrm{C}$, then
(a) $a=\frac{-1}{8}, b=\frac{7}{8}$
(b) $a=\frac{1}{8}, b=\frac{7}{8}$
(c) $a=\frac{-1}{8}, b=\frac{-7}{8}$
(d) $a=\frac{1}{8}, b=\frac{-7}{8}$
6. The intercepts made by the plane $2 x-3 y+4 z=12$ on the coordinate axes are
(a) $6,-4,3$
(b) 2, $-3,4$
(c) $\frac{1}{6},-\frac{1}{4}, \frac{1}{3}$
(d) $1,1,1$
7. If a line makes angles $\alpha, \beta, \gamma$ with the positive direction of co-ordinate axes, the the value of $\sin ^{2} \alpha+\sin ^{2} \beta+\sin ^{2} \gamma$ is
(a) 2
(b) 1
(c) -1
(d) -2
8. If a line has direction ratios $2,-1,-2$ then its direction cosines are
(a) $\frac{2}{3}, \frac{1}{3}, \frac{2}{3}$
(b) $\frac{2}{3}, \frac{1}{3},-\frac{2}{3}$
(c) $\frac{2}{3},-\frac{1}{3},-\frac{2}{3}$
(d) $-\frac{2}{3},-\frac{1}{3},-\frac{2}{3}$
9. Let X be a discrete random variable. The probability distribution of X is given below:

| X | 30 | 10 | -10 |
| :---: | :---: | :---: | :---: |
| $\mathrm{P}(\mathrm{X})$ | $\frac{1}{5}$ | $\frac{3}{10}$ | $\frac{1}{2}$ |

Then $E(X)$ is equal to
(a) 6
(b) 4
(c) 3
(d) -5
10. If $A$ and $B$ are two events such that $P(A) \neq 0$ and $P(B \mid A)=1$, then
(a) $\mathrm{A} \subset \mathrm{B}$
(b) $\mathrm{B} \subset \mathrm{A}$
(c) $\mathrm{B}=\varphi$
(d) $\mathrm{A}=\varphi$
11. The value of $\lambda$ when the projection of $\vec{a}=\lambda \hat{i}+\hat{j}+4 \hat{k}$ on $\vec{b}=2 \hat{i}+6 \hat{j}+3 \hat{k}$ is 4 units is $\qquad$

## OR

The angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes 1 and 2 respectively and when $|\vec{a} \times \vec{b}|=\sqrt{3}$ is $\qquad$
12. If $\mathrm{f}(\mathrm{x})=\left(4-(\mathrm{x}-7)^{3}\right\}$, then $\mathrm{f}^{-1}(\mathrm{x})=$ $\qquad$ .
13. Radius of a variable circle is changing at the rate of $5 \mathrm{~cm} / \mathrm{s}$. The radius of the circle at a time when its area is changing at the rate of $100 \mathrm{~cm}^{2} / \mathrm{s}$ is $\qquad$

## OR

The point on the curve $y=x^{2}$, where the rate of change of $x$-coordinate is equal to the rate of change of $y$-coordinate is $\qquad$
14. If $f(x)=\left\{\begin{array}{c}\frac{\sin 2 x}{5 x}, \text { if } x \neq 0 \\ k, \text { if } x=0\end{array}\right.$ is continuous at $\mathrm{x}=0$, then the value of k will be $\qquad$
15. If $A$ and $B$ are symmetric matrices of same order, then $A B$ is symmetric if and only if
$\qquad$ —.
16. Find $\int \cot x \log (\sin x) d x$

## OR

Evaluate: $\int \sqrt{1+\sin \frac{x}{4}} d x$
17. Given a square matrix $A$ of order $3 \times 3$, such that $|A|=12$, find the value of $|A \cdot \operatorname{adj} A|$.
18. If $\int_{0}^{a} 3 x^{2} d x=8$, write the value of a.
19. Find $\int_{-\pi}^{\pi} \sin ^{3} x \cos ^{2} x d x$.
20. Find the value of $m$ and $n$, where $m$ and $n$ are order and degree of differential equation $\frac{4\left(\frac{d^{2} y}{d x^{2}}\right)^{3}}{\frac{d^{3} y}{d x^{3}}}+\frac{d^{3} y}{d x^{3}}=x^{2}-1$.

## SECTION - B

## Questions 21 to 26 carry 2 marks each.

21. Let f and g be two real functions defined as $\mathrm{f}(\mathrm{x})=2 \mathrm{x}-3 ; \mathrm{g}(\mathrm{x})=\frac{3+x}{2}$. Find fog and gof. Can you say one is inverse of the other?

## OR

Express $\tan ^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right), x<\pi$ in simplest form.
22. Find the vector equation of a plane which is at a distance of 7 units from the origin and normal to the vector $3 \hat{i}+5 \hat{j}-6 \hat{k}$
23. Prove that the function given by $f(x)=\cos x$ is (a) strictly decreasing in $(0, \pi)$ (b) strictly increasing in $(\pi, 2 \pi)$
24. Form the differential equation of the family of parabolas having vertex at the origin and axis along positive y -axis.
25. If $\mathrm{P}(\mathrm{A})=\frac{7}{13}, \mathrm{P}(\mathrm{B})=\frac{9}{13}$ and $\mathrm{P}(\mathrm{A} \cap \mathrm{B})=\frac{4}{13}$, find $\mathrm{P}\left(\mathrm{A}^{\prime} / \mathrm{B}\right)$.
26. Find $\lambda$, if $(2 \hat{i}+6 \hat{j}+14 \hat{k}) \times(\hat{i}-\lambda \hat{j}+7 \hat{k})=\overrightarrow{0}$.

## OR

Find the volume of a parallelepiped whose continuous edges are represented by vectors $\vec{a}=2 \hat{i}-3 \hat{j}+\hat{k}, \vec{b}=\hat{i}-\hat{j}+2 \hat{k}$ and $\vec{c}=2 \hat{i}+\hat{j}-\hat{k}$

## SECTION - C

## Questions 27 to $\mathbf{3 2}$ carry 4 marks each.

27. Let $\mathrm{A}=\mathrm{R}-\{2\}$ and $\mathrm{B}=\mathrm{R}-\{1\}$. If $\mathrm{f}: \mathrm{A} \rightarrow \mathrm{B}$ is a function defined by $\mathrm{f}(\mathrm{x})=\frac{x-1}{x-2}$, show that f is one-one and onto. Hence, find $\mathrm{f}^{-1}$.

## OR

Find the value of $\tan ^{-1}\left[2 \cos \left(2 \sin ^{-1} \frac{1}{2}\right)\right]$.
28. Find $\frac{d y}{d x}$, if $\mathrm{y}=(\log \mathrm{x})^{\mathrm{x}}+\mathrm{x}^{\log \mathrm{x}}$.

## OR

Find $\frac{d y}{d x}$ of the functions expressed in parametric form $\sin \mathrm{x}=\frac{2 t}{1+t^{2}}, \tan \mathrm{y}=\frac{2 t}{1-t^{2}}$.
29. A company produces soft drinks that has a contract which requires that a minimum of 80 units of the chemical A and 60 units of the chemical B go into each bottle of the drink. The chemicals are available in prepared mix packets from two different suppliers. Supplier $S$ had a packet of mix of 4 units of A and 2 units of B that costs 10 . The supplier T has a packet of mix of 1 unit of A and 1 unit of $B$ that costs 4 . How many packets of mixes from $S$ and $T$ should the company purchase to honour the contract requirement and yet minimise cost? Make an LPP and solve graphically.
30. Evaluate the following integral $\int_{0}^{\pi} \log (1+\cos x) d x$
31. In a game, a man wins a rupee for a six and loses a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins / loses.

OR
Three bags contain balls as shown in the table below:

| Bag | Number of white balls | Number of black balls | Number of red balls |
| :---: | :---: | :---: | :---: |
| I | 1 | 2 | 3 |
| II | 2 | 1 | 1 |
| III | 4 | 3 | 2 |

A bag is chosen at random and two balls are drawn from it. They happen to be white and red. What is the probability that they came from the III bag?
32. Find the general solution of the following differential equation: $\left(1+y^{2}\right)+\left(x-e^{\tan ^{-1} y}\right) \frac{d y}{d x}=0$

## SECTION - D

## Questions 33 to 36 carry 6 marks each.

33. If $\mathrm{x} \neq \mathrm{y} \neq \mathrm{z}$ and $\Delta=\left|\begin{array}{lll}x & x^{2} & 1+x^{3} \\ y & y^{2} & 1+y^{3} \\ z & z^{2} & 1+z^{3}\end{array}\right|=0$, then show that $1+\mathrm{xyz}=0$.

## OR

Given $A=\left[\begin{array}{ccc}1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2\end{array}\right]$ and $B=\left[\begin{array}{ccc}2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5\end{array}\right]$, verify that $B A=6 I$, use the result to solve the system $\mathrm{x}-\mathrm{y}=3,2 \mathrm{x}+3 \mathrm{y}+4 \mathrm{z}=17, \mathrm{y}+2 \mathrm{z}=7$.
34. Prove that, the radius of the right circular cylinder of greatest curved surface which can be inscribed in a given cone, is half of that of the cone.

## OR

A wire of length 28 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined area of the square and the circle is minimum?
35. Using the method of integration find the area of the $\triangle A B C$, coordinates of whose vertices are $\mathrm{A}(2,0), \mathrm{B}(4,5)$ and $\mathrm{C}(6,3)$.
36. Find the coordinates of the point where the line through $(3,-4,-5)$ and $(2,-3,1)$ crosses the plane, passing through the points $(2,2,1),(3,0,1)$ and $(4,-1,0)$.

