KENVDRI YA VI DYALA $\mathcal{A}$ GACHIBO WLI, GPRA CAMPUS, $\mathcal{H Y D}-32$ $S \mathcal{A M P L E} \operatorname{PAPER}$ - 05 (2019-20)

S UBI ECT: MATHEMATICS (041)

BLULE PRINTI: CLASS XII

| Chapter | $\begin{gathered} \hline \text { MCQ } \\ (1 \mathrm{mark}) \end{gathered}$ | $\begin{gathered} \text { FIB } \\ (1 \text { mark }) \\ \hline \end{gathered}$ | $\begin{array}{\|c\|} \hline \text { VSA } \\ \text { (1 mark) } \\ \hline \end{array}$ | $\begin{array}{\|c} \hline \mathbf{S A} \\ \text { (2 marks) } \end{array}$ | $\begin{array}{\|c\|} \hline \text { LA - I } \\ (4 \text { marks }) \\ \hline \end{array}$ | $\begin{array}{\|c\|} \hline \text { LA- II } \\ (6 \text { marks }) \\ \hline \end{array}$ | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Relations and Functions | -- | 1(1) | -- | 2(1)* | 4(1) | -- | 5(2) |
| Inverse Trigonometric Functions | 1(1) | -- | -- |  | -- | -- | 3(2) |
| Matrices | 2(2) | 1(1) | -- | -- | -- | -- | 3(3) |
| Determinants | -- | -- | 1(1) | -- | -- | 6(1)* | 7(2) |
| Continuity \& Differentiability | -- | 1(1) | -- | -- | 4(1)* | -- | 5(2) |
| Applications of Derivatives | -- | 1(1)* | -- | 2(1) | -- | 6(1)* | 9(3) |
| Integrals | 1(1) | -- | $\begin{gathered} 2(2) \\ \mathbf{1 ( 1 ) *} \end{gathered}$ | -- | 4(1) | -- | 8(5) |
| Applications of the Integrals | -- | -- | -- | -- | -- | 6(1) | 6(1) |
| Differential Equations | -- | -- | 1(1) | 2(1) | 4(1) | -- | 7(3) |
| Vector Algebra | 1(1) | 1(1)* | -- | 2(1)* | -- | -- | 4(3) |
| Three-Dimensional Geometry | 3(3) | -- | -- | 2(1) | -- | 6(1) | 11(5) |
| Linear Programming | -- | -- | -- | -- | 4(1) | -- | 4(1) |
| Probability | 2(2) | -- | -- | 2(1) | 4(1)* | -- | 8(4) |
| Total | 10(10) | 5(5) | 5(5) | 12(6) | 24(6) | 24(4) | 80(36) |

## Note: * - Internal Choice Questions

$\mathcal{S U B I} \mathcal{E C T}: \mathcal{M A T H E \mathcal { M A }} \operatorname{ICS}$
CLASS : XII

## General Instruction:

(i) All the questions are compulsory.
(ii) The question paper consists of $\mathbf{3 6}$ questions divided into 4 sections A, B, C, and D.
(iii) Section A comprises of $\mathbf{2 0}$ questions of $\mathbf{1}$ mark each. Section $\mathbf{B}$ comprises of $\mathbf{6}$ questions of $\mathbf{2}$ marks each. Section C comprises of $\mathbf{6}$ questions of $\mathbf{4}$ marks each. Section D comprises of $\mathbf{4}$ questions of $\mathbf{6}$ marks each.
(iv) There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

## SECTION - A

Questions 1 to 20 carry 1 mark each.

1. The principal value of $\sin ^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ is
(a) $-\frac{2 \pi}{3}$
(b) $-\frac{\pi}{3}$
(c) $-\frac{4 \pi}{3}$
(d) $\frac{5 \pi}{3}$
2. If $\mathrm{A}=\left[\begin{array}{ll}5 & x \\ y & 0\end{array}\right]$ and $\mathrm{A}=\mathrm{A}^{\prime}$ then
(a) $x=0, y=5$
(b) $\mathrm{x}=\mathrm{y}$
(c) $x+y=5(d) x-y=5$
3. If $\mathrm{F}(\mathrm{x})=\left[\begin{array}{cc}\cos x & \sin x \\ -\sin x & \cos x\end{array}\right]$ then $\mathrm{F}(\mathrm{x}) \mathrm{F}(\mathrm{y})$ is equal to
(a) $F(x)$ (b) $F(x y)$
(c) $\mathrm{F}(\mathrm{x}+\mathrm{y})$
(d) $F(x-y)$
4. $\int \frac{x^{3}}{x+1} d x$ is equal to
(a) $x+\frac{x^{2}}{2}+\frac{x^{3}}{3}-\log |1-x|+C$
(b) $x+\frac{x^{2}}{2}-\frac{x^{3}}{3}-\log |1-x|+C$
(c) $x-\frac{x^{2}}{2}-\frac{x^{3}}{3}-\log |1+x|+C$
(d) $x-\frac{x^{2}}{2}+\frac{x^{3}}{3}-\log |1+x|+C$
5. Let the vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}|=3$ and $|\vec{b}|=\frac{\sqrt{2}}{3}$, then is a unit vector, if the angle between $\vec{a}$ and $\vec{b}$ is
(a) $\frac{\pi}{3}$
(b) $\frac{\pi}{2}$
(c) $\frac{\pi}{4}$
(d) $\frac{\pi}{6}$
6. If the directions cosines of a line are $\mathrm{k}, \mathrm{k}, \mathrm{k}$, then
(a) $\mathrm{k}>0$
(b) $0<k<1$
(c) $\mathrm{k}=1$
(d) $k=\frac{1}{\sqrt{3}}$ or $-\frac{1}{\sqrt{3}}$
7. The distance of the plane $\vec{r}$. $\left(\frac{2}{7} \hat{i}+\frac{3}{7} \hat{j}+\frac{6}{7} \hat{k}\right)=1$ from the origin is
(a) 1
(b) 7
(c) $\frac{1}{7}$
(d) None of these
8. The equation of a straight line parallel to the x -axis is given by
(a) $\frac{x-a}{1}=\frac{y-b}{1}=\frac{z-c}{1}$
(b) $\frac{x-a}{0}=\frac{y-b}{1}=\frac{z-c}{1}$
(c) $\frac{x-a}{0}=\frac{y-b}{0}=\frac{z-c}{1}$
(d) $\frac{x-a}{1}=\frac{y-b}{0}=\frac{z-c}{0}$
9. Suppose that two cards are drawn at random from a deck of cards. Let X be the number of aces obtained. Then the value of $E(X)$ is
(a) $\frac{37}{221}$
(b) $\frac{5}{13}$
(c) $\frac{1}{13}$
(d) $\frac{2}{13}$
10. Probability that A speaks truth is $\frac{4}{5}$. A coin is tossed. A reports that a head appears. The probability that actually there was head is
(a) $\frac{4}{5}$
(b) $\frac{1}{2}$
(c) $\frac{1}{5}$
(d) $\frac{2}{5}$
11. Let the relation $R$ be defined on the set $A=\{1,2,3,4,5\}$ by $R=\left\{(a, b):\left|a^{2}-b^{2}\right|<8\right.$. Then $R$ is given by $\qquad$ —.
12. If the rate of change of volume of a sphere is equal to the rate of change of its radius, then the radius is $\qquad$ OR
The side of an equilateral triangle is increasing at the rate of $0.5 \mathrm{~cm} / \mathrm{s}$ then the rate of increase of its perimeter is $\qquad$
13. The function $\mathrm{f}(\mathrm{x})=\frac{x+1}{1+\sqrt{1+x}}$ is continuous at $\mathrm{x}=0$ if $\mathrm{f}(0)$ is $\qquad$ .
14. If $A$ and $B$ are two skew symmetric matrices of same order, then $A B$ is symmetric matrix if
$\qquad$ —.
15. The projection of vector $2 \hat{i}+3 \hat{j}-\hat{k}$ along the vector $\hat{i}+\hat{j}$ is $\qquad$
OR
The value of $\lambda$ for which the two vectors $\vec{a}=2 \hat{i}+\lambda \hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+2 \hat{j}+3 \hat{k}$ are orthogonal is
$\qquad$
16. $A$ is invertible matrix of order $3 \times 3$ and $|A|=9$, then find value of $\left|A^{-1}\right|$.
17. Evaluate $\int_{0}^{2} \sqrt{4-x^{2}} d x$

## OR

Evaluate $\int_{-1 / 2}^{1 / 2} \cos x \cdot \log \left(\frac{1+x}{1-x}\right) d x$
18. Evaluate: $\int \frac{1}{4+9 x^{2}} d x$
19. Evaluate $\int \frac{d x}{\sin ^{2} x \cos ^{2} x}$
20. Write the order and degree of the differential equation $\left(\frac{d^{2} y}{d x^{2}}\right)^{3}-5 \frac{d y}{d x}+6=0$.

## SECTION - B

## Questions 21 to 26 carry 2 marks each.

21. Prove that the Greatest Integer Function $f: R \rightarrow R$, given by $f(x)=[x]$ is neither one-one nor onto, where $[\mathrm{x}]$ denotes the greatest integer less than or equal to x .

## OR

Express $\tan ^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right), x<\pi$ in simplest form
22. Find the point at which the tangent to the curve $\mathrm{y}=\sqrt{4 x-3}-1$ has its slope $\frac{2}{3}$.
23. Form the differential equation representing the curve $y^{2}=a(b-x)(b+x)$ where $a$ and $b$ are arbitrary constants.
24. Find the equation of the perpendicular drawn from the point $(1,-2,3)$ to the plane $2 x-3 y+4 z+$ $9=0$.
25. If $\hat{a}, \hat{b}$ and $\hat{c}$ are mutually perpendicular unit vectors, then find the value of $|2 \hat{a}+\hat{b}+\hat{c}|$.

OR
If $\vec{a}=\hat{i}-\hat{j}+7 \hat{k}$ and $\vec{b}=5 \hat{i}-\hat{j}+\lambda \hat{k}$, then find the value of $\lambda$, so that $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ are perpendicular vectors.
26. The probability of simultaneous occurrence of at least one of the two events $A$ and $B$ is $p$. If the probability that exactly one of $A, B$ occurs is $q$, then prove that $P\left(A^{\prime}\right)+P\left(B^{\prime}\right)=2-2 p+q$.

## SECTION - C

## Q uestions 27 to 32 carry 4 marks each.

27. Show that the function f in $\mathrm{A}=\mathrm{R}-\left\{\frac{2}{3}\right\}$ defined as $\mathrm{f}(\mathrm{x})=\frac{4 x+3}{6 x-4}$ is one-one and onto. Hence, find $\mathrm{f}^{-1}$.
28. Find $\frac{d y}{d x},(\cos \mathrm{x})^{\mathrm{y}}=(\sin \mathrm{y})^{\mathrm{x}}$

OR
If $\log y=\tan ^{-1} x$, show that $\left(1+x^{2}\right) y_{2}+(2 x-1) y_{1}=0$
29. Find the following integral $\int(x-3) \sqrt{x^{2}+3 x-18} d x$
30. Find the particular solution of the differential equation $\frac{d y}{d x}=1+x+y+x y$, given that $y=0$ when $\mathrm{x}=1$.
31. A farmer has a supply of chemical fertiliser of type A which contains $10 \%$ nitrogen and $5 \%$ phosphoric acid, and type B which contains $6 \%$ nitrogen and $10 \%$ phosphoric acid. After testing the soil conditions of the field, it was found that at least 14 kg of nitrogen and 14 kg of phosphoric acid is required for producing a good crop. The fertiliser of type A costs 5 per kg and the type B costs 3 per kg. How many kg of each type of the fertiliser should be used to meet the requirement at the minimum possible cost? Using an LPP solve the above problem graphically.
32. $A$ and $B$ throw a die alternatively till one of them gets a ' 6 ' and wins the game. Find their respective probabilities of winning, if A starts first.

OR
Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards. What is the probability that
(i) all the five cards are spades?
(ii) only 3 cards are spades?

## SECTION - D <br> Q uestions 33 to 36 carry 6 marks each.

33. If $\mathrm{A}=\left[\begin{array}{ccc}2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2\end{array}\right]$, find $\mathrm{A}^{-1}$. Using $\mathrm{A}^{-1}$ solve the following system of equations: $2 x-3 y+5 z=16 ; 3 x+2 y-4 z=-4 ; x+y-2 z=-3$

OR
The sum of three numbers is 6 . If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method.
34. An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. Show that the cost of the material will be the least when the depth of the tank is half of its width.

## OR

Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{ } 2$ time the radius of the base.
35. Using integration find the area of the region bounded by the parabola $y^{2}=4 x$ and the circle $4 x^{2}+4 y^{2}=9$.
36. Find the distance of the point $(-2,3,-4)$ from the line $\frac{x+2}{3}=\frac{2 y+3}{4}=\frac{3 z+4}{5}$ measured parallel to the plane $4 x+12 y-3 z+1=0$.

