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S UBI ECT: MATHEMATICS (041)

BLUE PRINTI: CLASS XII

| Chapter | $\begin{gathered} \text { MCQ } \\ (1 \text { mark }) \\ \hline \end{gathered}$ | $\begin{gathered} \text { FIB } \\ (1 \text { mark }) \end{gathered}$ | $\begin{array}{\|c\|} \hline \text { VSA } \\ (1 \text { mark }) \\ \hline \end{array}$ | $\begin{array}{\|c} \hline \text { SA } \\ \text { (2 marks) } \end{array}$ | $\begin{gathered} \text { LA - I } \\ (4 \text { marks }) \end{gathered}$ | $\begin{array}{\|c\|} \hline \text { LA - II } \\ (6 \text { marks }) \\ \hline \end{array}$ | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Relations and Functions | -- | 1(1) | -- | 2(1)* | 4(1) | -- | 5(2) |
| Inverse Trigonometric Functions | 1(1) | -- | -- |  | -- | -- | 3(2) |
| Matrices | 2(2) | 1(1) | -- | -- | -- | -- | 3(3) |
| Determinants | -- | -- | 1(1) | -- | -- | 6(1)* | 7(2) |
| Continuity \& Differentiability | -- | 1(1) | -- | -- | 4(1)* | -- | 5(2) |
| Applications of Derivatives | -- | 1(1)* | -- | 2(1) | -- | 6(1)* | 9(3) |
| Integrals | 1(1) | -- | $\begin{aligned} & 2(2) \\ & \mathbf{1}(\mathbf{1})^{*} \end{aligned}$ | -- | 4(1) | -- | 8(5) |
| Applications of the Integrals | -- | -- | -- | -- | -- | 6(1) | 6(1) |
| Differential Equations | -- | -- | 1(1) | 2(1) | 4(1) | -- | 7(3) |
| Vector Algebra | 1(1) | 1(1)* | -- | 2(1)* | -- | -- | 4(3) |
| Three-Dimensional Geometry | 3(3) | -- | -- | 2(1) | -- | 6(1) | 11(5) |
| Linear Programming | -- | -- | -- | -- | 4(1) | -- | 4(1) |
| Probability | 2(2) | -- | -- | 2(1) | 4(1)* | -- | 8(4) |
| Total | 10(10) | 5(5) | 5(5) | 12(6) | 24(6) | 24(4) | 80(36) |

## Note: * - Internal Choice Questions

$\mathcal{S U B I} \mathcal{E C T}: \mathcal{M A T \mathcal { H E M A }} \operatorname{ICS}$
MAX. MARKS : 80
CLASS : XII

## General Instruction:

(i) All the questions are compulsory.
(ii) The question paper consists of $\mathbf{3 6}$ questions divided into 4 sections A, B, C, and D.
(iii) Section A comprises of $\mathbf{2 0}$ questions of $\mathbf{1}$ mark each. Section $\mathbf{B}$ comprises of $\mathbf{6}$ questions of $\mathbf{2}$ marks each. Section C comprises of $\mathbf{6}$ questions of $\mathbf{4}$ marks each. Section D comprises of $\mathbf{4}$ questions of $\mathbf{6}$ marks each.
(iv) There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

## SECTION - A

Questions 1 to 20 carry 1 mark each.

1. The domain of the function defined by $f(x)=\sin ^{-1} x+\cos x$ is
(a) $[-1,1]$
(b) $[-1, \pi+1]$
(c) $(-\infty, \infty)$
(d) $\varphi$
2. If a matrix $A$ is both symmetric and skew symmetric then matrix $A$ is
(a) a scalar matrix
(b) a diagonal matrix
(c) a zero matrix of order $\mathrm{n} \times \mathrm{n}$
(d) a rectangular matrix.
3. If $A=\left[\begin{array}{lll}0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0\end{array}\right]$, then $A^{6}$ is equal to
(a) zero matrix (b) A (c) I
(d) none of these
4. $\int_{a+c}^{b+c} f(x) d x$ is equal to
(a) $\int_{a}^{b} f(x) d x$
(b) $\int_{a}^{b} f(x-c) d x$
(c) $\int_{a}^{b} f(x+c) d x$
(d) $\int_{a-c}^{b-c} f(x) d x$
5. The vector in the direction of the vector $\hat{i}-2 \hat{j}+2 \hat{k}$ that has magnitude 9 is
(a) $\hat{i}-2 \hat{j}+2 \hat{k}$
(b) $3(\hat{i}-2 \hat{j}+2 \hat{k})$
(c) $9(\hat{i}-2 \hat{j}+2 \hat{k})$
(d) $\frac{\hat{i}-2 \hat{j}+2 \hat{k}}{3}$
6. The angle between straight lines whose direction cosines are $\left(\frac{1}{2},-\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$ and $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}},-\frac{1}{\sqrt{3}}\right)$ is
(a) $\cos ^{-1}\left(\frac{2}{\sqrt{3}}\right)$
(b) $\cos ^{-1}\left(\frac{1}{\sqrt{6}}\right)$
(c) $\cos ^{-1}\left(-\frac{1}{\sqrt{6}}\right)$
(d) None of these
7. The plane $2 x-3 y+6 z-11=0$ makes an angle $\sin ^{-1}(\alpha)$ with $x$-axis. The value of $\alpha$ is equal to
(a) $\frac{\sqrt{3}}{2}$
(b) $\frac{\sqrt{2}}{3}$
(c) $\frac{2}{7}$
(d) $\frac{3}{7}$
8. Which one of the following is best condition for the plane $a x+b y+c z+d=0$ to intersect the $x-$ and $y$-axis at equal angle?
(a) $|\mathrm{a}|=|\mathrm{b}|$
(b) $\mathrm{a}=-\mathrm{b}$
(c) $a=b$
(d) $a^{2}+b^{2}=1$
9. If $\mathrm{P}(\mathrm{A})=\frac{1}{2}, \mathrm{P}(\mathrm{B})=0$, then $\mathrm{P}(\mathrm{A} \mid \mathrm{B})$ is
(a) 0
(b) $\frac{1}{2}$
(c) not defined
(d) 1
10. Two events A and B will be independent, if
(a) A and $B$ are mutually exclusive
(b) $\mathrm{P}\left(\mathrm{A}^{\prime} \mathrm{B}^{\prime}\right)=[1-\mathrm{P}(\mathrm{A})][1-\mathrm{P}(\mathrm{B})]$
(c) $\mathrm{P}(\mathrm{A})=\mathrm{P}(\mathrm{B})$
(d) $\mathrm{P}(\mathrm{A})+\mathrm{P}(\mathrm{B})=1$
11. The value of $x$ and $y$, if $3\left[\begin{array}{ll}2 & 3 \\ 1 & x\end{array}\right]-\left[\begin{array}{cc}y & 0 \\ -1 & 2\end{array}\right]=\left[\begin{array}{cc}5 & 9 \\ 4 & 10\end{array}\right]$ is $\qquad$
12. The points of discontinuity for the function $f(x)=[x]$, in $-3<x<3$ are $\qquad$
13. The point on the curve $y=x^{2}-4 x+5$, where tangent to the curve is parallel to the $x$-axis is
$\qquad$

## OR

If the radius of a sphere is measured as 9 cm with an error of 0.03 cm , then the approximate error in calculating its surface area is $\qquad$
14. The vector $\vec{a}+\vec{b}$ bisects the angle between the non-collinear vectors $\vec{a}$ and $\vec{b}$ if $\qquad$

## OR

The value of ' $a$ ' for which the vectors $2 \hat{i}-3 \hat{j}+4 \hat{k}$ and $a \hat{i}+6 \hat{j}-8 \hat{k}$ are collinear is $\qquad$
15. The interval for which $\mathrm{f}(\mathrm{x})=\sin \mathrm{x}$ is one-one in $[0, \pi]$ is $\qquad$
16. Find the Integrating Factor of the differential equation $x \frac{d y}{d x}-y=2 x^{2}$
17. If $A^{2}-3 A+I=O$ and $A$ is a non-singular matrix, then write $A^{-1}$ in terms of $I$ and $A$.
18. Evaluate: $\int x^{x}(1+\log x) d x$

## OR

Evaluate: $\int \frac{x^{2}}{1+x^{3}} d x$
19. Evaluate: $\int \frac{1}{\sqrt{4-x^{2}}} d x$
20. Evaluate: $\int \frac{2-3 \sin x}{\cos ^{2} x} d x$

## SECTION - B

## Questions 21 to 26 carry 2 marks each.

21. If f is an invertible function defined as $\mathrm{f}(\mathrm{x})=\frac{3 x-4}{5}$, write $\mathrm{f}^{-1}(\mathrm{x})$.

## OR

If $\tan ^{-1} \alpha+\tan ^{-1} \beta=\frac{\pi}{4}$, then write the value of $\alpha+\beta+\alpha \beta$.
22. Show that the tangents to the curve $y=7 x^{3}+11$ are parallel at the points, where $x=2$ and $x=-2$.
23. Let $\vec{a}=a_{1} \hat{i}+b_{1} \hat{j}+c_{1} \hat{k}$ and $=\vec{b}=a_{2} \hat{i}+b_{2} \hat{j}+c_{2} \hat{k}$.

Prove that $\left(a_{1} a_{2}+b_{1} b_{2}+c_{1} c_{2}\right)^{2} \leq\left(a_{1}^{2}+b_{1}^{2}+c_{1}^{2}\right)\left(a_{2}^{2}+b_{2}^{2}+c_{2}^{2}\right)$.

## OR

If $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=4 \hat{i}-2 \hat{j}+3 \hat{k}$ and $\vec{c}=\hat{i}-2 \hat{j}+\hat{k}$, find a vector of magnitude 6 units which is parallel to the vector $2 \vec{a}-\vec{b}+3 \vec{c}$.
24. Find the angle $\theta$, between the line $\frac{x-2}{3}=\frac{y-3}{5}=\frac{z-4}{4}$ and the plane $2 x-2 y+z-5=0$.
25. Solve the differential equation $\frac{d y}{d x}=1+\mathrm{x}+\mathrm{y}+\mathrm{xy}$.
26. $A$ and $B$ are two independent events of an experiment. If $P($ not $A)=0.65, P(A \cup B)=0.65$ and $P(B)=p$, find $p$.

## SECTION - C

## Questions 27 to 32 carry 4 marks each.

27. Let a relation $\mathrm{R}_{1}$ on the set R of real numbers be defined $(a, b) \in R_{1} \Rightarrow 1+a b>0$ for all $\mathrm{a}, \mathrm{b} \in \mathrm{R}$. Show that $\mathrm{R}_{1}$ is reflexive and symmetric but not transitive.
28. A gardener has a supply of fertilizers of type I which consists of $10 \%$ nitrogen and $6 \%$ phosphoric acid and type II which consists of $5 \%$ nitrogen and $10 \%$ phosphoric acid. After testing the soil conditions, he finds that he needs at least 14 kg of nitrogen and 14 kg of phosphoric acid for his crop. If type I fertilizer costs 600 paise per kg and type II fertilizer costs 400 paise per kg, find how many kilograms of each fertilizer should be used so that nutrient requirements are met at a minimum cost?
29. If $\mathrm{y}=\frac{\sin ^{-1} x}{\sqrt{1-x^{2}}}$, show that $\left(1-x^{2}\right) \frac{d^{2} y}{d x^{2}}-3 x \frac{d y}{d x}-y=0$

OR
If $\mathrm{y}=e^{a \cos ^{-1} x},-1 \leq x \leq 1$, show that $\left(1-\mathrm{x}^{2}\right) \mathrm{y}_{2}-\mathrm{xy}_{1}-\mathrm{a}^{2} \mathrm{y}=0$.
30. Show that the equation $(x+y) d y+(x-y) d x=0$ is homogeneous. Also, find the particular solution given that $\mathrm{x}=1$ when $\mathrm{y}=1$.
31. Evaluate $\int \frac{\cos x}{(2+\sin x)(3+4 \sin x)} d x$.
32. If a fair coin is tossed 10 times, find the probability of (i) exactly six heads (ii) at least six heads OR
A problem based on statistics is given to three students A, B and C whose chances of solving it are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively,
(i) find the probability that only one of them solves it correctly.
(ii) find the probability that problem is solved.

## SECTION - D <br> Questions 33 to 36 carry 6 marks each.

33. The RWA (Resident Welfare Association) of a colony has three different subcommittees with total of 12 members. First subcommittee is adult education committee, which looks after the literacy needs of the adults, the second subcommittee is health and cleanliness committee, which looks after health and cleanliness needs of the colony and third subcommittee is safety committee, which looks after the safety needs of the colony. The number of members of the first subcommittee is half the sum of the members of the other two subcommittees and the number of members of the second subcommittee is the sum of the members of the other two subcommittees. Reduce the information in the form of algebraic statements and solve using matrices.

OR
Using properties of determinant, prove that $\left|\begin{array}{lll}1 & a & a^{3} \\ 1 & b & b^{3} \\ 1 & c & c^{3}\end{array}\right|=(a-b)(b-c)(c-a)(a+b+c)$
34. A given quantity of metal sheet is to be cast into a solid half circular cylinder with rectangular base and semi-circular ends. Show that in order that the total surface area may be minimum, the ratio of length of the cylinder to the diameter of its circular ends is $\pi: \pi+2$.

## OR

Prove that the volume of the largest cone that can be inscribed in a sphere of radius $R$ is $\frac{8}{27}$ of the volume of the sphere.
35. Find the area of the circle $x^{2}+y^{2}=6 x$, lying above the $x$-axis and bounded by the parabola $y^{2}=$ $3 x$.
36. Find the equation of the plane containing the lines $\mathrm{r}=\hat{i}+2 \hat{j}-\hat{k}+\lambda(2 \hat{i}-\hat{j}+\hat{k})$ and $\mathrm{r}=$ $\hat{i}+2 \hat{j}-\hat{k}+\mu(\hat{i}-\hat{j}+2 \hat{k})$. Also find the distance of this plane from origin.

