

UNIT-I.

Electrical Circuits & Measurements:-

All matter (solid, liquid & gas) are composed of minute particles called molecules. Each molecule is made up of atoms.

The central part of an atom is called an nucleus. The nucleus contains protons & neutron. Proton are +vely charged particles. Neutrons have no charge. Both have same mass.

The outer part contains e^- . Mass of e^- is small when compared to mass of protons & neutrons.

An e^- is -vely charged particle having -ve charge equal to the +ve charge of a proton.

Free e^- .

All the e^- in one atom move around its nucleus in different orbits. In inner orbits, the e^- are tightly bound to the nucleus. The e^- in the last orbit are loosely bound to the nucleus. These e^- are called valence e^- .

In metals, the valence e^- are so weakly attached to their nuclei that they can be removed. Not all the e^- are free e^- . But one can expect a large no. of free e^- in a small piece of metal.

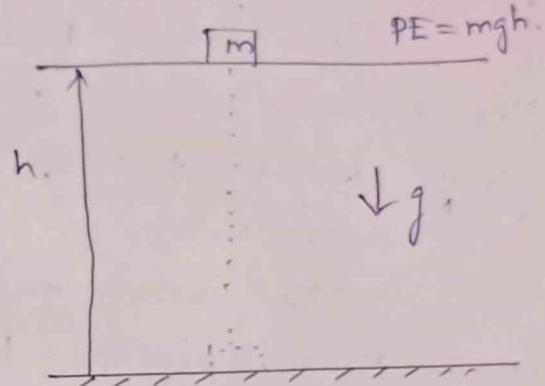
Electric Potential :-

When a body is charged, certain amount of work is done in charging it.

This workdone is stored in the form of potential energy. The charged body has the capacity to do work by moving the other charges by either attraction or repulsion.

The ability of the charged body to do work is called potential electric potential.

[The electric potential is capable of doing work just as the mass which was raised against gravitational force g to a height ' h ' above the ground plane.



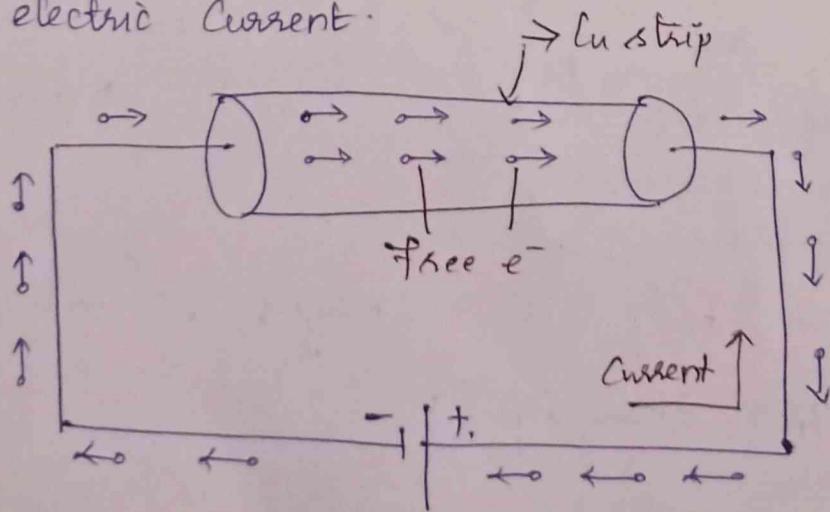
The potential mgh represents an ability to do work when mass m is released. As the mass falls, it accelerates & its potential energy is converted to KE]

Greater the capacity of a charged body to do work, the greater is its electric potential.

$$\text{Electric potential, } V = \frac{\text{Workdone}}{\text{charge}} = \frac{W}{Q}$$

Electric Current:

The flow of free e^- in a metal is called electric current.



The Cu strip has a large no. of free e^- . At any section of the strip, the no. of e^- that move from right to left equals the no. of e^- that move from left to right. The net no. of e^- crossing the section is zero. Hence, there is no flow of electric current under normal condition.

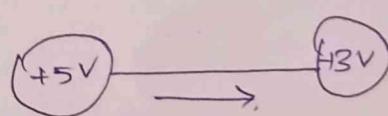
When electric pressure or voltage is applied then the free e^- which are very charged will start moving towards the terminal. The direction of current is from -ve terminal to +ve terminal.

$I \Rightarrow$ Rate of flow of e^- (ie) charge flowing per second.

$$I = \frac{dq}{dt} \text{ (or) } \frac{Q}{t}$$

Potential difference:-

The difference b/w ϕ in the potential of two charged bodies is called potential difference.



Unit of potential difference is same as that of electric potential (volt).

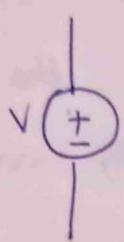
Active and Passive elements:-

Active elements produce energy in the form of voltage or current.

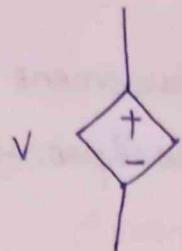
Eg: Voltage source, Current Sources

[Passive element stores energy in the form of voltage or current] Passive elements take energy from source.

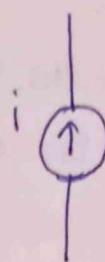
Eg: Resistors, Inductors, Capacitors



Independent voltage
Source



Dependent
Voltage Source



Indep.
Current Sour.



Dependent
Current Source

Resistance:-

The opposition offered by a substance to the flow of electric current is called resistance. (or) resistance may be defined as the physical property of a substance due to which it opposes or restricts the flow of electricity through it.

(Unit: Ohm (Ω)).

The resistance R of a conductor depends on the following factors

→ Length
 $R \propto l$.

→ Cross sectional area
 $R \propto l/a$.

→ Nature of Material

→ Temperature → Resistance changes with temp.

Assume const temp:-

$$R \propto l/a$$

$$R = \rho l/a$$

$\rho \Rightarrow$ constant & is known as a resistivity or specific resistance. Its value depends on the nature of material

Unit of specific resistivity \Rightarrow ohm-m.

Conductance:

Conductance is the inducement to the flow of current. Conductance is the reciprocal of resistance

$$G = 1/R.$$

$$R = \frac{\rho l}{A}.$$

$$G = \frac{1}{R} = \frac{1}{\rho l/A} = \frac{1}{\rho} \frac{A}{l} = \sigma \frac{A}{l}.$$

$$[\because \sigma = 1/\rho]$$

\Rightarrow Conductivity or specific conductance of the material.

Unit: mho (Ω^{-1})

Inductance:-

The circuit element that stores energy in a magnetic field is an inductor.

↓
is the area around a magnet in which the effect of magnetism is felt.

$$P = VI = L \frac{di}{dt} i = \frac{d}{dt} \left[\frac{1}{2} L i^2 \right]$$

$$N_{L0} = \int_{t_1}^{t_2} P dt = \int_{t_1}^{t_2} L i dt = \frac{1}{2} \left[i^2 \right]_{t_1}^{t_2} \\ = \frac{1}{2} [i]$$

$$W_L = \frac{1}{2} L i^2.$$

Conductance Capacitance:

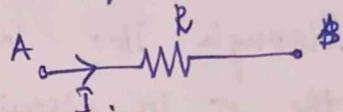
The circuit element stores some energy in an electric field is a capacitor

Ohm's law:

The relationship b/w the potential diff., current & resistance in a d.c circuit \Rightarrow discovered by Scientist George Simon Ohm

At constant temperature, the voltage or potential diff b/w two points of a conductor is directly proportional to the current flowing through it

$$V \propto I.$$



$$V = IR.$$

R = Proportionality constant
↓
Resistance.

Limitations:-

\Rightarrow Ohm's law does not apply to all ^{non-}
metallic conductors. ^{graphite}

\Rightarrow It also does not apply to non-linear devices such as Zener diode, voltage-regulator and etc.

\Rightarrow Ohm's law is true for metal conductors at constant temperature. If the temperature changes, the law is not applicable.

By Ohm's law

$$V = IR ; \quad R = \frac{V}{I} ; \quad I = \frac{V}{R}.$$

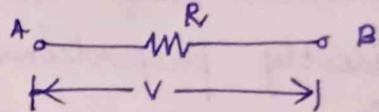
\Rightarrow If Ohm's law is expressed graphically.
Y-axis \Rightarrow Voltage & Current in x-axis. The graph will be a st. line passing through the origin. The slope of the graph will give the resistance.

Electric Power:-

The rate at which work is done in an electric circuit is called electric power.

$$\text{Electric Power} = \frac{\text{Workdone in electric circuit}}{\text{Time}}$$

When voltage is applied to circuit, it causes current to flow through it. Work is done in moving the e in circuit.



$$V = \frac{\text{Workdone}}{\alpha}$$

$$I = \frac{\alpha}{E}$$

$$\alpha = I \times t$$

$$V = \frac{W}{IE}$$

$$W = VIE$$

$$P = \frac{W}{t} = \frac{VIE}{t} = VI \text{ Joules/s (or) Watt}$$

$$P = VI \text{ Watts.}$$

$$P = IRI = I^2 R$$

$$P = V^2/R$$

Other Units of Power:-

⇒ Power is also measured in horse power (HP)

$$1 \text{ H.P.} = 735 \text{ Watts.}$$

$$\Rightarrow \text{Power} = \frac{2\pi NT}{60 \times 735} \text{ H.P.}$$

T → Torque (Nm)

N → Speed in rpm

$$\text{Workdone/Minute} = 2\pi NT$$

$$W/s = \frac{2\pi NT}{60} \text{ (Watts)}$$

Electrical Energy:-

The total workdone in an electric Ckt is called electrical energy.

$$\begin{aligned}\text{Elec. Energy} &= P \cdot I \times \text{Total charge flow} \\ &= V Q \\ &= VIE \text{ Joules (or) Watt \cdot sec.}\end{aligned}$$

$$\begin{aligned}\text{Elec. energy} &= VIE \\ &= I^2 R E \\ &= V^2 / R \times E.\end{aligned}$$

Electrical energy is the product of electric power & time for which current flows in the circuit

Network Terminology:-

1. Circuit:-

The closed path followed by an electric current is called an electrical circuit.

2. Parameters:

The various elements of an electric circuit are called its parameters. & these may be lumped or distributed

Eg: Resistance, Capacitance &
Inductance.

Lumped element \Rightarrow physically separable elements are termed as lumped elements.

Distributed elements \Rightarrow If parameters are distributed among the line, then it is termed as distributed elements.

Node:

Node is a junction where two or more circuit elements are connected together.

Branch:

A branch is the part of a circuit which lies between two junction points.

Loop:

A loop is any closed path of a circuit.

Mesh:

A mesh is the most elementary form of a loop and cannot be further divided into loops.

Kirchoff's Current law:-

The algebraic sum of currents meeting at a junction or node in an electrical circuit is zero.

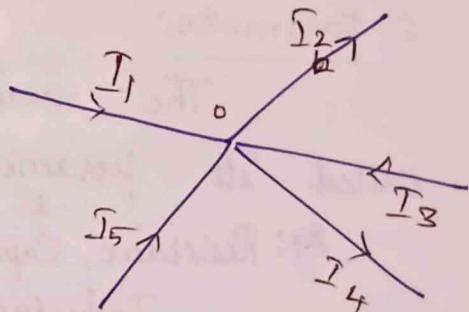
The algebraic sum of currents flowing towards any junction in an electric circuit is equal to the algebraic sum of the currents flowing away from the junction.

By applying KCL at junction o,
we get

$$(+I_1) + (-I_2) + (+I_3) + (-I_4) + (+I_5) = 0.$$

$$I_1 - I_2 + I_3 - I_4 + I_5 = 0$$

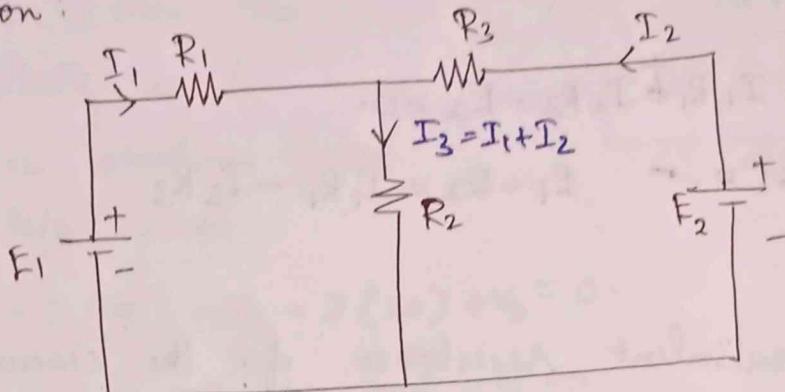
$$\boxed{I_1 + I_3 + I_5 = I_2 + I_4}$$



Kirchhoff's voltage law:

In an closed circuit or mesh or loop, the algebraic sum of all the voltages taken around is zero.

Illustration:



A rise in potential can be assumed +ve while fall in potential is assumed -ve

→ If we go from +ve terminal to -ve, there is a fall in potential & so that the voltage should be -ve. If we go from -ve to +ve of the battery source, then there is rise in potential and so it should be +ve sign.

→ When current flows through a resistor, there is a voltage drop across it. If the resistance is in same direction as that of the current, then there is fall in potential. So, the sign of the voltage drop is -ve. If we go opposite to the direction of current flow, there is rise in potential & the voltage drop is +ve.

Loop ABCFA: $E_1 \rightarrow +ve \Rightarrow -ve \text{ to } +ve$.

$$E_1 - I_1 R_1 - I_3 R_2 = 0$$

$$E_1 = I_1 R_1 + I_3 R_2.$$

Loop EDCFE :-

$$E_3 - I_2 R_3 + - I_3 R_2 = 0$$

$$E_3 = I_2 R_3 + I_3 R_2.$$

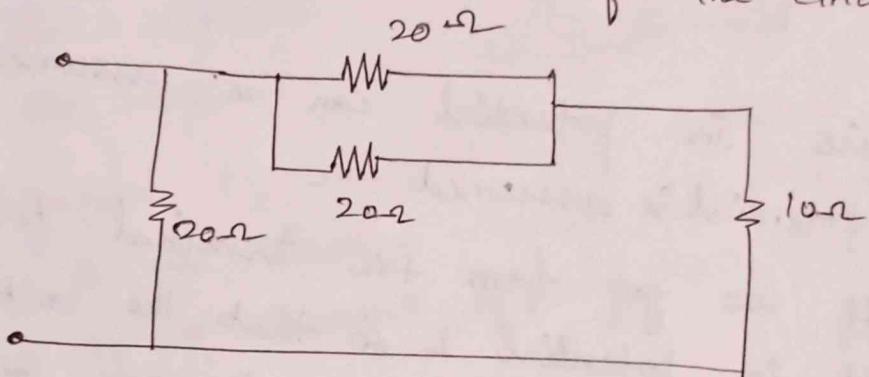
Loop ABCDEFA:-

$$E_1 - I_1 R_1 + I_2 R_3 - E_2 = 0.$$

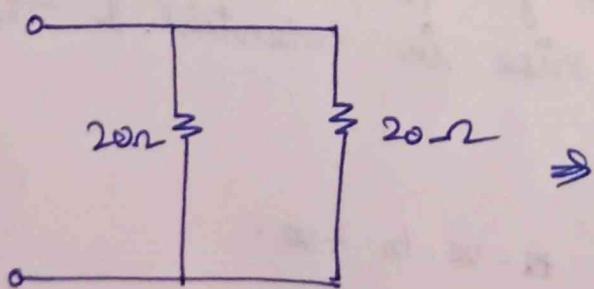
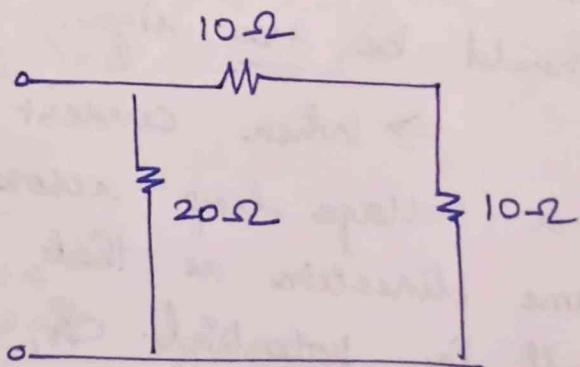
$$E_1 = E_2 \quad E_1 - E_2 = I_1 R_1 - I_2 R_3$$

Problem :-

Find the equivalent resistance for the circuit



$$\frac{20\Omega}{20\Omega} = \frac{20 \times 20}{20+20} = \frac{400}{40} = 10\Omega$$



\Rightarrow ~~AAA~~

$$\frac{20 \times 20}{20+20} = \frac{400}{40} = 10\Omega_{//}$$

Problem:

Find V_3 and its polarity if the current I in the circuit given is 0.40A

Assume V_3 has the same polarity as V_1 .

Apply KVL starting from lower left corner

$$V_1 - I(5) - V_2 - I(20) + V_3 = 0$$

$$V_1 = -5I - V_2 - 20I + V_3 = 0$$

$$50 - 5I - 10 - 20I + V_3 = 0$$

$$40 - 25I + V_3 = 0$$

$$V_3 = -40 + 25I$$

$$V_3 = -40 + 25(0.4)$$

$$V_3 = -40 + 10 = -30$$

$V_3 = -30\text{V}$

Problem:

Obtain the currents I_1 & I_2 for the network shown in figure

Sol:

a & b comprise one node.

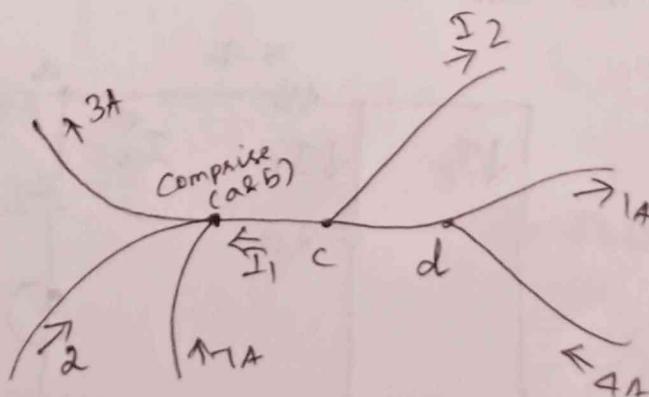
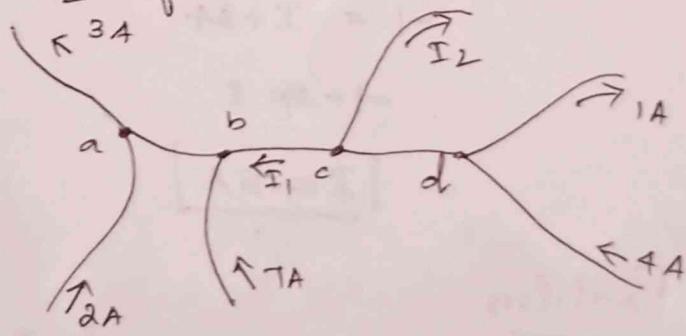
Applying KCL,

$$2 + 7 + I_1 = 3$$

$$9 + I_1 = 3$$

$$I_1 = 3 - 9$$

$I_1 = -6\text{ A}$



C & D comprise a single node

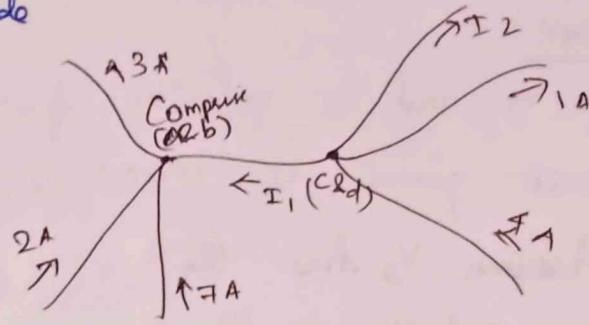
$$I_1 + I_2 + I = 4$$

$$-6 + I_2 + I = 4$$

$$-5 + I_2 = 4$$

$$I_2 = 4 + 5$$

$$\boxed{I_2 = 9A}$$



Problem:

Find the Current I for the circuit shown in figure

The branch currents within the enclosed area cannot be calculated since no values of the resistors are given.

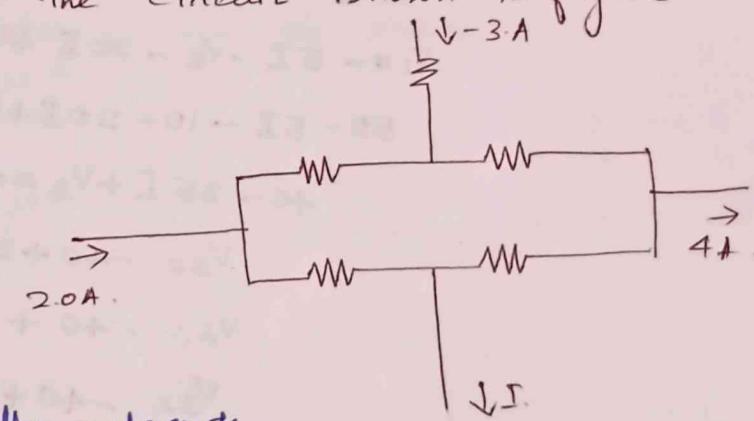
Hence KCL applies to the network taken as a single node. Thus

$$2 + (-3) = I + 4$$

$$-1 = I + 4$$

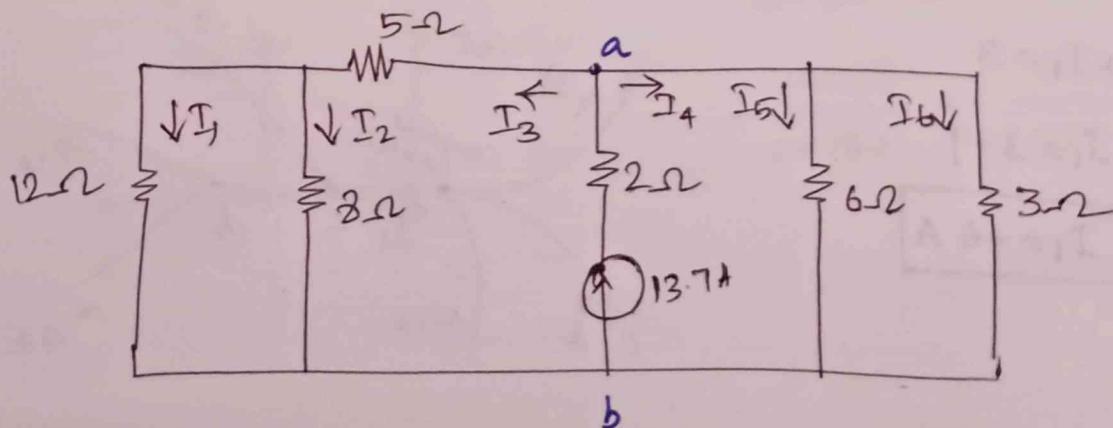
$$-1 - 4 = I$$

$$\boxed{I = -5A}$$



Problem

Find all branch currents in the network



The equivalent resistances to the left & right of nodes a & b are

$$R_{eq\ (left)} = 5 + (12||8)$$

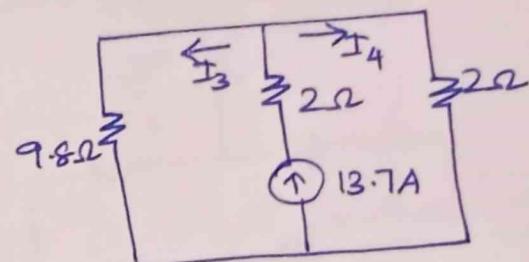
$$= 5 + \frac{12 \times 8}{12+8} = 5 + \frac{96}{20} = 5 + 4.8 = 9.8 \Omega$$

$$R_{eq\ (right)} = 6||3$$

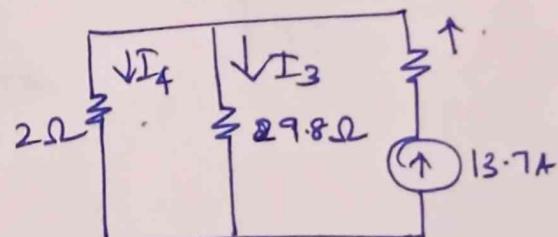
$$= \frac{6 \times 3}{6+3} = \frac{18}{9} = 2 \Omega$$

By current division rule,

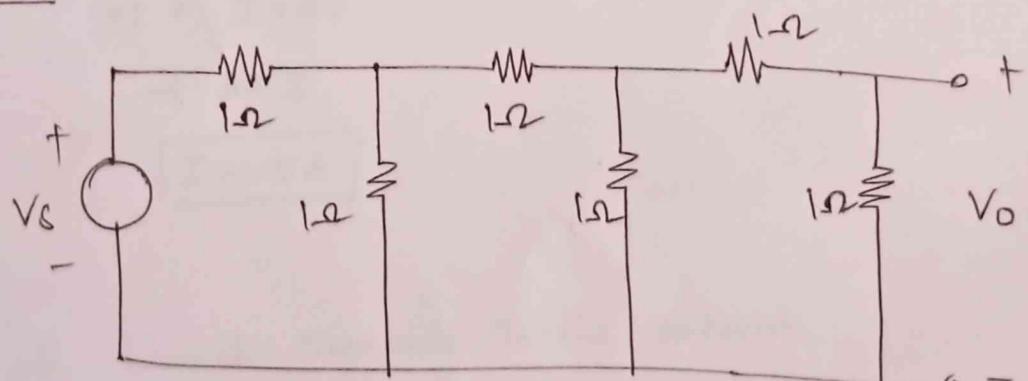
$$I_3 = 13.7 \times \frac{9.8}{11.8} = 2.32 A$$



$$I_4 = 13.7 \times \frac{9.8}{11.8} = 11.38 A$$



Problem :



$$\text{Find } \frac{V_S}{V_0}$$

Sol:

Voltage division:

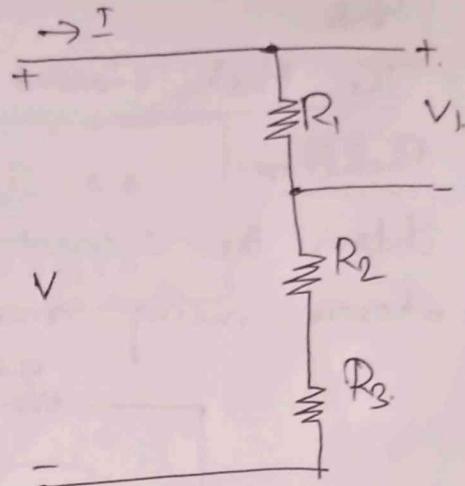
$$V_1 = i R_1 \rightarrow ①$$

$$V = i (R_1 + R_2 + R_3)$$

$$V_1 = V \left[\frac{R_1}{R_1 + R_2 + R_3} \right]$$

$$V = i = \frac{V}{R_1 + R_2 + R_3}$$

Sub i in ①



Current division:-

$$i = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$i_1 = i \times \frac{R_2}{R_1 + R_2} \quad i_1 = V \left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right] +$$

$$V = \frac{i}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

$$i_1 = \frac{V}{R_1} \quad \Rightarrow \frac{i}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) R_1}$$

Problem:

A voltage divider circuit of two resistors is designed with a total resistance of the two resistors equal to 50Ω . If the output voltage is 10 percent of the input voltage, obtain the values of the two resistors in the circuit

Sol:

$$V_o = 10\% \text{ of } V_i$$

$$R_1 + R_2 = 50\Omega$$

$$V_o = \frac{10}{100} V_i$$

$$R_1 + R_2 = 50\Omega$$

$$V_o = 0.1 V_i$$

$$R_2 = 50 - 5$$

$$V_o = V_i \frac{R_1}{R_1 + R_2}$$

$$\boxed{R_2 = 45\Omega}$$

$$\frac{0.1 V_i}{V_i} = \frac{R_1}{R_1 + R_2}$$

$$0.1 = \frac{R_1}{50}$$

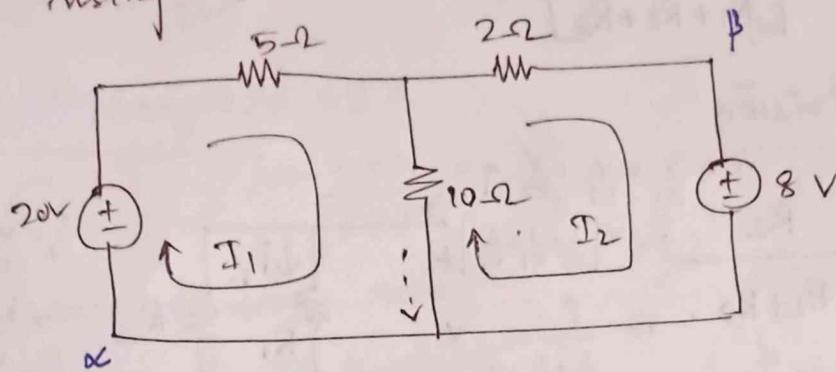
$$R_1 = 5\Omega$$

Prob

The Mesh Current Method:-

Problem:

Obtain the current in each branch of the network shown using mesh current method



The currents I_1 & I_2 are shown in the circuit diagram.

Applying KVL

$$20 - 5I_1 - 10(I_1 - I_2) = 0$$

$$20 - 5I_1 - 10I_1 + 10I_2 = 0$$

$$20 - 15I_1 + 10I_2 = 0$$

$$15I_1 - 10I_2 = 20 \rightarrow ①$$

Apply KVL around right loop

$$-10I_2 - 2I_1 - 10(I_2 - I_1) - 2I_2 - 8 = 0$$

$$-10I_2 + 10I_1 - 2I_2 - 8 = 0$$

$$10I_1 - 12I_2 = 8 \rightarrow ②$$

Solve ① & ②

$$① \times 10$$

$$150I_1 - 100I_2 = 200$$

$$I_2 = 80/80$$

$$② \times 15$$

$$\begin{array}{r} \cancel{150I_1} - 180I_2 = \cancel{120} \\ (+) \qquad \qquad \qquad (-) \\ 80I_2 = 80 \end{array}$$

$$I_2 = 1 \text{ A}$$

Sub I_1 in ①

$$15I_1 - 10I_2 = 20$$

$$15I_1 - 10(1) = 20$$

$$15I_1 = 30$$

$$I_1 = \frac{30}{15}$$

$$I_1 = 2A$$

$$I_2 = 1A$$

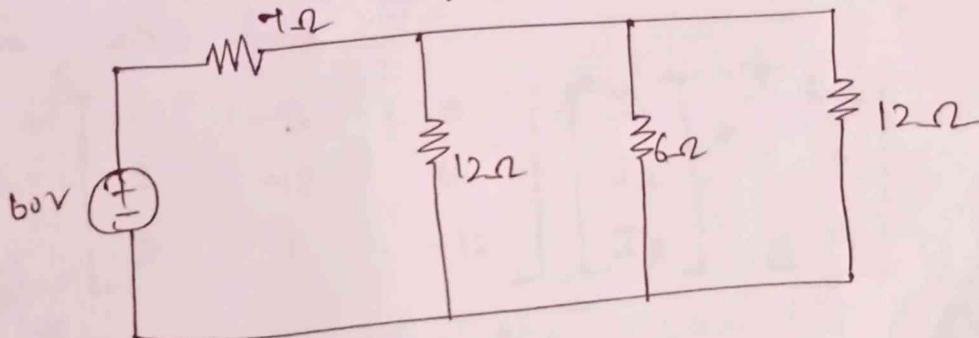
$$\boxed{I_1 = 2A}$$

Current through 10Ω

$$I_1 - I_2 = 2 - 1 \Rightarrow 1A$$

Problem :-

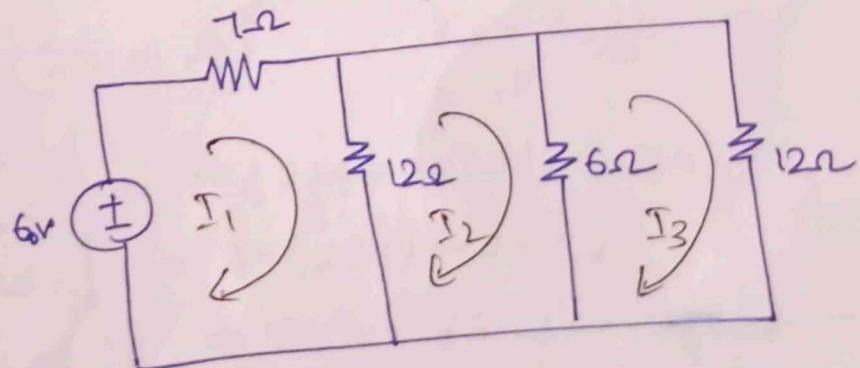
Solve the problem by mesh current method.



Sol:

By applying KVL for
3 loops

Loop 1 :-



$$60 - 7I_1 - 12(I_1 - I_2) = 0$$

$$60 - 7I_1 - 12I_1 + 12I_2 = 0$$

$$60 - 19I_1 + 12I_2 = 0$$

$$19I_1 - 12I_2 = 60 \rightarrow \textcircled{1}$$

Loop 2:

$$-12(I_2 - I_1) - 6(I_2 - I_3) = 0$$

$$-12I_2 + 12I_1 - 6I_2 + 6I_3 = 0$$

$$12I_1 - 18I_2 + 6I_3 = 0 \rightarrow ②$$

Loop 3:

$$-6(I_3 - I_2) - 12I_3 = 0$$

$$-6I_3 + 6I_2 - 12I_3 = 0$$

$$6I_2 - 18I_3 = 0 \rightarrow ③$$

$$19I_1 - 12I_2 = 60$$

$$12I_1 - 18I_2 + 6I_3 = 0$$

$$6I_2 - 18I_3 = 0$$

In Matrix form

$$\begin{bmatrix} 19 & -12 & 0 \\ 12 & -18 & 6 \\ 0 & 6 & -18 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 0 \\ 0 \end{bmatrix}$$

By Using Cramer's rule

$$I_1 = \Delta_1 / \Delta$$

$$I_2 = \Delta_2 / \Delta$$

$$I_3 = \Delta_3 / \Delta$$

$$\Delta = \begin{vmatrix} 19 & -12 & 0 \\ 12 & -18 & 6 \\ 0 & 6 & -18 \end{vmatrix} = 19(324 - 36) + 12(-216) \\ = 19(288) - 2592 \\ \Rightarrow 5472 - 2592 = 2880$$

$$\Delta_1 = \begin{vmatrix} 60 & -12 & 0 \\ 0 & -18 & 6 \\ 0 & 6 & -18 \end{vmatrix} = 60(324 - 36) + 12(0) + 0 \\ = 60(288) = 17280$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{17280}{2880}$$

$$\boxed{I_1 = 64}$$

$$\Delta_2 = \begin{vmatrix} 19 & 60 & 0 \\ 12 & 0 & 6 \\ 0 & 0 & -18 \end{vmatrix} = 19(0) - 60(-216) + 0 = 12960$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{12960}{2880} = 4.5A$$

$$\boxed{I_2 = 4.5A}$$

$$\Delta_3 = \begin{vmatrix} 19 & -12 & 60 \\ 12 & -18 & 0 \\ 0 & 6 & 0 \end{vmatrix} = 19(0) + 12(0) + 60(72) = 4320$$

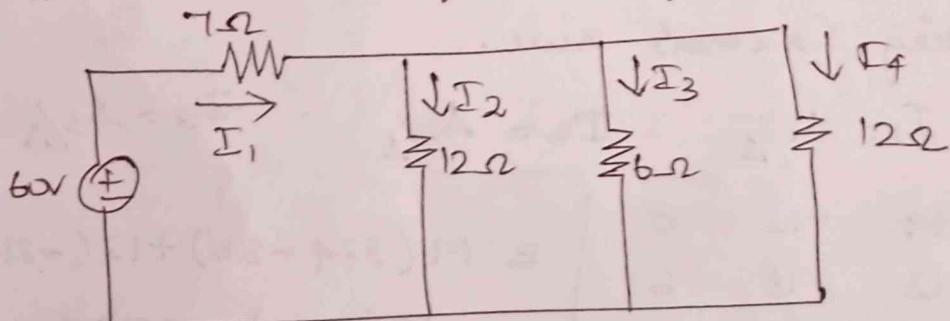
$$I_3 = \frac{\Delta_3}{\Delta} = \frac{4320}{2880} = 1.5A$$

$$\boxed{I_3 = 1.5A}$$

$$\boxed{\begin{aligned} I_1 &= 6A \\ I_2 &= 4.5A \\ I_3 &= 1.5A \end{aligned}}$$

Problem:

Find the current supplied by 60V source



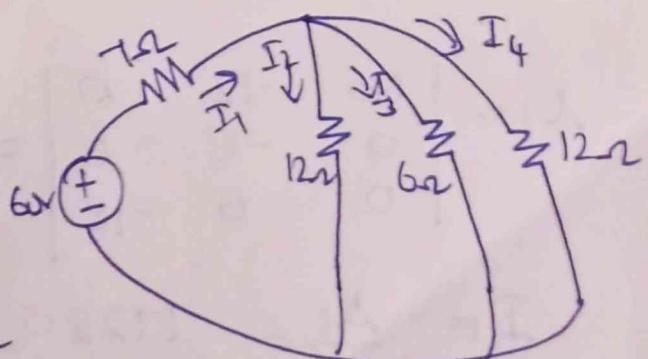
By using KCL

$$I_1 = I_2 + I_3 + I_4 \rightarrow ①$$

Since 12Ω, 6Ω & 12Ω are in 1st voltage will be same

$$2I_2 = I_3$$

$$2I_2 = I_4 \rightarrow ②$$



$$12I_2 = 12I_4$$

$$I_2 = I_4 \rightarrow \textcircled{3}$$

Apply KVL for first loop

$$+60 - 7I_1 - 12I_2 = 0$$

$$7I_1 + 12I_2 = 60 \rightarrow \textcircled{4}$$

Sub \textcircled{2} & \textcircled{3} in \textcircled{1}

$$I_1 = I_2 + I_3 + I_4.$$

$$I_1 = I_2 + 2I_2 + I_2 = 4I_2$$

$$\boxed{I_1 = 4I_2}$$

Sub I_1 from above eqn in \textcircled{4}

$$7(4I_2) + 12I_2 = 60$$

$$28I_2 + 12I_2 = 60$$

$$I_1 = 4I_2$$

$$16I_2 = 60$$

$$I_2 = \frac{60}{16}$$

$$I_2 = 3.75A$$

$$40I_2 = 60$$

$$\boxed{I_2 = 1.5A}$$

$$I_1 = 4 \times 1.5$$

$$\boxed{I_1 = 6A.}$$

$$I_3 = 2I_2$$

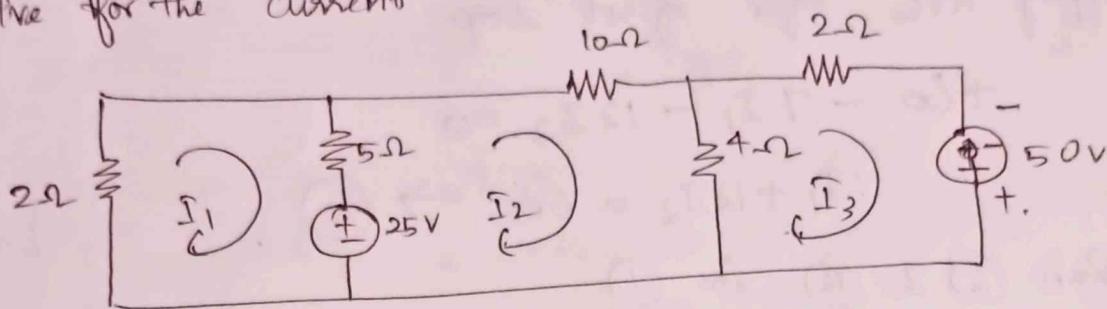
$$\boxed{I_3 = 3A}$$

$$I_2 = I_4$$

$$\boxed{I_2 = 1.5A}$$

Problem:

Write the mesh current matrix equation for the network & solve for the currents



Sol:

Loop 1:-

$$-2I_1 - 5(I_1 - I_2) - 25 = 0$$

$$-2I_1 - 5I_1 + 5I_2 - 25 = 0$$

$$-7I_1 + 5I_2 - 25 = 0$$

$$-7I_1 + 5I_2 = 25 \rightarrow ①$$

Loop 2:

~~$$25 - 5(I_2 - I_1) - 10I_2 = 0$$~~

~~$$25 - 5I_2 + 5I_1 - 10I_2 = 0$$~~

~~$$5I_1 - 15I_2 + 25 = 0$$~~

$$25 - 5(I_2 - I_1) - 10I_2 - 4(I_2 - I_3) = 0$$

$$25 - 5I_2 + 5I_1 - 10I_2 - 4I_2 + 4I_3 = 0.$$

$$5I_1 - 19I_2 + 4I_3 = -25 \rightarrow ②$$

Loop 3:

$$-4(I_3 - I_2) - 2I_3 + 50 = 0.$$

$$-4I_3 + 4I_2 - 2I_3 = -50.$$

$$4I_2 - 6I_3 = -50 \rightarrow ③$$

$$\begin{bmatrix} -7 & 5 & 0 \\ 5 & -19 & 4 \\ 0 & 4 & -6 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 25 \\ -25 \\ -50 \end{bmatrix}$$

By Cramers Rule

$$\Delta = \begin{vmatrix} -7 & 5 & 0 \\ 5 & -19 & 4 \\ 0 & 4 & -6 \end{vmatrix} = -7(114 - 16) - 5(-30) + 0 = -7(98) + 150 = -686 + 150 = -536$$

$$\Delta_1 = \begin{vmatrix} 25 & 5 & 0 \\ -25 & -19 & 4 \\ -50 & 4 & -6 \end{vmatrix} = 25(114-16) - 5(150+200) + 0 \\ = 25(98) - 5(350) = 2450 + 250 \\ = 2700 \quad 2450 - 1750 \\ = 700 \parallel.$$

$$\Delta_2 = \begin{vmatrix} -7 & 25 & 0 \\ 5 & -25 & 4 \\ 0 & -50 & -6 \end{vmatrix} = -7(150+200) - 25(-30) \\ = -7(350) + 750 = -2450 + 750 \\ = -1700$$

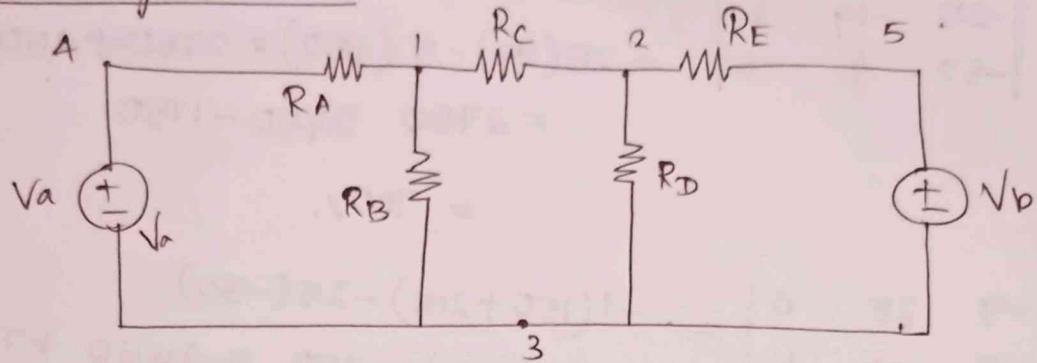
$$\Delta_3 = \begin{vmatrix} -7 & 5 & 25 \\ 5 & -19 & -25 \\ 0 & 4 & -50 \end{vmatrix} = -7(950+100) - 5(-250) + 25(20) \\ = -7(1050) + 1250 + 500 = -7350 + 1750 \\ = -5600$$

$$I_1 = A_1 / \Delta = \frac{700}{-536} = -1.305 \parallel = -1.31 \text{ A} \cdot$$

$$I_2 = A_2 / \Delta = \frac{-1700}{-536} = 3.17 \text{ A}$$

$$I_3 = A_3 / \Delta = \frac{-5600}{-536} = 10.45 \text{ A} \parallel.$$

Node Voltage Method :-



It has 5 nodes

1 & 2 & 3 → Principal nodes.

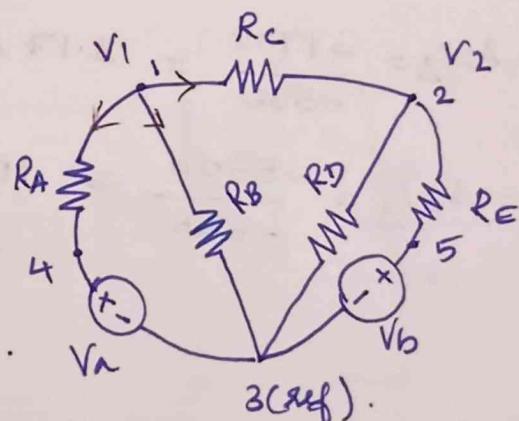
4, 5 → Simple nodes.

In ~~one~~ principal node, one node is taken as reference. 2 equations based on KCL are written at the other pair of principal nodes.

Node 3 is taken as ref for voltages V_1 & V_2 .

Apply KCL:

Assume current at the particular node as outgoing.



At node 1:

Apply KCL

$$\frac{V_1 - V_a}{R_A} + \frac{V_1}{R_B} + \frac{V_1 - V_2}{R_C} = 0.$$

At Node 2:

Apply KCL

$$\frac{V_2 - V_1}{R_C} + \frac{V_2}{R_D} + \frac{V_2 - V_b}{R_E} = 0.$$

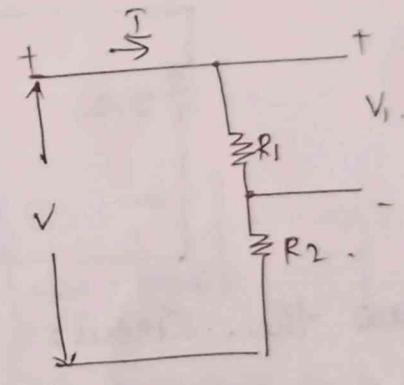
Voltage division

$$V_1 = I R_1$$

$$V = I(R_1 + R_2) \quad [\because I = \frac{V}{R_1 + R_2}]$$

$$V_1 = \frac{V}{R_1 + R_2} R_1$$

$$\boxed{V_1 = V \cdot \frac{R_1}{R_1 + R_2}}$$

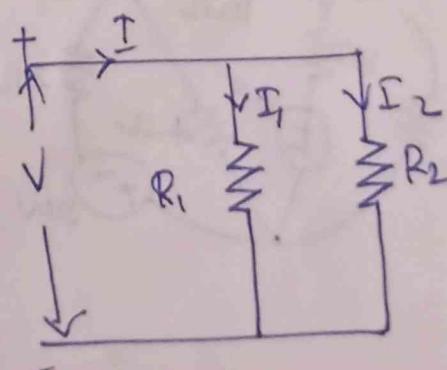


Current division:

$$I = \frac{V}{R_1} + \frac{V}{R_2}$$

$$I_1 = \frac{V}{R_1}$$

$$I = V \left[\frac{1}{R_1} + \frac{1}{R_2} \right]$$



$$V = \frac{I}{\frac{1}{R_1} + \frac{1}{R_2}}$$

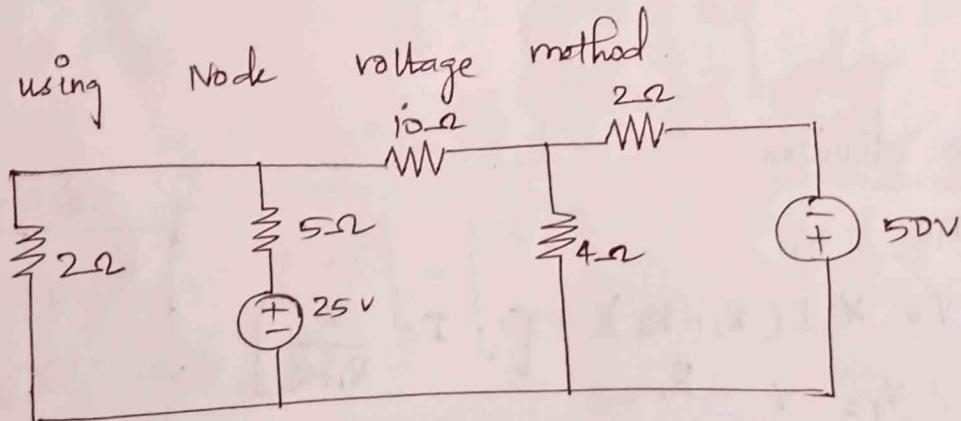
$$I_1 = \frac{V}{R_1} = \frac{\frac{I}{\frac{1}{R_1} + \frac{1}{R_2}}}{R_1} = \frac{\frac{I}{R_2 + R_1}}{\frac{R_1 R_2}{R_1 + R_2}} = \frac{I}{R_1 + R_2}$$

$$= \frac{I}{\frac{R_2 + R_1}{R_1 R_2} \cdot R_1} = I \frac{R_2}{R_1 + R_2}$$

$$I_1 = I \left[\frac{R_2}{R_1 + R_2} \right]$$

Problem:

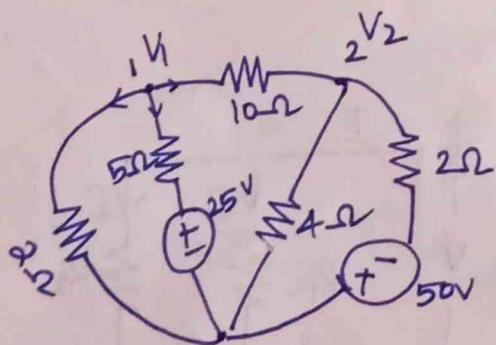
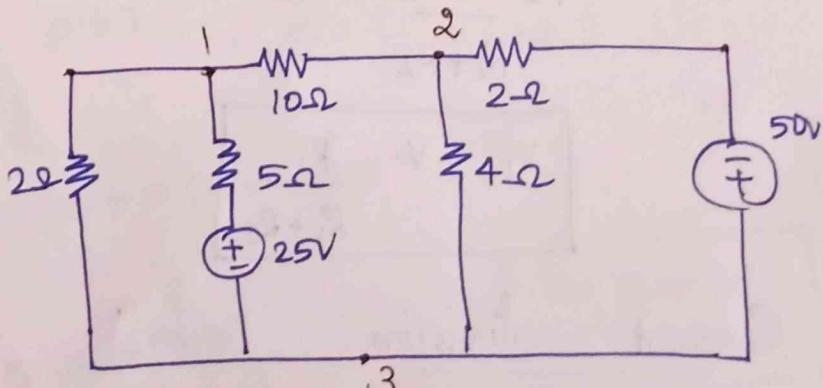
Solve by using Node voltage method.



Redraw the circuit

3, 1 & 2 \Rightarrow Principal Node

3 \Rightarrow taken as ref



Applying KCL.

$$\frac{V_1}{2} + \frac{V_1 - 25}{5} + \frac{V_1 - V_2}{10} = 0$$

$$\frac{V_1}{2} + \frac{V_1}{5} - \frac{25}{5} + \frac{V_1}{10} - \frac{V_2}{10} = 0$$

$$\frac{V_1}{2} + \frac{V_1}{5} + \frac{V_1}{10} - \frac{V_2}{10} = 5$$

$$V_1 \left[\frac{1}{2} + \frac{1}{5} + \frac{1}{10} \right] - \frac{V_2}{10} = 5 \quad \rightarrow ①$$

At Node 2:

Applying KCL.

$$\frac{V_2}{4} + \frac{V_2 - V_1}{10} + \frac{V_2 + 50}{\frac{1}{4}+2} = 0$$

$$\frac{V_2}{4} + \frac{V_2}{10} - \frac{V_1}{10} + \frac{V_2}{\frac{1}{4}+2} + \frac{50}{\frac{1}{4}+2} = 0$$

$$-\frac{V_1}{10} + V_2 \left[\frac{1}{4} + \frac{1}{10} + \frac{1}{\frac{1}{4}+2} \right] = -\frac{50}{\frac{1}{4}+2} \quad \rightarrow ②$$

Write ① & ② in matrix form.

$$\begin{bmatrix} \frac{1}{2} + \frac{1}{5} + \frac{1}{10} & -\frac{1}{10} \\ -\frac{1}{10} & \frac{1}{4} + \frac{1}{10} + \frac{1}{2} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 5 \\ -25 \end{bmatrix}$$

$$\begin{bmatrix} 0.8 & -0.1 \\ -0.1 & 0.85 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 5 \\ -25 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 0.8 & -0.1 \\ -0.1 & 0.85 \end{vmatrix} \Rightarrow 0.67 \quad \Delta_1 = \begin{vmatrix} 5 & -0.1 \\ -25 & 0.85 \end{vmatrix} = 1.75$$

$$\Delta_2 = \begin{vmatrix} 0.8 & 5 \\ -0.1 & -25 \end{vmatrix} = -20 + 0.5 = -19.5$$

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{1.75}{0.67} = 2.612 \text{ V}$$

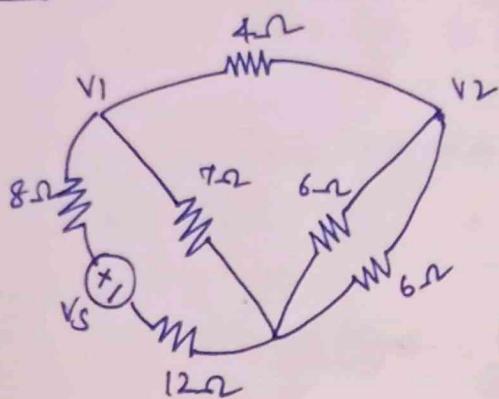
$V_1 = 2.612 \text{ V}$
$V_2 = -29.1 \text{ V}$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{-19.5}{0.67} = -29.1 \text{ V}$$

Problem:

For the network shown, find V_s which makes $\text{out } I_o = 9.5 \text{ mA}$

Sol..



Node 1:

Apply KCL

$$\frac{V_1 - V_s}{8+12} + \frac{V_1}{7} + \frac{V_1 - V_2}{4} = 0.$$

$$\frac{V_1 - V_s}{20} + \frac{V_1}{7} + \frac{V_1 - V_2}{4} = 0.$$

$$\frac{V_1}{20} - \frac{V_s}{20} + \frac{V_1}{7} + \frac{V_1}{4} - \frac{V_2}{4} = 0$$

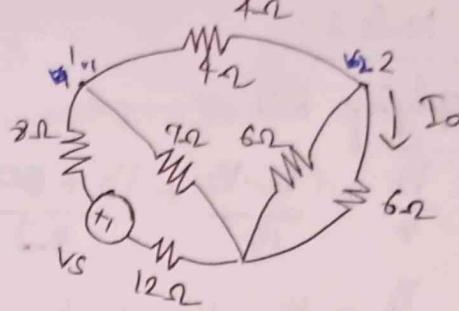
$$V_1 \left[\frac{1}{20} + \frac{1}{7} + \frac{1}{4} \right] - \frac{V_2}{4} = \frac{V_s}{20}$$

$$\begin{bmatrix} \frac{1}{20} + \frac{1}{7} + \frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} + \frac{1}{6} + \frac{1}{6} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_s/20 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 0.443 & -0.25 \\ -0.25 & 0.583 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_s/20 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 0.443 & -0.25 \\ -0.25 & 0.583 \end{vmatrix} = 0.258 + 0.196$$

$$\Delta_1 = \begin{vmatrix} V_s/20 & -0.25 \\ 0 & 0.583 \end{vmatrix} = \left(\frac{V_s}{20} \times 0.583 \right) - 0 = 0.029 V_s$$



Node 2:

Apply KCL

$$\frac{V_2 - V_1}{4} + \frac{V_2}{6} + \frac{V_2}{6} = 0$$

$$\frac{V_2}{4} - \frac{V_1}{4} + \frac{V_2}{6} + \frac{V_2}{6} = 0$$

$$-\frac{V_1}{4} + V_2 \left[\frac{1}{4} + \frac{1}{6} + \frac{1}{6} \right] = 0$$

$$V_1 = \frac{A_1}{\Delta} = \frac{0.029 V_s}{0.196}$$

$$\Delta_2 = \begin{vmatrix} 0.443 & V_s/20 \\ -0.25 & 0 \end{vmatrix} = 0 + \frac{V_s}{20} (+0.25) = +0.0125 V_s$$

$$V_2 = \frac{\Delta_2}{\Delta} = +\frac{0.0125 V_s}{0.196} = 0.06377 V_s = 0.0638 V_s$$

$$I_0 = 7.5 \text{ mA}$$

$$I_0 = \frac{V_2}{6} = \frac{0.0638 V_s}{6}$$

$$7.5 \times 10^{-3} = \frac{0.0638 V_s}{6}$$

$$\frac{(7.5 \times 10^{-3}) 6}{0.0638} = V_s$$

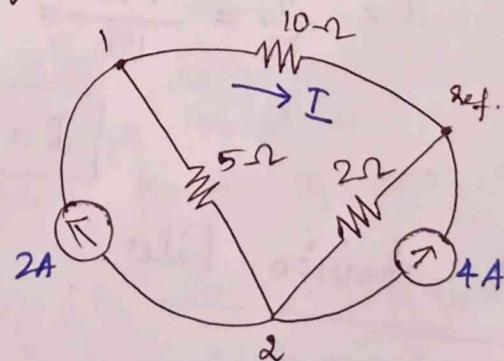
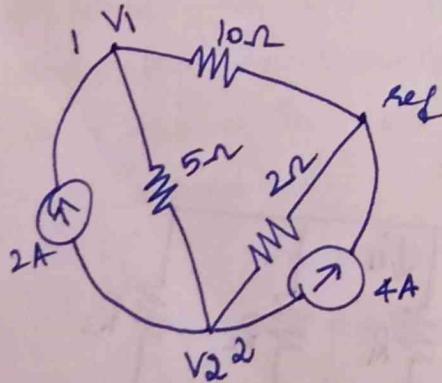
$$0.7053 = V_s$$

$$\boxed{V_s = 0.7053 \text{ V}}$$

Problem :-

In the network shown, find the current in 10Ω

Sol :-



At Node 1:

$$\frac{V_1}{5} + V = \frac{V_1 - V_2}{5} + \frac{V_1}{10} - 2 = 0$$

$$\frac{V_1}{5} - \frac{V_2}{5} + \frac{V_1}{10} = 2$$

$$V_1 \left[\frac{1}{5} + \frac{1}{10} \right] - \frac{V_2}{5} = 2$$

At Node 2:

$$\frac{V_2 - V_1}{5} + \frac{V_2}{2} + 2 + 4 = 0.$$

$$\frac{V_2}{5} - \frac{V_1}{5} + \frac{V_2}{2} = -6.$$

$$-\frac{V_1}{5} + V_2 \left[\frac{1}{5} + \frac{1}{2} \right] = -6.$$

$$\begin{bmatrix} \frac{1}{5} + \frac{1}{10} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{1}{5} + \frac{1}{2} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 0.3 & -0.2 \\ -0.2 & 0.7 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 0.3 & -0.2 \\ -0.2 & 0.7 \end{vmatrix} = 0.21 - 0.04 = 0.17.$$

$$\Delta_{11} = \begin{vmatrix} 2 & -0.2 \\ -6 & 0.7 \end{vmatrix} = 1.4 - 1.2 = 0.2$$

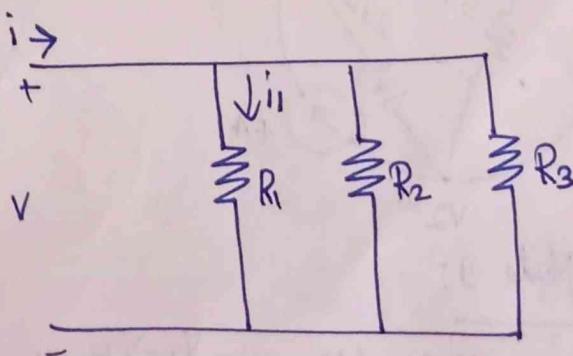
$$V_1 = \frac{\Delta_{11}}{\Delta} = \frac{0.2}{0.17} = 1.176 = 0.118 \text{ V}$$

$$I = \frac{V_1}{10} = \frac{0.118}{10} = 0.0118 \text{ A}$$

$$\boxed{I = 0.0118 \text{ A}}$$

Current division Rule:

$$i_1 = i \cdot \frac{R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$



Problems:

1. Two resistors of 4Ω & 6Ω are connected in parallel. If the total current is $30A$. Find the current through each resistor.

Sol:

$$I = 30A$$

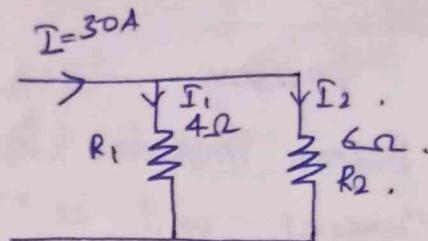
$$R_1 = 4\Omega, R_2 = 6\Omega$$

By current division rule:

$$I_1 = I \cdot \frac{R_2}{R_1 + R_2}$$

$$= 30 \cdot \frac{6}{6+4} = 30 \cdot \frac{6}{10} = 18A.$$

$$\boxed{I_1 = 18A}$$



$$I_2 = I \cdot \frac{R_{21}}{R_1 + R_2} = 30 \cdot \frac{4}{6+4} = 30 \cdot \frac{4}{10} = 12A$$

$$\boxed{I_2 = 12A}$$

(m)

$$I_2 = I - I_1 = 30 - 18 = 12A.$$

- ② Four resistors of $2\Omega, 3\Omega, 4\Omega$ & 5Ω respectively are connected in parallel. What voltage must be applied to the group in order that total power of $100W$ may be absorbed.

Sol:-

$$P = 100W.$$

$$P = VI$$

$$P = \frac{V^2}{R}$$

$$V^2 = PR.$$

$$V^2 = 100 \times \frac{0.779\Omega}{2+3+4+5} = 12.83V$$

$$V^2 = 77.922$$

$$\boxed{V = 8.827V}$$

$$R_{eq} = 2||3||4||5$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

$$\frac{1}{R_{eq}} = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$$

$$\frac{1}{R_{eq}} = \frac{30+20+15+12}{60}$$

$$\frac{1}{R_{eq}} = \frac{77}{60} = 1.283\Omega$$

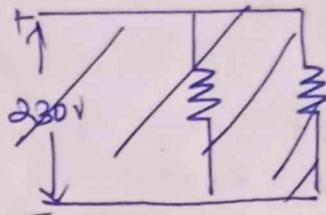
$$R_{eq} = \frac{60}{77} = 0.779\Omega$$

③ When a resistor is placed across a 230V supply, the current is 12A. What is the value of the resistor that must be placed in parallel to increase the load to 16A.

Sol:

$$V = 230V$$

Before increase in current, the current will be $12A = I_1$



After the increase in load current is $16A = I_2$

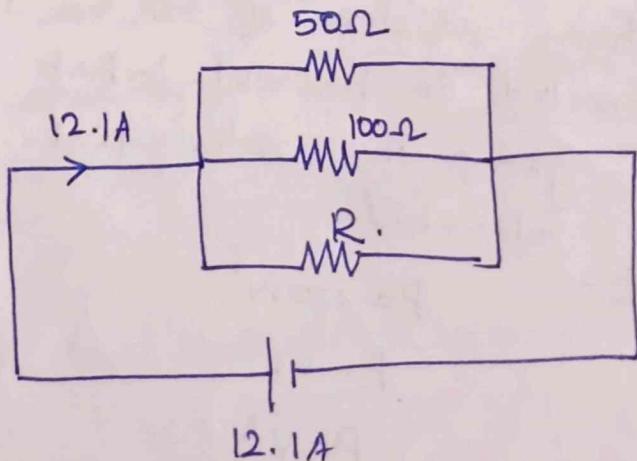
So, the resistor should be designed for $(16 - 12)A \Rightarrow 4A$ current.

$$I = I_2 - I_1 = 16 - 12 = 4A$$

$$R = \frac{V}{I} = \frac{230}{4} = 57.4\Omega$$

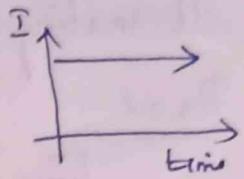
④ A 50Ω resistor is in parallel with a 100Ω resistor. The current in 50Ω resistor is 7.2A. What is the value of third resistance to be added in parallel to make the line current as 12.1A?

Sol:

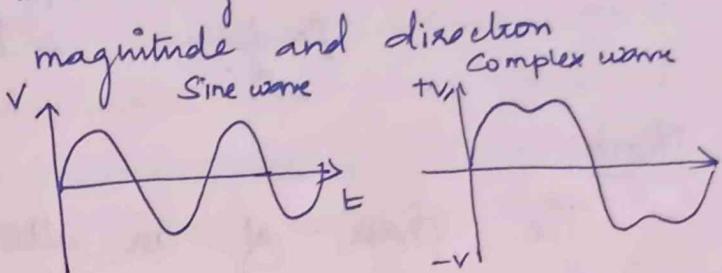


Introduction to A.C.

A direct Current (D.C) is a current which remains constant in magnitude with respect to time

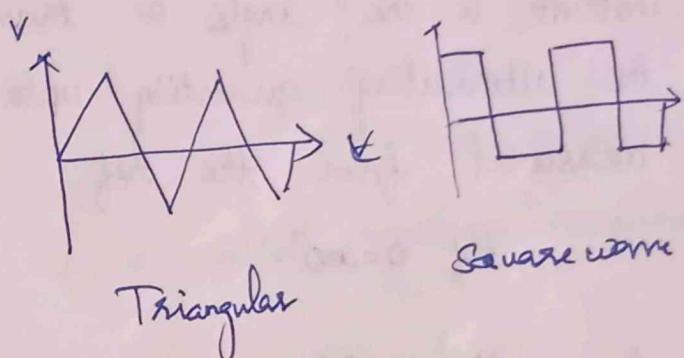


An alternating Current (A.C) is a current which changes periodically with respect to time both in magnitude and direction



Advantage of A.C

→ The voltages in a.c system can be raised or lowered with the help of transformer. In d.c system, raising & lowering of voltages is not so easy.



→ Electrical transmission of a.c signals at high voltage is possible.

Triangular

Square wave

→ AC electrical motors are simple in construction, are cheaper & require less maintenance

→ Whenever it is necessary A.C supply can be easily converted to obtain d.c supply.

Frequency:

No of cycles per second.

Unit: Hertz.

Cycle:

One complete set of +ve and -ve halves constitute a cycle.

Amplitude:

The maximum +ve or -ve value of an alternating quantity is called amplitude.

Period:

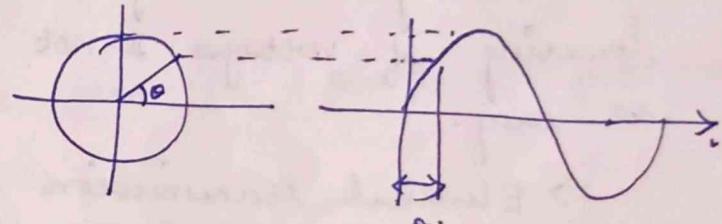
Time taken to complete one cycle.

$$T = \frac{1}{f} \quad \text{Expressed in seconds.}$$

Phase:

The phase of an alternating quantity at any instant is the angle θ travelled by the phasor representing the alternating quantity upto the instant of consideration, measured from the ref

$$\text{Ref } \theta = 0^\circ.$$



Phase difference:

It is used to compare phase of two waveforms or alternating quantities.

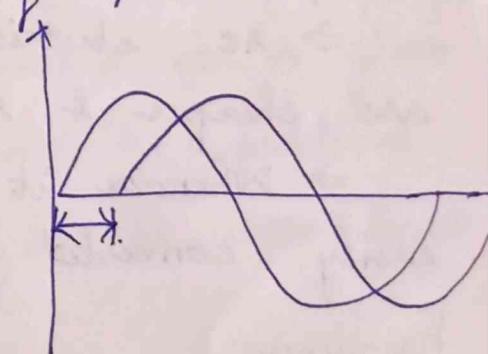
EMF [Electromotive Force]

Potential difference (or) voltage is needed to make e^- flow from one position to another.

EMF maintains the potential.

Measured in volts.

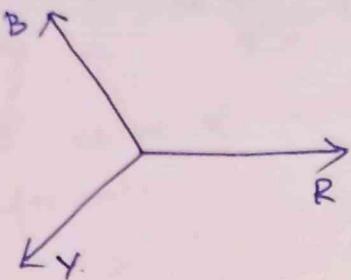
The EMF (Electromotive force) is defined as the energy per unit charge that is converted from other forms of energy to electrical energy to move a charge across the whole circuit.



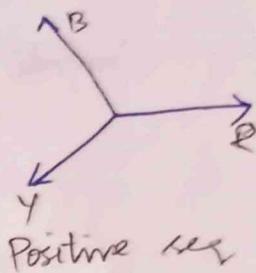
Phase Sequence:

Phase sequence or phase rotation is the order in which the voltage waveforms of a polyphase AC source reach their respective peaks.

For eg., the phase sequence is given as RYB.



The sequence follows as in same with the phase sequence RYB (anticlockwise), it is called as positive phase sequence



Negative Seq:

The sequence follows in opposite direction, RBY, then it is called negative sequence

Zero seq:

Here the 3 phases are in phase (i.e. same phase)

RMS Value:-

Effective or RMS volt value of an alternating current is defined by that steady value of current (dc) which when flowing in a given circuit for a given time produces the same heat as would be produced by the alternating current flowing in the same circuit for same time

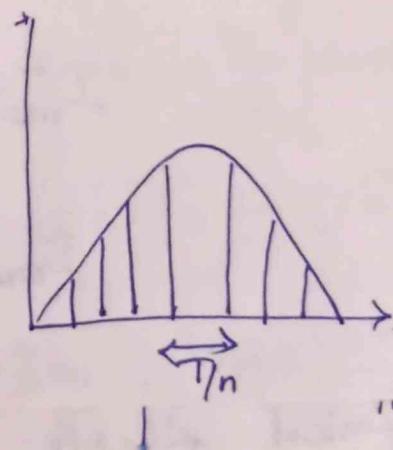
Determination of RMS Value to AC

$$\text{Heat Produced } W_1 = \frac{i_1^2 R T}{n}$$

$$W_2 = \frac{i_2^2 R T}{n}$$

$$W_n = \frac{i_n^2 R T}{n}$$

$$\therefore \text{avg} = \frac{1}{\pi}$$



Total heat Produced } \Rightarrow Sum of heat produced in n intervals

$$= \frac{RT}{n} \left[i_1^2 + i_2^2 + i_3^2 + \dots + i_n^2 \right] R \frac{T}{n} + i_2^2 R \frac{T}{n} + \dots + i_n^2 R \frac{T}{n}$$

$$= \frac{RT}{n} [i_1^2 + i_2^2 + \dots + i_n^2]$$

$$= \left[\frac{i_1^2 + i_2^2 + \dots + i_n^2}{n} \right] RT \rightarrow ①$$

$$I^2 \cdot W = I_{\text{rms}}^2 R T \rightarrow ②$$

Compare ① & ②

$$I_{\text{rms}}^2 = \left[\frac{i_1^2 + i_2^2 + \dots + i_n^2}{n} \right]$$

$$I_{\text{rms}} = \sqrt{\frac{i_1^2 + i_2^2 + \dots + i_n^2}{n}}$$

Let the alternating current be represented by.

$$i = I_m \sin \omega t$$

$$= I_m \sin \theta$$

$$\text{Mean Square of AC} \cdot \text{per sec} = \frac{1}{2\pi} \int_0^{2\pi} \frac{I_m^2 \sin^2 \theta}{2\pi} d\theta$$

$$= \frac{1}{2\pi} \int_0^{2\pi} \left[I_m^2 \sin^2 \theta d\theta \right] = \frac{I_m^2}{2\pi} \int_0^{2\pi} \frac{(1 - \cos 2\theta)}{2} d\theta$$

$$= \frac{I_m^2}{4\pi} \left[\theta - \frac{\sin 2\theta}{2} \right]_0^{2\pi}$$

$$= \frac{I_m^2}{4\pi} \left\{ \left(2\pi - \frac{\sin 2\pi}{2} \right) - \left(0 - \frac{\sin 0}{2} \right) \right\}$$

$$I_{\text{rms}}^2 \Rightarrow \frac{I_m^2 \times 2\pi}{4\pi \cdot 2} = \frac{I_m^2}{2}$$

$$I_{\text{rms}} = \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}} = 0.707 I_m //$$

Similarly, for alternating Voltage

$$V_{RMS} = V = \sqrt{\frac{V_1^2 + V_2^2 + \dots + V_n^2}{n}}$$

For sinusoidal voltage } $V = \frac{V_m}{\sqrt{2}} = 0.707 V_m$

RMS value of a wave can also be obtained by the formula.

$$\Rightarrow \sqrt{\frac{\text{Area Under the square cosine for one cycle}}{\text{Period}}}$$

Average Value of AC.

The average value of an AC is given by that steady current which transfers across a circuit the same charge as would be transferred by the AC across the same circuit

$$I_{av} = \frac{i_1 + i_2 + i_3 + \dots + i_n}{n}$$

$$I_{av} = \frac{1}{T} \int_0^T i dt$$

Analytical Method to obtain the average value for Sinusoidal Current:

$$I = I_m \sin \theta$$

Symmetrical waves \rightarrow it has 2 halves \rightarrow +ve & -ve values.

Considering one half cycle for this symm. wave, the avg. value is

$$I_{av} = \frac{1}{\pi} \int_0^\pi i d\theta = \frac{1}{\pi} \int_0^\pi I_m \sin \theta d\theta = \frac{I_m}{\pi} \int_0^\pi \sin \theta d\theta$$

$$I_m \rightarrow \text{Max. current} = \frac{I_m}{\pi} [-\cos \theta]_0^\pi = \frac{I_m}{\pi} [-\cos \pi + \cos 0] = \frac{I_m}{\pi} [1 + 1]$$

$$I_{avg} = \frac{2I_m}{\pi} \Rightarrow 0.637 \times I_m. \quad \left| \quad V_{avg} = \frac{2V_m}{\pi} \right.$$

$$\text{Form factor} = \frac{\text{RMS Value}}{\text{Average Value}}$$

$$\text{Peak factor} = \frac{\text{Peak Value}}{\text{RMS value}}$$

For sinusoidal wave.

$$\text{Form factor (Kf)} = \frac{0.707 \text{ Im.}}{0.637 \text{ Im.}} = 1.11.$$

$$\text{Peak factor (Kf)} = \frac{I_{\text{p}}}{I_{\text{m}}/\sqrt{2}} = \sqrt{2} = 1.414.$$

Measuring Instruments:

A measuring instrument is a piece of apparatus used to measure a quantity such as voltage, current, power, energy, resistance etc.

Classification of Instruments:

① Depending on the quantity measured.

Eg: Voltmeter, Ammeter, Wattmeter, Energy meter, Ohmmeter

② Depending on the different principles used for their working

Eg: Moving Iron type, Moving coil type, Dynamometer type, Induction type

③ Depending on how the quantity is measured.

Eg: Deflecting type, Integrating type, Recording type.

Deflecting torque:-

This torque acts on the moving system of the instrument to give the required deflection. It exists as long as the instrument is connected to supply.

Produced by magnetic, electromagnetic, induction, chemical effects.

Essential Torques Required for Instruments:

- ⇒ Deflecting Torque [Operating Torque]
- ⇒ Controlling Torque [Restoring Torque]
- ⇒ Damping Torque

Controlling Torque:

If the deflecting torque were acting alone, the pointer will continue to move indefinitely & would swing over to the maximum deflected position irrespective of the magnitude of the electrical quantity to be measured.

The controlling torque should increase with the deflection of the moving system.

The pointer will be brought to rest at a position where two opposing torques are equal $T_d = T_c$.

Controlling Torque performs two functions:

⇒ It increases with the deflection of the moving system so that the final position of the pointer on the scale will be according to the magnitude of the electrical quantity to be measured.

⇒ It brings back to zero position, when the deflecting torque is removed. If it was not provided, the pointer once deflected could not return to zero position on removing deflecting torque.

The following two methods are commonly used to provide controlling torque in indicating instruments:

- ① Spring Control.
- ② Gravity Control.

Spring Control:-

One end of the spring is attached to the spindle, while the other end is attached to a fixed point in the instrument.

The two springs provide the necessary torque controlling torque as well as they provide electrical connection to the operating coil.

When the instrument is not in use, the controlling torque is zero.

When the instrument is in the process of measuring of an electrical quantity with production of the deflection torque, the pointer moves & one of the springs is unwound while the other gets twisted.

More the deflection, more is the twist & hence greater will be the controlling torque

$$T_c \propto \theta$$

Advantages

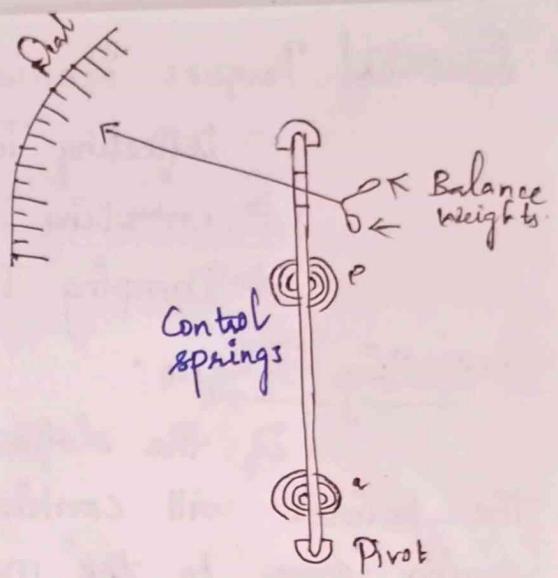
⇒ It is suitable for portable instrument as it works in any position of the instrument.

⇒ There is practically no increase in the weight of the moving system.

Disadvantages:

⇒ change in temperature may affect the spring length & hence the controlling torque.

⇒ The controlling torque cannot be adjusted normally.



Moving Iron Instruments

- ① Moving Iron Attraction type instruments
- ② Moving Iron repulsion type instrument

Moving Iron instruments are mainly used to measure voltage or current.

Moving Iron Attraction Type Instrument:

Principle:

The soft iron is magnetized when it is brought in the magnetic field produced by permanent magnet.

The iron piece is attracted towards the portion where the magnetic flux density is more.

Construction:

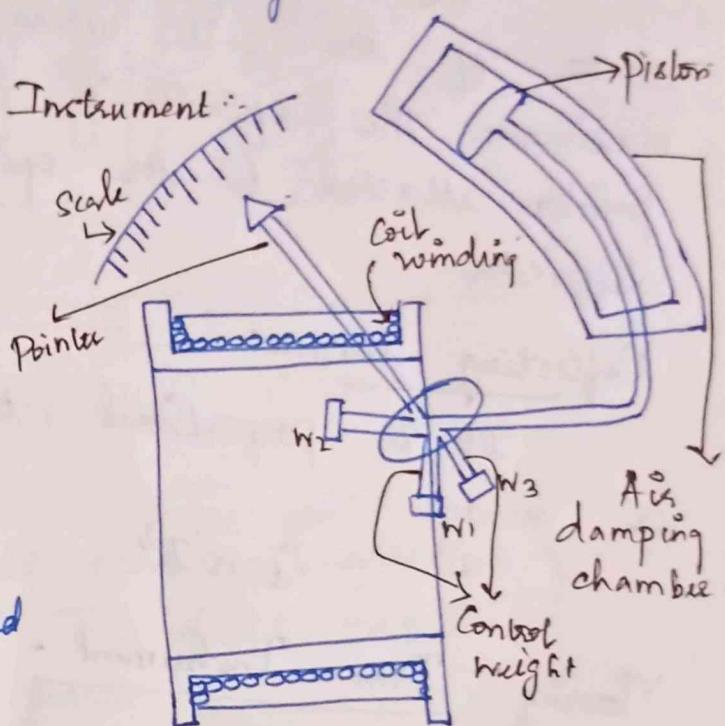
It consists of fixed coil and moving iron piece.

The moving iron is a flat disc which is mounted on the spindle. The spindle carries a pointer which moves over a graduated scale.

The number of turns of the fixed coil are dependent on the range of the instrument. For passing large current through the coil only few turns are required.

The damping torque is provided by the air friction.

The controlling torque is provided by spring. Gravity control may also be used for vertically mounted panel type instruments.



Working:

The working coil carries a current which produces a magnetic field.

The moving disc is attracted towards the centre of the coil where the flux density is maximum. The spindle is therefore moved. Thus the pointer attached to the spindle gives a proportional deflection.

Deflecting torque:

It is proportional to the square of the current or voltage

$$T_d \propto I^2$$

Moving Iron Instrument - Repulsion type:

Two iron pieces kept with close proximity in the magnetic field get magnetized to the same polarity. Repulsive force is produced.

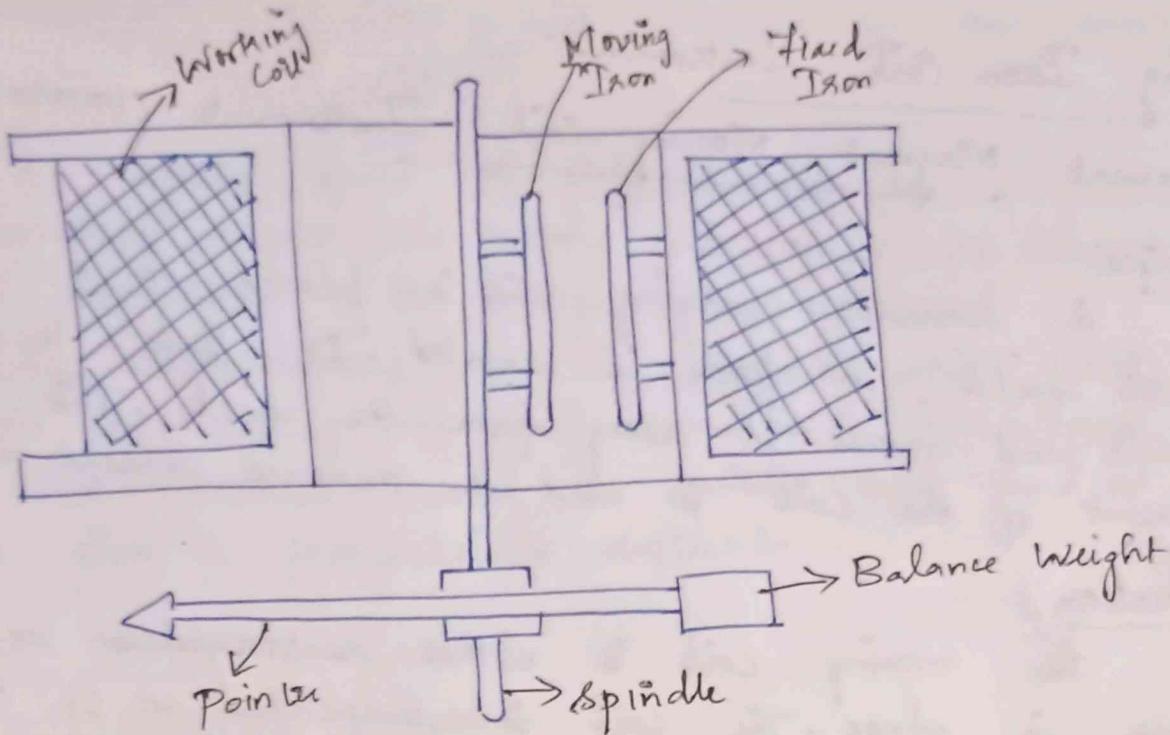
If one of the two pieces is made movable, the repulsive force will act on it & move it on one side.

Construction:-

There are two iron pieces. One of the iron piece is fixed and the other one is movable.

This instrument consists of a working coil which carries current

The moving iron is connected to spindle to which is attached a pointer. It is made to move over a calibrated scale.



Working:

When the operating coil carries current, a magnetic field is produced. This field magnetises similarly both the soft iron pieces.

Thus a repulsive force is produced which acts on moving iron & pushes it away from its rest position.

Thus the spindle moves & hence the pointer gives deflection. The iron are similarly magnetised for any direction of current in the coil.

Deflecting torque:

$$T_d \propto I^2.$$

Moving Iron Coil Instruments:

Permanent Magnet Moving coil Instruments (PMTIIC)

Principle:

A current carrying coil is placed in a magnetic field, a force is exerted. It tends to act on the coil and moves it away from the field. This movement of the coil is used to measure current or voltage.

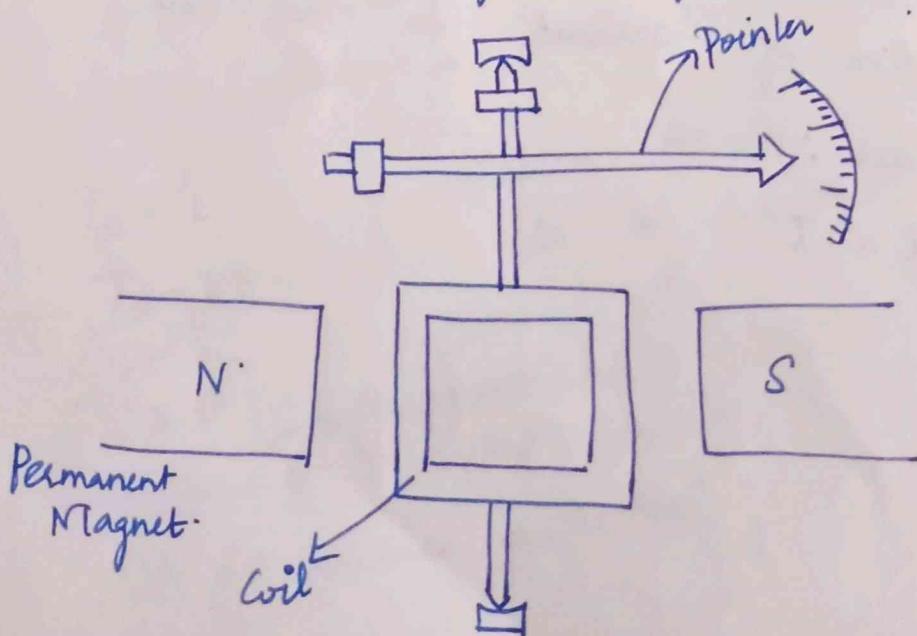
Construction:

The moving coil is either rectangular or circular in shape. The coil is suspended so it is free to turn about its vertical axis.

The controlling torque is provided by two phosphor bronze hair springs.

The damping torque is provided by eddy current damping. The pointer is carried by the spindle and it moves over a graduated scale.

The pointer deflection is directly proportional to the current passing through the coil.



Working:

A magnetic field of sufficient density is produced by the permanent magnet. The moving coil carries the current or a current proportional to the voltage to be measured.

The electromagnetic force is produced which tends to act on the moving coil and moves it away from the field. This movement makes the spindle move and so the pointer gives a proportionate deflection.

Torque Formation:

$$F = BIl \text{ (Newtons)}$$

$B \Rightarrow$ Flux density is Wb/m^2 .

$l \Rightarrow$ length or depth of coil in metres

Deflecting torque (T_d) = Force \times d perpendicular distance

$$= NBIl \times b$$

$$= NBI(l \times b) = NBIA \text{ (Nm)}.$$

where

$N \Rightarrow$ Number of turns of the coil

$A \Rightarrow$ Effective coil area (m^2).

$I \Rightarrow$ Current in moving coil (Amperes).

$$T_d \propto I$$

Deflecting torque is directly proportional
to the current

$$T_d = kI$$

$$T_c \propto \theta$$

(proportionality constant)

$$T_c = c\theta \cdot \text{spring const.}$$

$\theta \Rightarrow$ Angular deflection

$$T_c = T_d$$

$$k\theta c\theta = kI_{\parallel}$$

Advantages of Permanent Magnetic Moving coil Instrument

- It has uniform scale
- Sensitivity is high
- It has high accuracy.
- It is free from hysteresis error
- Not affected by external magnetic field called stray magnetic field
 - With a powerful magnet, its torque to weight ratio is very high. So, operating current is small

Disadvantages of PNTMC:

- Suitable for DC measurements only
- Cost is high due to delicate construction & accurate machining.
- Friction and temperature might introduce errors as in case of other instruments

Dynamometer Type Wattmeter

Principle:-

A current carrying conductor is placed in a magnetic field, a force is exerted. It tends to act on the coil & moves it away from the field.

Working principle is similar to that of permanent magnet moving coil type. But there is no permanent magnet in the instrument. It is replaced by fixed coils

Construction:

It consists of two coils namely moving coil & fixed coil. The fixed coils are placed parallel to each other.

The moving coil is pivoted between the two fixed coils & is placed on the spindle to which the pointer is attached

The fixed coils are connected in series with load & carry the circuit current. It is therefore also called as current coil

The moving coil is connected across the load and carries current proportional to the voltage. It is called as potential coil (or) pressure coil

A high resistance is connected in series with potential coil to limit the current through it.

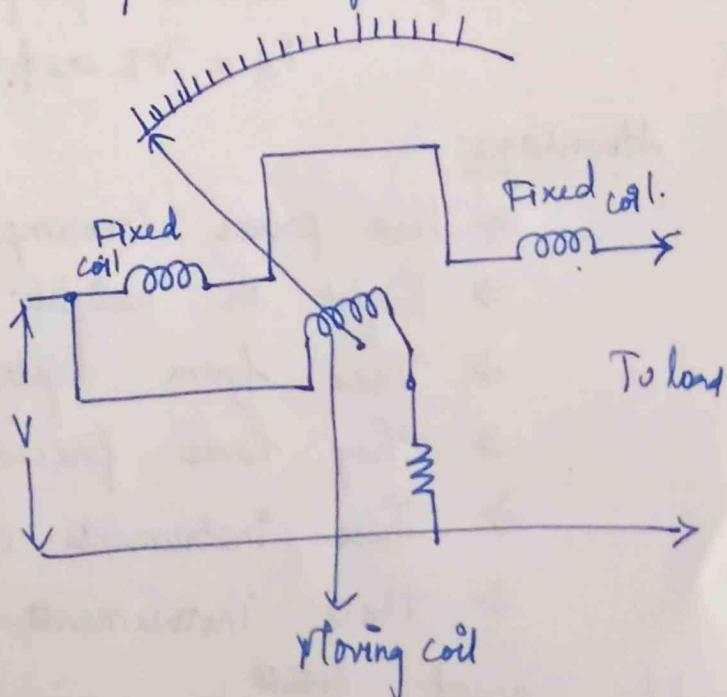
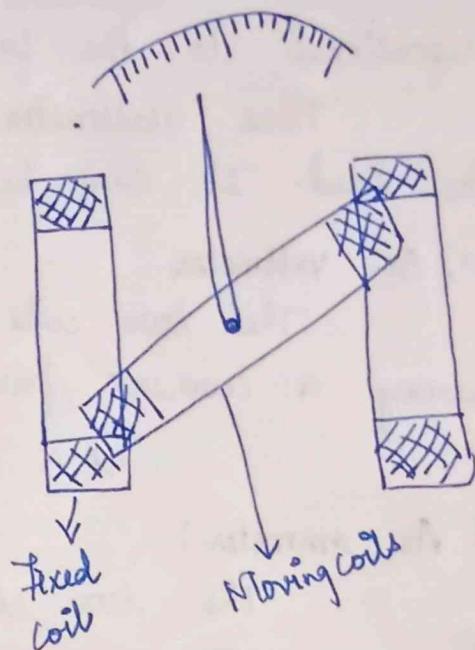
The controlling torque is provided by the springs

Air friction damping is employed in such instruments

Working:

When current is passes through the fixed and moving coils, both coils produce the magnetic fields.

The field produced by the fixed coil is proportional to the load current while the field produced by the moving coil is proportional to the voltage



As the deflecting torque is produced due to the interaction of these two fields, the deflection is proportional to the power supplied to the load.

Thus wattmeter indicates the power consumption of the load. It can be used for both AC & DC

(a) As voltmeter:

The two coils are electrically in series. They carry a current proportional to voltage to be measured.

$$T_d \propto V^2.$$

(b) As ammeter:

The two coils are electrically in series. They carry the current to be measured.

$$T_d \propto I^2.$$

(c) As wattmeter:

Fixed coils carry the system current. Moving coil carries a current proportional to the system voltage.

$$T_d \propto VI \cos\phi.$$

Advantages:

- low power consumption
- light in weight.
- free from hysteresis losses.
- they have precision grade accuracy.
- these instruments can be used on both AC & DC
- these instruments is free from hysteresis & eddy current losses

Disadvantages:

- These instrument have a low sensitivity due to low torque to weight ratio

- They are more expensive than other type of instruments.
- They have non-uniform scale.

Energy Meters:

The meter which is used for measuring the energy utilized by the electrical load is known as the energy meter.

The energy is that the total power consumed & utilized by the load at a particular interval of time.

It is used in domestic & industrial AC circuit for measuring the power consumption.

There are two varieties of AC energy meters:

→ Single phase Energy meters are used to measure electrical energy in AC single-phase circuits

→ Three phase Energy meters are used to measure energy in three phase circuits. They consist of two - Single phase energy meters working on two discs fixed to the same spindle

Single Phase Energy Meter:

When a conducting metal part is placed in an alternating magnetic field, eddy currents are induced in the metal part. The magnetic flux produced by these eddy currents are made to interact with another magnetic field. The required operating torque is produced.

Construction:-

The four main parts of the operating mechanism are:-

1. Driving system.
2. Moving system.
3. Braking system
4. Registering system

1. Driving system:-

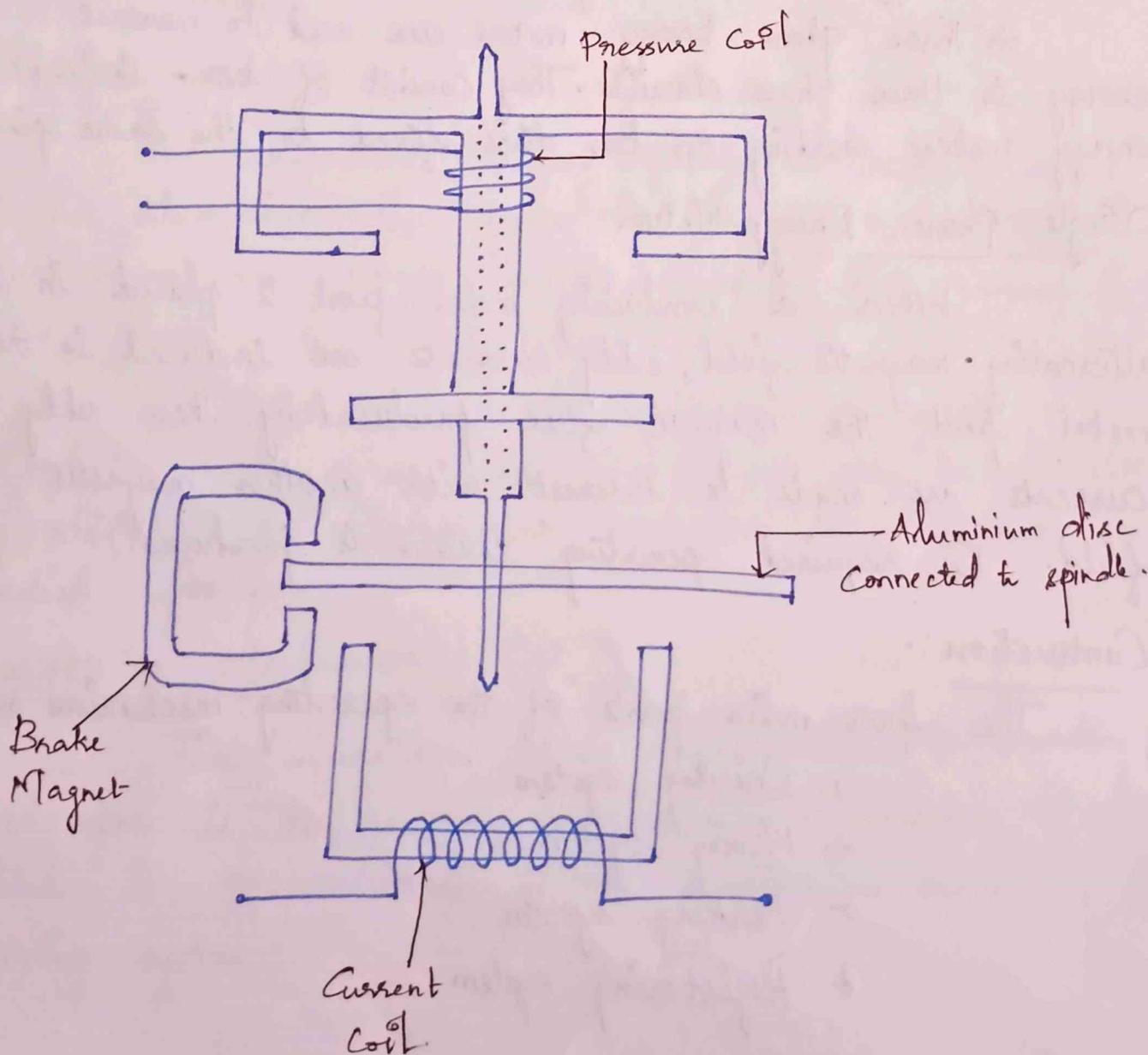
The driving system of the meter consists of two

electro-magnets. The core of these electromagnets is made up of silicon steel laminations

The coil of one of the electromagnets is excited by the load current. This coil is called current coil.

The coil of second electromagnet is connected across the supply & therefore, carries a current proportional to the supply voltage. This coil is called the pressure coil. Consequently the two electromagnets are known as series & shunt magnets respectively.

Copper shading bands are provided on central limb.



Moving system:

This consists of an aluminium disc mounted on a light alloy shaft. This disc is positioned in the air gap between series & shunt magnets.

The upper bearing of the moving system is a steel pin located in a hole in the bearing cap fixed to the top of the shaft.

The eddy current is induced in the disc because of the change of the magnetic field.

This eddy current is cut by the magnetic flux. The interaction of the flux and the disc induces the deflecting torque.

Braking system:

A permanent magnet positioned near the edge of the aluminium disc forms the braking system. The aluminium disc moves in the field of this magnet & thus provides a braking torque.

The position of the permanent magnet is adjustable & therefore, braking torque can be adjusted by shifting the permanent magnet to different radial positions.

Registering Mechanism:

The function of a registering or counting mechanism is to record continuously a number of which is proportional to the revolutions made by the moving system.

Working:

The current coil produces the alternating flux ϕ_1 , which is proportional and in phase with the current coil.

The pressure coil carries the current I and produces the flux ϕ_2 which is proportional to the supply voltage and lags behind it by 90° which is achieved by the copper shading bands.

Major portion of the flux ϕ_2 crosses the narrow gap between the central and the side limbs of the shunt magnet and only small amount passes through the disc which is the useful flux.

Both the fluxes ϕ_1 and ϕ_2 induce e.m.f.s in the disc which produce the eddy currents in the disc.

The interaction between these fluxes and the eddy currents produce the necessary driving torque and the disc starts rotating.

The speed of disc is controlled by the C shaped magnet called braking magnet. When the disc rotates in the air gap, eddy currents are induced in the disc which oppose the cause that produced them. Hence braking torque is generated.

By adjusting the position of the magnet, desired speed of disc is obtained.

Spindle connected to recording mechanism through gears which record the energy supplied

$$T_d \propto V I_{\text{avg}} \cos \phi.$$

$$T_d \propto N$$

For constant speed

$$T_b = T_d \quad (\text{ie}) \quad N \propto V I, \cos \phi.$$

$$Nt \propto Pt \propto \text{energy}.$$

Number of revolutions in time (t) \propto Energy supplied

Advantages of Energy meters

- Its construction is simple & strong.
- It is cheap in cost
- It has more accuracy.
- It requires less maintenance
- It has high torque to weight ratio, so frictional errors are less & we can get accurate reading

Disadvantages:

- It can be used only in AC circuits
- Lack of symmetry in magnetic circuit can cause errors

UNIT-2.

ELECTRICAL MACHINES

An electrical machine which converts mechanical energy into an electrical energy is called an electric generator.

An electrical machine which converts electrical energy into mechanical energy is called an electrical motor.

D.C Generators:

These machines which convert mechanical input power into dc electrical power.

Working Principle:

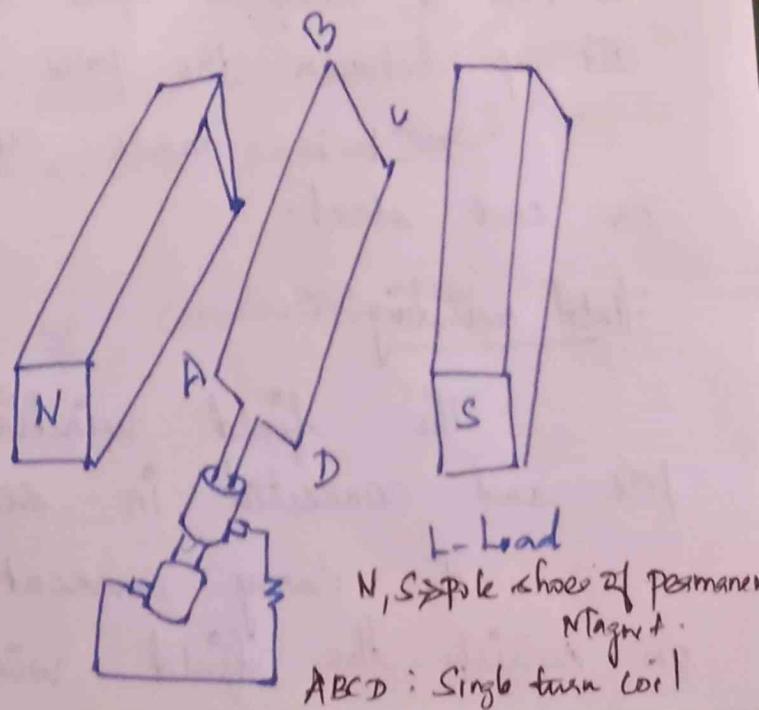
It operates on the principle based on the Faraday's law of electromagnetic induction. It states that whenever flux linking with a conductor or a coil changes, an electromotive force (emf) is set up in that conductor or coil.

The basic requirements for the dynamically induced emf to exist are the following.

- ① A steady magnetic field
- ② A conductor capable of carrying current.
- ③ The conductor to move in the magnetic field.

The steady magnetic field produced by the pole pieces of a magnet N and S. A single turn coil ABCD is placed in the field produced between the pole pieces.

The coil is rotated by means of a prime mover



By Faraday's law, an emf is induced in the coil. This bidirectional emf is made unidirectional using the commutator.

Construction:

Yoke:

It is the outermost solid metal part of a machine. It forms a part of magnetic circuit & protects all the inner parts from mechanical damage.

It provides a path of low reluctance for magnetic flux. The low reluctance of path is important to avoid wastage of power to provide same flux.

It is prepared by using cast iron because it is cheapest & provides low reluctance path. For large machines, rolled steel, cast steel is used.

Poles:

Each pole is divided into two parts namely

pole core

pole shoe.

→ Pole core basically carries a field winding which is necessary to produce the flux.

→ The pole shoe is in the form of horse shoe so that a uniform flux distribution is obtained in the air gap between the poles and rotating part.

→ It is made up of material like cast iron or cast steel.

Field winding:

The field windings are placed over each pole and connected in series.

To carry current due to which pole core on which the field winding is placed behaves as an

electromagnet, producing necessary flux.

The materials used ~~be~~ copper or aluminum

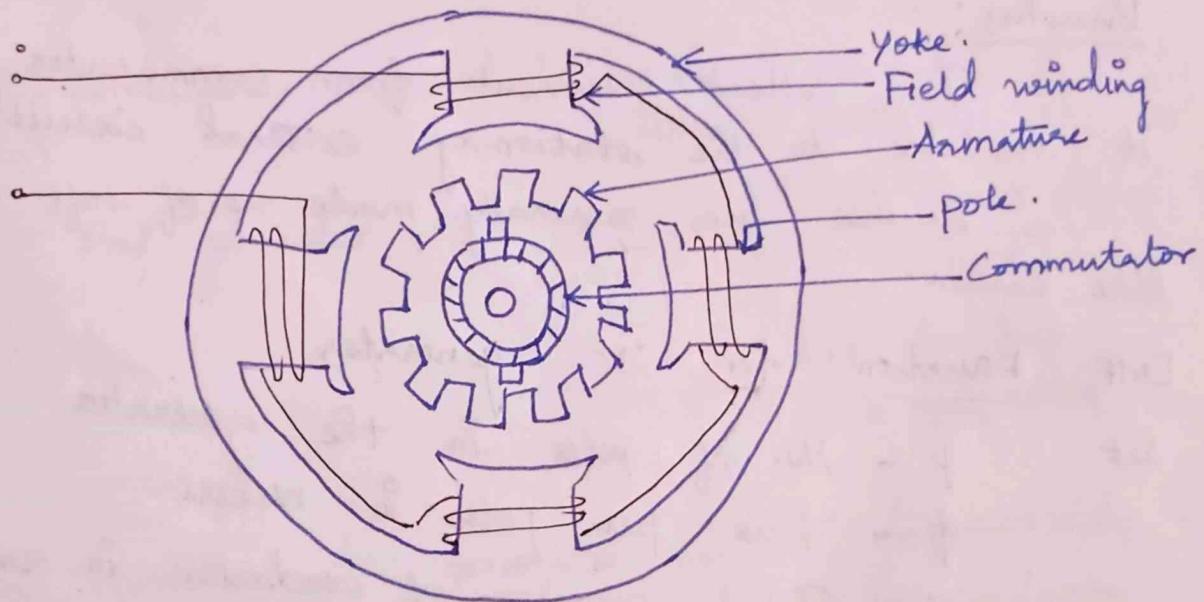
Armature:

Armature Core:

It is cylindrical in shape mounted on the shaft.

Armature core provides base for armature winding (i.e) armature conductors.

It provides low reluctance path to the flux.
It is made of materials like cast iron or cast steel.



Armature Winding:

Armature winding is nothing but the interconnection of the armature conductors.

When armature is rotated in case of generator magnetic flux gets cut by armature conductors & emf is induced in them.

It is made up of conductor material like Copper.

Commutator:

The basic nature of e.m.f induced in the armature conductors is alternating. This needs rectification in case of d.c generator which is done by commutator.

It is used to collect the current from armature conductors.

It is similar in shape of armature. But it has less diameter than that of armature. There is an electrical insulation between every pair of segments.

Brushes:

To collect current from commutator & make it available to the stationary external circuit.

Brushes are normally made up of soft material like carbon.

EMF Formation for DC generator.

Let P - No. of poles in the generator

ϕ - Flux per pole in webers.

Z - Total number of conductors in armature armature conductors.

N - Speed of armature in r.p.m.

A - Number of parallel paths formed by the armature winding between the armature terminals

$A=2$, for wave winding wound armature winding

$A=p$, for lap wound armature winding

E = emf Induced across the armature terminal

According to Faradays law of electromagnetic induction, average emf induced in one conductor is

$$e = \frac{d\phi}{dt}$$

Consider one revolution of conductor. In one conductor will cut the total flux produced by all the poles

Time required to complete one revolution is $\frac{60}{N}$ seconds

$d\phi = \text{flux cut by conductor in one revolution} = P\phi (\text{wb})$

$dt = \text{Time taken by the conductor for } \frac{1}{P} \text{ one revolution} = \frac{60}{N} \text{ (seconds)}$

$$e = \frac{d\phi}{dt} = \frac{P\phi}{\frac{60}{N}} = \frac{\phi PN}{60}$$

No. of conductors connected in series in one parallel path $\left\{ \right\} = \frac{Z}{A}$.

$$\text{Emf generated } E = \frac{\phi PN}{60} \times \frac{Z}{A} \Rightarrow \frac{\phi PN Z}{60A}$$

$A = p$ for lap
 $A = 2$ for wave

Problem:

A 4 pole, 1500 rpm dc generator has a lap winding wound armature having 24 slots with 10 conductor per slot. If the flux per pole is 0.04 wb. Calculate the emf generated in the armature. What would be the generated emf if the winding is wave connected

sol: $P=4, N=1500 \text{ rpm}$

$$A(\text{Lap winding}) = P = 4.$$

$$\phi = 0.04 \text{ wb.}$$

$$Z = \text{slots} \times (\text{Conductor/slot}) \\ = 24 \times 10 = 240.$$

$E_g = \frac{\phi PN Z}{60A}$

$$= \frac{0.04 \times 4 \times 1500 \times 240}{60 \times 4}$$

$$\boxed{E_g = 240 \text{ V}}$$

If winding is wave winding, $A=2$.

$$E_g = \frac{0.04 \times 0.4 \times 1500 \times 240}{60 \times 2}$$

$$= 480 \text{ V}$$

④ 1 + pole, lap wound, dc generators has 42 coils with 8 turns per coils. It is driven at 1120 rpm. If useful flux per pole is 21 mWb. Calculate the generated emf. Find the speed at which it is to be driven to generate same emf as calculated above with wave wound armature.

Sol:

$$P=4, \phi = 21 \text{ mWb} = 21 \times 10^{-3} \text{ WB}$$

$$N = 1120 \text{ rpm}$$

$$\text{Coils} = 42 \text{ & turns/coil} = 8$$

$$\begin{aligned} \text{Total Conductors} &= \text{Coils} \times \text{Turns/coil} \\ &= 42 \times 8 = 336 \end{aligned}$$

$$\text{Total Conductors (Z)} = 2 \times 336 = 672$$

$$\textcircled{1} \text{ Lap wound ; } A=P, E = \frac{P\phi NZ}{60A} = \frac{21 \times 10^{-3} \times 1120 \times 672 \times 4}{60 \times 4}$$

$$= 263.424 \text{ V}$$

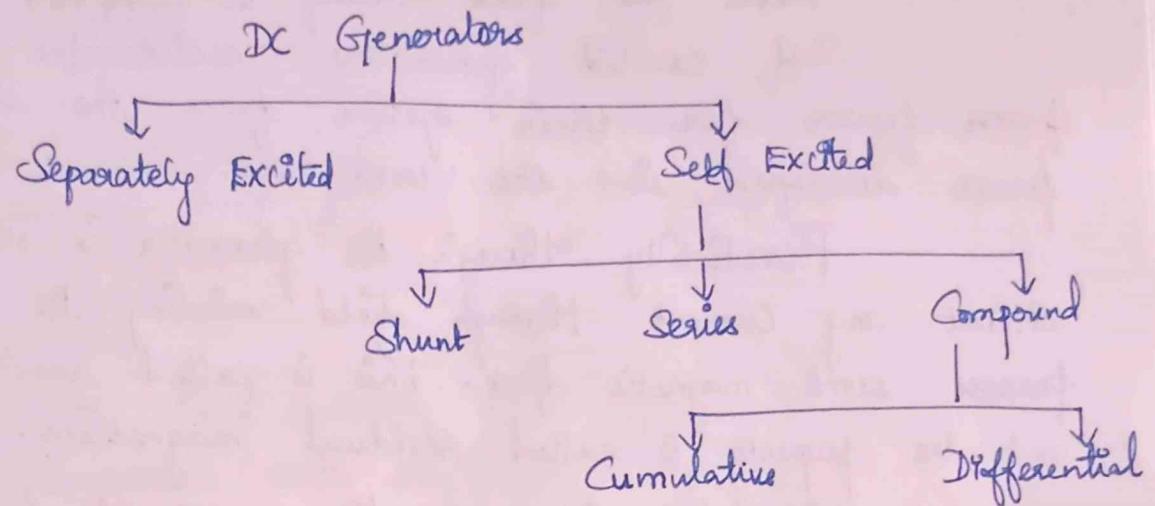
$$\textcircled{2} \text{ For wave wound, } A=2, E = \frac{P\phi NZ}{60A}$$

$$E = 263.424$$

$$263.424 = \frac{21 \times 10^{-3} \times 4 \times N \times 672}{120}$$

$$\boxed{N = 560 \text{ rpm}}$$

Types of DC Generators :-



Separately Excited Generator:

In separately excited generator, a separate external DC supply is used to provide exciting current through the field winding.

The field winding is connected to separate DC source. The load is connected across the armature terminals.

The field winding of this type of generator has large number of turns of thin wire. Resistance of this field winding is high in order to limit the field current.

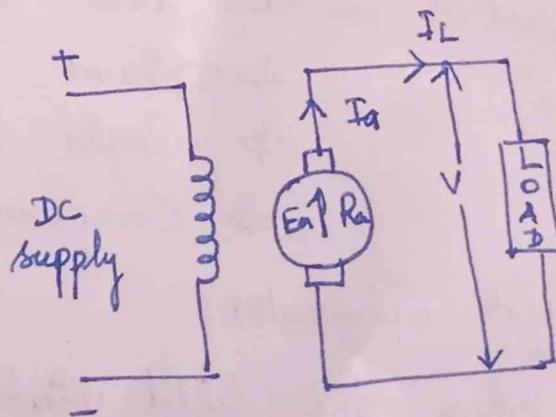
I_a = Armature Current.

I_L = Load Current

I_f = Field Current.

$$V = E - I_a R_a$$

$I_a R_a \rightarrow$ Armature drop.



There is some voltage drop at the contacts of the brush called brush contact drop.

$$V = E_a - I_a R_a - 2 V_b$$

But this brush contact drop is negligible and hence generally neglected

Self Excited Generator:

When the field winding is supplied from the self excited generators receive the required power from their system from the electrical power developed in the armature.

Practically though the generator is not working, without any current through field winding, the field pole possess some magnetic flux. This is called residual flux and the property is called residual magnetism.

The following are the causes of failure of self excited generators

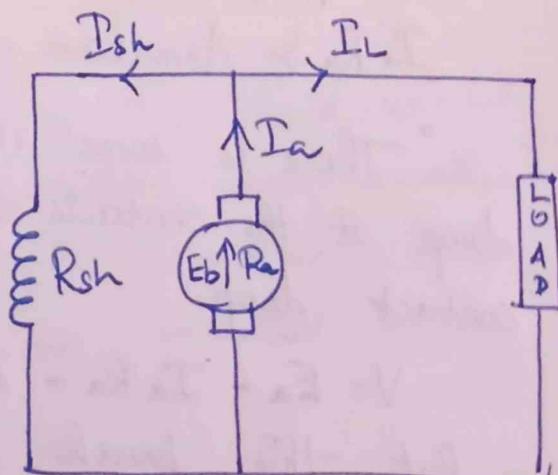
- Absence of residual magnetism
- Wrong connections of field winding so as to cancel residual magnetism.
- Driven in opposite direction so as to cancel residual magnetism.

Based on how field winding is connected to the armature to derive its excitation, this type is further divided into following 3 types

- Shunt generator
- Series Generator
- Compound Generator

Shunt Generator:

When the field winding is connected in parallel with the armature and the combination across the load, then the generator is called shunt generator



$$I_a = I_h + I_{sh}$$

$$I_{sh} = \frac{V}{R_{sh}}$$

$$V = E_g - I_a R_a$$

$$V = E_g - I_a R_a - 2V_b$$

Series Generator:

When the field winding is connected in series with the armature winding while supplying the load the generator is called series generator.

The resistance of series field winding is very small & hence it has less number of turns of thick cross-section wire.

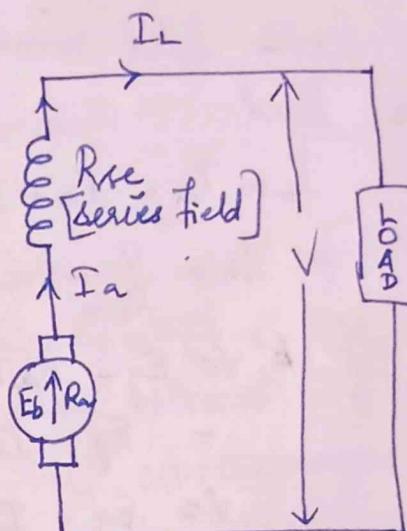
$$I_{se} = I_r = I_a$$

$$V = E_g - I_a R_a - I_{se} R_{se}$$

$$V = E_g - I_a R_a - I_a R_{se}$$

$$V = E_g - I_a (R_a + R_{se})$$

$$V = E_g - I_a (R_a + R_{se}) - 2V_b$$



Compound Generator:

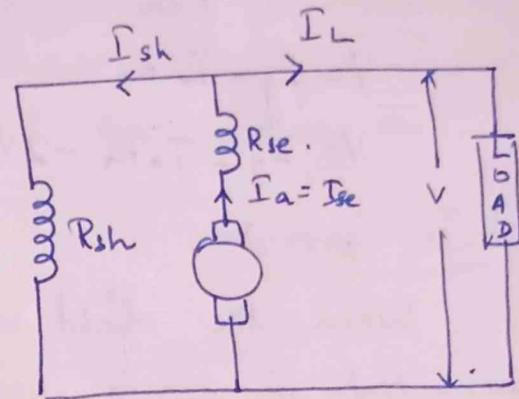
In this type, the part of the field winding is connected in parallel with armature and another part is connected in series with armature. Both series and shunt field windings are mounted on the same poles.

Depending upon the connection of shunt & series winding, compound generator is further classified as

- ① Long shunt Compound Generator
- ② Short shunt Compound Generator

Long shunt Compound Generator:

The shunt field winding is connected across the series combination of armature & series field winding across the load



$$I_L = I_{sh} = I_a + I_{sh}$$

$$I_{sh} = \frac{V}{R_{sh}}$$

$$\therefore V = E_g - I_a R_a - I_{sh} R_{se}$$

$$= E_g - I_a R_a - I_a R_{se} = E_g - I_a (R_a + R_{se})$$

$$V = E_g - I_a (R_a + R_{se}) - 2V_b$$

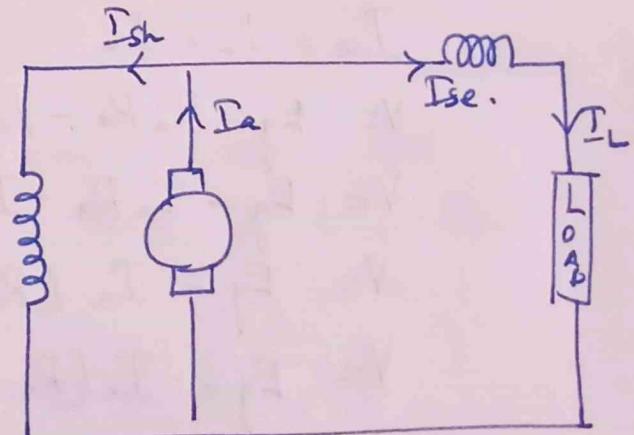
Short shunt Compound Generator:

Shunt field winding is connected across the series combination of load & series field winding and across the armature terminal.

$$I_L = I_{se}$$

$$I_a = I_L + I_{sh}$$

$$I_{sh} = \frac{V_{sh}}{R_{sh}}, \quad V_{sh} = V$$



$$V = E_g - I_a R_a - I_L R_{se} = E_g - I_a R_a - I_L R_{se} - 2V_b$$

DC Motor:

Principle of Operation:

The principle of operation of a DC motor can be stated as 'when a current carrying conductor is placed in a magnetic field, it experiences a mechanical force'.

Construction:

Refer. Construction of DC Generator.

Working:

In a DC motor, both armature and field windings are connected to a DC supply. Thus, we have current carrying armature conductors placed in stationary magnetic field. Due to the electromagnetic torque on the armature conductors, the armature starts revolving.

As per Faraday's law of electromagnetic induction, an emf is induced in the armature conductors. As per Lenz's law, this induced emf opposes the voltage applied to the armature. Hence it is called as back emf or counter emf.

The voltage equation for DC motor can be obtained as

$$V = E_b + I_a R_a + \text{Brush drop}$$

$$I_a = \frac{V - E_b}{R_a}$$