Electric Field

The electric field intensity or the electric field strength at a point is defined as the force perunit charge. That is

$$\vec{E} = \lim_{Q \to 0} \frac{\vec{F}}{Q} \qquad \vec{E} = \frac{\vec{F}}{Q}$$

Or ,................................(2.4)

The electric field intensity E at a point r (observation point) due a point charge Q located at $\vec{r}\,'$ (source point) is given by:

$$\vec{E} = \frac{Q(\vec{r} - \vec{r}\,')}{4\pi\varepsilon_0 \left|\vec{r} - \vec{r}\,'\right|^3}$$

....*(2.5)*

For a collection of N point charges $Q_1, Q_2, ...Q_N$ located at $\vec{r}_1, \vec{r}_2, ...\vec{r}_N$, the electric field intensity at point $\vec{r}\,'$ is obtained as

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{Q_k(\vec{r} - \vec{r}_i)}{\left|\vec{r} - \vec{r}_i\right|^3}$$

....(2.6)

The expression *(2.6)* can be modified suitably to compute the electric filed due to acontinuous distribution of charges.

In figure 2.2 we consider a continuous volume distribution of charge $\rho(t)$ in the regiondenoted as the source region.

For an elementary charge $dQ = \rho(\vec{r}\,')dv'$, i.e. considering this charge as point charge, wecan write the field expression as:

$$d\vec{E} = \frac{dQ(\vec{r} - \vec{r}\,')}{4\pi\varepsilon_0 \left|\vec{r} - \vec{r}\,'\right|^3} = \frac{\rho(\vec{r}\,')dv'(\vec{r} - \vec{r}\,')}{4\pi\varepsilon_0 \left|\vec{r} - \vec{r}\,'\right|^3}$$

.............(2.7)

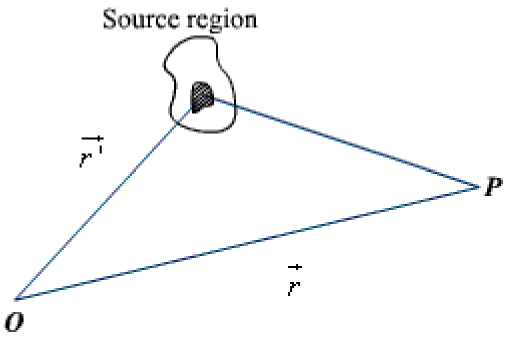

Fig : Continuous Volume Distribution of Charge

When this expression is integrated over the source region, we get the electric field at the point P due to this distribution of charges. Thus the expression for the electric field at P canbe written as:

$$\overrightarrow{E(r)} = \int_V \frac{\rho(\vec{r}\,')(\vec{r}-\vec{r}\,')}{4\pi\varepsilon_0 \left|\vec{r}-\vec{r}\,'\right|^3} dv' \qquad ....(2.8)$$

Similar technique can be adopted when the charge distribution is in the form of a line charge density or a surface charge density.

$$\overrightarrow{E(r)} = \int_L \frac{\rho_L(\vec{r}\,')(\vec{r}-\vec{r}\,')}{4\pi\varepsilon_0 \left|\vec{r}-\vec{r}\,'\right|^3} dl'$$

....(2.9)

$$\overrightarrow{E(r)} = \int_S \frac{\rho_s(\vec{r}\,')(\vec{r}-\vec{r}\,')}{4\pi\varepsilon_0 \left|\vec{r}-\vec{r}\,'\right|^3} ds'$$

1. An infinite line charge

As the first example of illustration of use of Gauss's law, let consider the problem of determination of the electric field produced by an infinite line charge of density ρ_LC/m. Let us consider a line charge positioned along the z-axis as shown in Fig. 2.4(a) (next slide). Since the line charge is assumed to be infinitely long, the electric field will be of the form as shown in Fig. 2.4(b) (next slide).

If we consider a close cylindrical surface as shown in Fig. 2.4(a), using Gauss's theorm wecan write,

$$\rho_l l = Q = \oint_S \varepsilon_0 \vec{E}.d\vec{s} = \int_{S_1} \varepsilon_0 \vec{E}.d\vec{s} + \int_{S_2} \varepsilon_0 \vec{E}.d\vec{s} + \int_{S_3} \varepsilon_0 \vec{E}.d\vec{s} \qquad ....(2.15)$$

Considering the fact that the unit normal vector to areas S_1 and S_3 are perpendicular to the electric field, the surface integrals for the top and bottom surfaces evaluates to zero. Hence we can write, $\rho_l l = \varepsilon_0 E.2\pi\rho l$

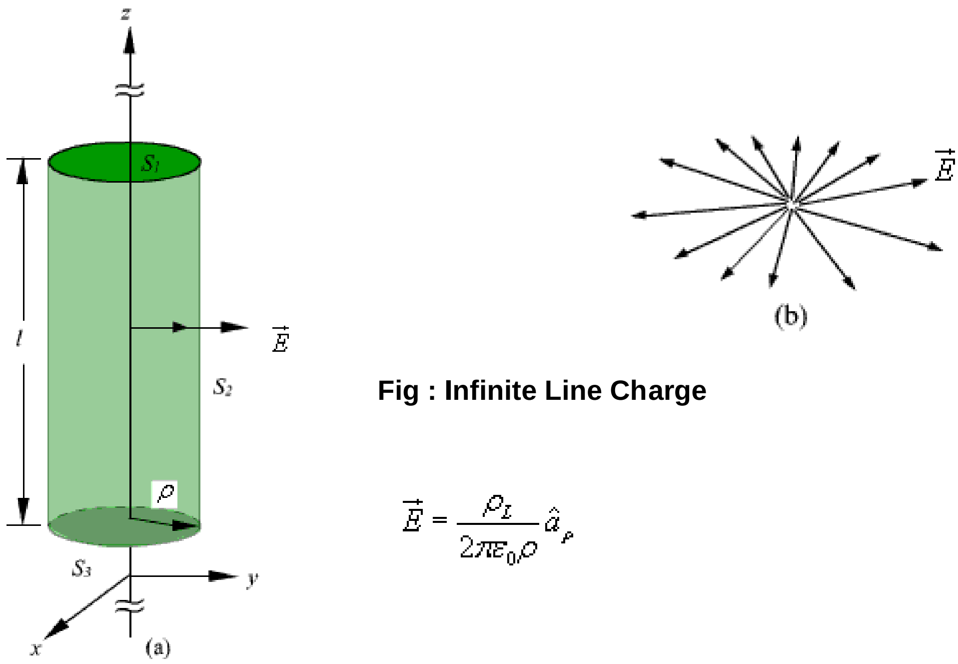

Fig : Infinite Line Charge

$$\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0\rho}\hat{a}_\rho$$