Electrostatic Potential

In the previous sections we have seen how the electric field intensity due to a charge or a charge distribution can be found using Coulomb's law or Gauss's law. Since a charge placed in the vicinity ofanother charge (or in other words in the field of other charge) experiences a force, the movement of the charge represents energy exchange. Electrostatic potential is related to the work done in carrying a charge from one point to the other in the presence of an electric field.

Let us suppose that we wish to move a positive test charge Δq from a point P to anotherpoint Q as shown in the Fig. 2.8.The force at any point along its path would cause the particle to accelerate and move it out of the region if unconstrained. Since we are dealing with an electrostatic case, a force equal to the negative of that acting on the charge is to be applied while Δq moves from P to Q. Thework done by this external agent in moving the charge by a distance $d\vec{l}$ is given by:

$$dW = -\Delta q \vec{E}.d\vec{l}$$ ....(2.23)

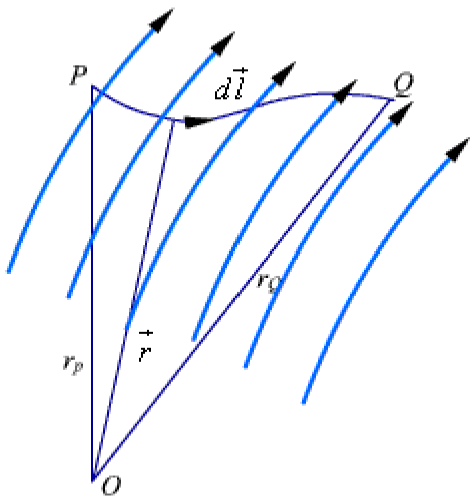

Fig : Movement of Test Charge in Electric Field

The negative sign accounts for the fact that work is done on the system by the externalagent.

$$W = -\Delta q \int_P^Q \vec{E} \cdot d\vec{l}$$

....................................(2.24)

The potential difference between two points P and Q , V_{PQ}, is defined as the work done perunit charge, i.e.

$$V_{PQ} = \frac{W}{\Delta Q} = -\int_P^Q \vec{E} \cdot d\vec{l}$$ ....(2.25)

It may be noted that in moving a charge from the initial point to the final point if the potentialdifference is positive, there is a gain in potential energy in the movement, external agent performs the work against the field. If the sign of the potential difference is negative, work isdone by the field.

We will see that the electrostatic system is conservative in that no net energy is exchanged if the test charge is moved about a closed path, i.e. returning to its initial position. Further, the

potential difference between two points in an electrostatic field is a point function; it is

independent of the path taken. The potential difference is measured in Joules/Coulomb which is referred to as **Volts**.

Let us consider a point charge Q as shown in the Fig. 2.9.

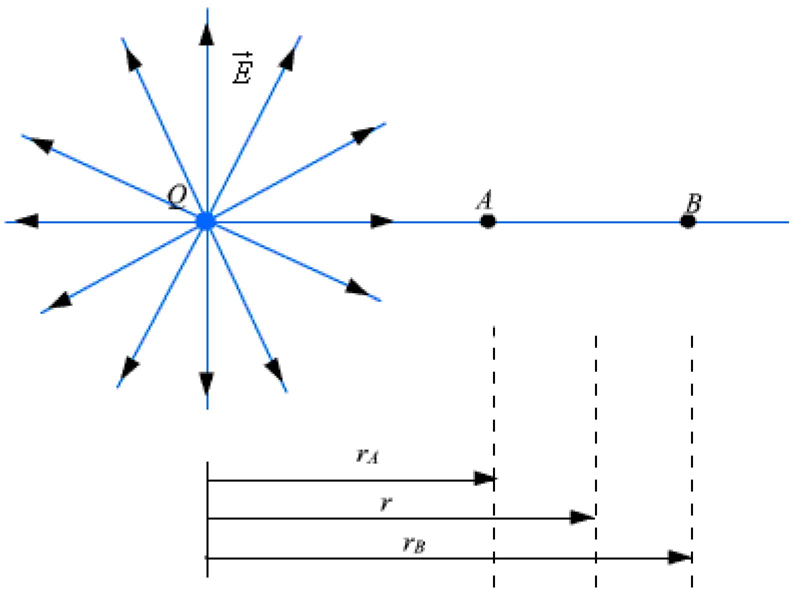

Fig : Electrostatic Potential calculation for a point charge

Further consider the two points A and B as shown in the Fig. 2.9. Considering the movementof a unit positive test charge from B to A , we can write an expression for the potential difference as:

$$V_{BA} = -\int_B^A \vec{E} \cdot d\vec{l} = -\int_{r_B}^{r_A} \frac{Q}{4\pi\varepsilon_0 r^2} \hat{a}_r \cdot dr\hat{a}_r = \frac{Q}{4\pi\varepsilon_0} \left[\frac{1}{r_A} - \frac{1}{r_B} \right] = V_A - V_B$$

..................................*(2.26)*

It is customary to choose the potential to be zero at infinity. Thus potential at any point ($r_A = r$) due to a point charge Q can be written as the amount of work done in bringing a unit positive charge from infinity to that point (i.e. $r_B = 0$).

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

..................................*(2.27)*

Or, in other words,

$$V = -\int_{\infty}^{r} E.dl$$

..................................(2.28)

Let us now consider a situation where the point charge Q is not located at the origin asshown in Fig. *2*.10.

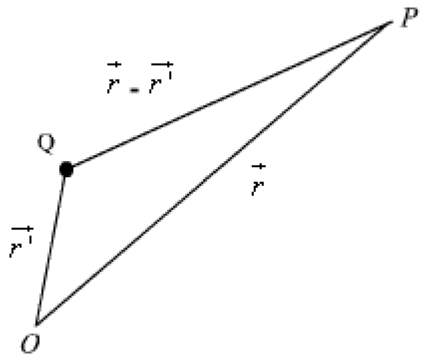

Fig : Electrostatic Potential due a Displaced Charge

The potential at a point P becomes

$$V(r) = \frac{Q}{4\pi\varepsilon_0}\frac{1}{\left|\vec{r} - \vec{r}\,'\right|}$$

..................................(2.29)

So far we have considered the potential due to point charges only. As any other type of charge distribution can be considered to be consisting of point charges, the same basic ideas now can be extended to other types of charge distribution also.

Let us first consider N point charges Q_1, Q_2,. Q_N located at points with position vectors $\vec{r}_1$, $\vec{r}_2$,...... $\vec{r}_N$. The potential at a point having position vector $\vec{r}$ can be written as:

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{\left|\vec{r}-\vec{r_1}\right|} + \frac{Q_2}{\left|\vec{r}-\vec{r_2}\right|} + \ldots\ldots \frac{Q_N}{\left|\vec{r}-\vec{r_N}\right|}\right) \quad (2.30a)$$

or,

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{i=n}^{N}\frac{Q_n}{\left|\vec{r}-\vec{r_n}\right|} \quad (2.30b)$$

For continuous charge distribution, we replace point charges Q_n by corresponding charge elements $\rho_L dl$ or $\rho_S ds$ or $\rho_V dv$ depending on whether the charge distribution is linear, surface or a volume charge distribution and the summation is replaced by an integral. Withthese modifications we can write: