POTENTIAL DUE TO LINE CHARGE AND DIPOLE

Further, in our discussion so far we have used the reference or zero potential at infinity. If any other point is chosen as reference, we can write:

$$V = \frac{Q}{4\pi\varepsilon_0 r} + C$$

.................................(2.34)

where C is a constant. In the same manner when potential is computed from a known electric field we can write:

$$V = -\int \vec{E} \cdot d\vec{l} + C$$ ....(2.35)

The potential difference is however independent of the choice of reference.

$$V_{AB} = V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{l} = \frac{W}{Q}$$

.......................(2.36)

We have mentioned that electrostatic field is a conservative field; the work done in moving a charge from one point to the other is independent of the path. Let us consider moving a charge from point P_1 to P_2 in one path and then from point P_2 back to P_1 over a different path.If the work done on the two paths were different, a net positive or negative amount of work would have been done when the body returns to its original position P_1. In a conservative field there is no mechanism for dissipating energy corresponding to any positive work neitherany source is present from which energy could be absorbed in the case of negative work.

Hence the question of different works in two paths is untenable, the work must have to be independent of path and depends on the initial and final positions.

Since the potential difference is independent of the paths taken, $V_{AB} = - V_{BA}$, and over a closed path, $V_{BA} + V_{AB} = \oint \vec{E} \cdot d\vec{l} = 0$ ....*(2.37)*

Applying Stokes's theorem, we can write:

$$\oint \vec{E} \cdot d\vec{l} = \int_s \left(\nabla \times \vec{E}\right) \cdot d\vec{s} = 0 \qquad \text{.............................}(2.38)$$

from which it follows that for electrostatic field,

$$\text{..}(2.39) \quad \nabla \times \vec{E} = 0$$

Any vector field $\vec{A}$ that satisfies $\nabla \times \vec{A} = 0$ is called an irrotational field. From our definition of potential,

$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial x}dz = -\vec{E} \cdot d\vec{l}$$

we can write

$$\left(\frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z\right) \cdot \left(dx\hat{a}_x + dy\hat{a}_y + dz\hat{a}_z\right) = -\vec{E} \cdot d\vec{l}$$

$$\text{.................................}(2.40) \quad \nabla V \cdot d\vec{l} = -\vec{E} \cdot d\vec{l}$$

from which we obtain,

$$\text{...}(2.41) \quad \vec{E} = -\nabla V$$

From the foregoing discussions we observe that the electric field strength at any point is thenegative of the potential gradient at any point, negative $\vec{E}$ sign shows that $\vec{V}$ is directed from higher to lower values of . This gives us another method of computing the electric field, i. e. if we know the potential function, the electric field may be computed. We may note here $\vec{V}$ that that one scalar function $\vec{E}$ contain all the information that three components $\vec{E}$ of $\nabla \times \vec{E}$ carry,the same is possible because of the fact that three components of are interrelated by therelation .

Example: Electric Dipole

An electric dipole consists of two point charges of equal magnitude but of opposite sign andseparated by a small distance.

As shown in figure 2.11, the dipole is formed by the two point charges Q and -Q separated bya distance d , the charges being placed symmetrically about the origin. Let us consider a point P at a distance r, where we are interested to find the field.

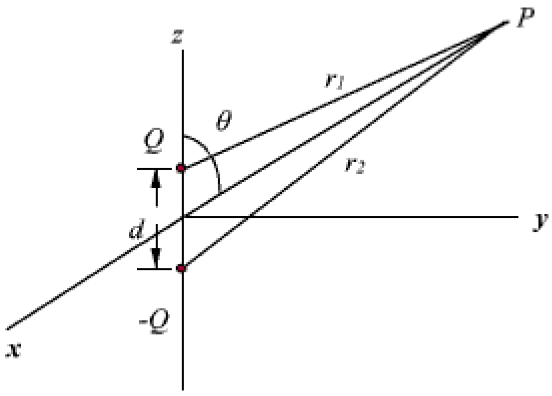

Fig : Electric Dipole

The potential at P due to the dipole can be written as:

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r_1} - \frac{Q}{r_2}\right] = \frac{Q}{4\pi\varepsilon_0}\left[\frac{r_2 - r_1}{r_1 r_2}\right] \text{..........................}(2.42)$$

When r_1 and $r_2 >> d$, we can write $r_2 - r_1 = 2\times\frac{d}{2}\cos\theta = d\cos\theta$ and $r_1 \cong r_2 \cong r$.

Therefore,

$$V = \frac{Q}{4\pi\varepsilon_0}\frac{d\cos\theta}{r^2} \text{..}(2.43)$$

We can write,

$$Qd\cos\theta = Qd\hat{a}_z \cdot \hat{a}_r \text{..}(2.44)$$

The quantity $\vec{P} = Q\vec{d}$ is called the **dipole moment** of the electric dipole. Hence the expression for the electric potential can now be v

$$V = \frac{\vec{P}\cdot\hat{a}_r}{4\pi\varepsilon_0 r^2}$$

................................*(2.45)*

It may be noted that while potential of an isolated charge varies with distance as $1/r$ that ofan electric dipole varies as $1/r^2$ with distance.

If the dipole is not centered at the origin, but the dipole center lies at $\vec{r}'$, the expression forthe potential can be written as:

$$V = \frac{\vec{P}\cdot(\vec{r}-\vec{r}')}{4\pi\varepsilon_0 \left|\vec{r}-\vec{r}'\right|^3}$$

........................*(2.46)*

The electric field for the dipole centered at the origin can be computed as

$$\vec{E} = -\nabla V = -\left[\frac{\partial V}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{a}_\theta\right]$$

$$= \frac{Qd\cos\theta}{2\pi\varepsilon_0 r^3}\hat{a}_r + \frac{Qd\sin\theta}{4\pi\varepsilon_0 r^3}\hat{a}_\theta$$

$$= \frac{Qd}{4\pi\varepsilon_0 r^3}\left(2\cos\theta\hat{a}_r + \sin\theta\hat{a}_\theta\right)$$

$$\vec{E} = \frac{\vec{P}}{4\pi\varepsilon_0 r^3}\left(2\cos\theta\hat{a}_r + \sin\theta\hat{a}_\theta\right)$$

........................(2.47)

$\vec{P} = Q\vec{d}$ is the magnitude of the dipole moment. Once again we note that the electric field ofelectric dipole varies as $1/r^3$ where as that of a point charge varies as $1/r^2$.