Gradient of a Scalar function:

In Cartesian coordinates:

$$\nabla = \frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z$$
....(1.44)

In cylindrical coordinates:

$$\nabla = \frac{\partial}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial}{\partial \phi}\hat{a}_\phi + \frac{\partial}{\partial z}\hat{a}_z$$
....(1.45)

and in spherical polar coordinates:

$$\nabla = \frac{\partial}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\hat{a}_\phi$$
.................................(1.46)

Let us consider a scalar field $V(u,v,w)$, a function of space coordinates.

Gradient of the scalar field V is a vector that represents both the magnitude and directionof the maximum space rate of increase of this scalar field V.

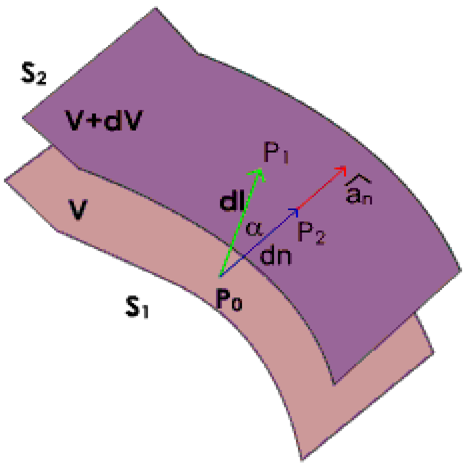

Fig 1.17 : Gradient of a scalar function

As shown in figure 1.17, let us consider two surfaces $S_1$and S_2 where the function V has constant magnitude and the magnitude differs by a small amount dV. Now as one moves from S_1 to S_2, the magnitude of spatial rate of change of V i.e. dV/dl depends on the direction of elementary path length dl, the maximum occurs when one traverses from S_1toS_2along a path normal to the surfaces as in this case the distance is minimum.

By our definition of gradient we can write:

$$\text{grad}V = \frac{dV}{dn}\hat{a}_n = \nabla V$$
....(1.47)

since $d\vec{n}$ which represents the distance along the normal is the shortest distance betweenthe two surfaces.

For a general curvilinear coordinate system

$$d\vec{l} = \hat{a}_u\, dl_u + \hat{a}_v\, dl_v + \hat{a}_w\, dl_w = \left(h_1 du\,\hat{a}_u + h_2 dv\,\hat{a}_v + h_3 dw\,\hat{a}_w\right)$$
....................(1.48)

Further we can write

$$\frac{dV}{dl} = \frac{dV}{dn}\frac{dn}{dl} = \frac{dV}{dn}\cos\alpha = \nabla V \cdot \hat{a}_l$$
....(1.49)

Hence,

$$dV = \nabla V . dl = \nabla V . (h_1 du\,\hat{a}_u + h_2 dv\,\hat{a}_v + h_3 dw\,\hat{a}_w)$$ ....(1.50)

Also we can write,

$$\begin{aligned} dV &= \frac{\partial V}{\partial l_u} dl_u + \frac{\partial V}{\partial l_v} dl_v + \frac{\partial V}{\partial l_w} dl_w \\ &= \left(\frac{\partial V}{\partial l_u}\hat{a}_u + \frac{\partial V}{\partial l_v}\hat{a}_v + \frac{\partial V}{\partial l_w}\hat{a}_w\right) \cdot (dl_u\hat{a}_u + dl_v\hat{a}_v + dl_w\hat{a}_w) \\ &= \left(\frac{\partial V}{h_1\partial u}\hat{a}_u + \frac{\partial V}{h_2\partial v}\hat{a}_v + \frac{\partial V}{h_3\partial w}\hat{a}_w\right) \cdot (h_1 du\hat{a}_u + h_2 dv\hat{a}_v + h_3 dw\hat{a}_w) \end{aligned}$$
............................(1.51)

By comparison we can write,

$$\nabla V = \frac{1}{h_1}\frac{\partial V}{\partial u}\hat{a}_u + \frac{1}{h_2}\frac{\partial V}{\partial v}\hat{a}_v + \frac{1}{h_3}\frac{\partial V}{\partial w}\hat{a}_w$$

....(1.52)

Hence for the Cartesian, cylindrical and spherical polar coordinate system, theexpressions for gradient can be written as:
In Cartesian coordinates:

$$\nabla V = \frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z$$ ....(1.53)

In cylindrical coordinates:

$$\nabla V = \frac{\partial V}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\hat{a}_\phi + \frac{\partial V}{\partial z}\hat{a}_z$$ ....(1.54)

and in spherical polar coordinates:

$$\nabla V = \frac{\partial V}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{a}_\phi$$ ....(1.55)

The following relationships hold for gradient operator.

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$$\nabla(U+V) = \nabla U + \nabla V$$
$$\nabla(UV) = V\nabla U + U\nabla V$$
$$\nabla\left(\frac{U}{V}\right) = \frac{V\nabla U - U\nabla V}{V^2}$$ ....(1.56)
$$\nabla V^n = nV^{n-1}\nabla V$$

where *U* and *V* are scalar functions and *n* is an integer.

It may further be noted that since magnitude of $\frac{dV}{dl}(=\Delta V.\hat{a}_1)$ depends on the direction of d*l*, it is called the **directional derivative**. If $A = \Delta V$, V is called the scalar potential function of the vector function $\vec{A}$

Electric flux density:

As stated earlier electric field intensity or simply 'Electric field' gives the strength of the field at a particular point. The electric field depends on the material media in which the field is being considered. The flux density vector is defined to be independent of the material media(as we'll see that it relates to the charge that is producing it).For a linear isotropic medium under consideration; the flux density vector is defined as:

$$\vec{D} = \varepsilon\vec{E}$$ ....*(2.11)*

We define the electric flux □ as

.....................................*(2.12)*

$$\psi = \int_S \vec{D}.d\vec{s}$$