



Gauss's Law: Gauss's law is one of the fundamental laws of electromagnetism and it states that the total electric flux through a closed surface is equal to the total charge enclosed by the surface.

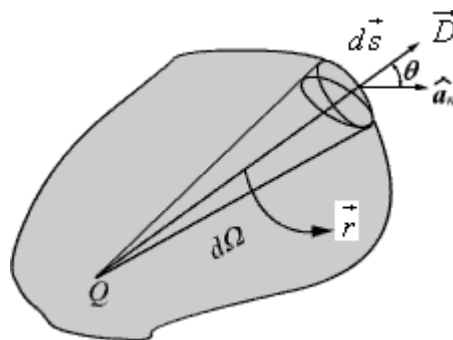


Fig : Gauss's Law

Let us consider a point charge Q located in an isotropic homogeneous medium of dielectric constant ϵ . The flux density at a distance r on a surface enclosing the charge is given by

$$\vec{D} = \epsilon \vec{E} = \frac{Q}{4\pi r^2} \hat{a}_r$$

If we consider an elementary area ds , the amount of flux passing through the elementary area is given by

$$d\psi = \vec{D} \cdot ds = \frac{Q}{4\pi r^2} ds \cos \theta \tag{2.14}$$

But $\frac{ds \cos \theta}{r^2} = d\Omega$, is the elementary solid angle subtended by the area ds at the location of

Q . Therefore we can write $d\psi = \frac{Q}{4\pi} d\Omega$

$$\psi = \oint d\psi = \frac{Q}{4\pi} \oint d\Omega = Q$$

For a closed surface enclosing the charge, we can write

which can be seen to be same as what we have stated in the definition

of Gauss's Law. Application of Gauss's Law

$$\vec{E} \quad \vec{D}$$

Gauss's law is particularly useful in computing \vec{E} or \vec{D} where the charge distribution has some symmetry. We shall illustrate the application of Gauss's Law with some examples.

.....(2.16)

1. Infinite Sheet of Charge

As a second example of application of Gauss's theorem, we consider an infinite charged sheet covering the x - z plane as shown in figure 2.5.

Assuming a surface charge density of ρ_s for the infinite surface charge, if we consider a cylindrical volume having sides placed symmetrically as shown in figure 5, we can write:

$$\oint_S \vec{D} \cdot d\vec{s} = 2D\Delta s = \rho_s \Delta s$$

$$\therefore \vec{E} = \frac{\rho_s}{2\epsilon_0} \hat{a}_y$$

.....(2.17)

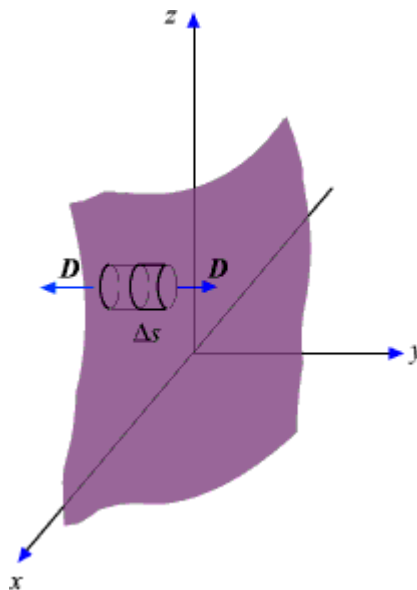


Fig Infinite Sheet of Charge

It may be noted that the electric field strength is independent of distance. This is true for the infinite plane of charge; electric lines of force on either side of the charge will be perpendicular to the sheet and extend to infinity as parallel lines. As number of lines of force per unit area gives the strength of the field, the field becomes independent of distance. For a finite charge sheet, the field will be a function of distance.