Conditional probability :-

The Conditional probability of $A|B$ (given) is $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$P(B) \neq 0$

$B|A$ (given) is $P(B|A) = \frac{P(A \cap B)}{P(A)}$, $P(A) \neq 0$

Note:

Multiplication rule:

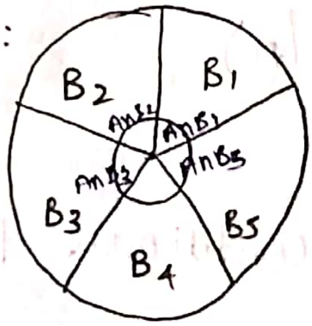
$$P(A \cap B) = \begin{cases} P(A|B) \cdot P(B), & P(B) \neq 0 \\ P(B|A) \cdot P(A) & P(A) \neq 0 \end{cases}$$

Thm of total probability :-

If $B_1, B_2 \ldots B_n$ be a set of exhaustive and mutually Exclusive event and A is another event associated withe B_i then

$$P(A) = \sum_{i=1}^{n} P(B_i) \cdot P(A/B_i)$$

Proof:



The inner circle represents the event A

A can occur along with $B_1, B_2 \ldots B_n$ that are exhaustive & mutually exclusive.

$\therefore$ $AB_1, AB_2, AB_3 \ldots AB_n$ are also mutually exclusive

$\therefore A = AB_1 + AB_2 + AB_3 + AB_n$ (By addition thm)

$$P(A) = P(\sum AB_i)$$

$$= \sum P(AB_i)$$

$$= \sum_{i=1}^{n} P(B_i) \times P(A/B_i)$$

Baye's theorem: ⊗ 8MK

If $B_1, B_2 \dots B_n$ be a set of exhaustive and mutually exclusive events associated with random experiment and A is another event associate with B_i

Then $$P(B_i/A) = \frac{P(B_i) \cdot P(A/B_i)}{\sum_{i=1}^{n} P(B_i) \cdot P(A/B_i)}$$

Proof:

$$P(B_i \cap A) = P(B_i) \cdot P(A/B_i)$$
$$= P(A) \cdot P(B_i/A)$$

$$P(B_i/A) = \frac{P(B_i \cap A)}{P(A)}$$ [By conditional probability]

$$= \frac{P[B_i] \cdot P(A/B_i)}{\sum_{i=1}^{n} P(B_i) \cdot P(A/B_i)}$$ ⟹ By Total probability

20/1/20 problem based on Baye's theorem :-

8MK

1) Suppose there are 3 urns containing 2 white 3 black & $\overset{3W,\,2B}{\wedge}$ 4 W & 1 b respectively.

There is equal probability of each urn being choosen. One ball is drawn from

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an urn choosen at random. What is the prob that white ball is drawn from the 1st urn?

Soln

Let B_1 be the event that 1st urn choosen

Let B_2 be the event that 2nd urn choosen

Let B_3 be the event that 3rd urn choosen.

Let A be the event that a W ball is drawn.

$$P(B_1) = P(B_2) = P(B_3) = \frac{1}{3}$$

$$P(A|B_1) = \frac{2}{5} \; ; \; P(A|B_2) = \frac{3}{5} \; ; \; P(A|B_3) = \frac{4}{5}$$

∴ By baye's thm probab of WB being drawn out of the 1st urn is given by

$$P(B_1|A) = \frac{P(B_1) P(A|B_i)}{\sum_{i=1}^{3} P(B_i) P(A|B_i)}$$

$$= \frac{P(B_1) P(A|B_1)}{P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + P(B_3)P(A|B_3)}$$

$$= \frac{\frac{1}{3} \cdot \frac{2}{5}}{\frac{1}{3}\left[\frac{2}{5} + \frac{3}{5} + \frac{4}{5}\right]} = \frac{\frac{2}{5}}{9/5}$$

$$= \frac{2}{5} \times \frac{5}{9} = \frac{2}{9}.$$

2) A bag contains 5 balls & it is not known how many of them are white. 2 Balls are drawn at random from the bag and they are noted to be white. What is the chance that all the balls in the bag are white.

Soln:

Since 2 W balls have drawn out, The Bag must have contain 2, 3, 4 (or) 5 W balls.

Let B_1 event of bag containing 2 W Balls

B_2 " " 3 W Balls

B_3 " " 4 W balls

B_4 " " 5 W balls

Let A be the event of drawing 2 white balls.

Since no. of W balls in the bag is not known, B_i's are equally likely

$$P(B_1) = P(B_2) = P(B_3) = P(B_4) = \frac{1}{4}$$

$B_1 \Rightarrow 2W = P(B_1)$

$B_2 = 3W = P(B_2)$

$B_3 = 4W = P(B_3)$

$B_4 = 5W = P(B_4)$

$$P(B_4/A) = \frac{P(B_4) \cdot P(A|B_4)}{\sum_{i=1}^{4} P(B_i) \cdot P(A|B_i)}$$

$$P(A|B_i) = \frac{{}^2C_2}{{}^5C_2} = \frac{\cancel{2} \times 1}{1 \times \cancel{2}} = \frac{1}{10} \qquad \frac{5 \times 4}{1 \times 2} = \frac{20}{2}$$

$$P(A|B_2) = \frac{3C_2}{5C_2} = \frac{3 \times 2}{1 \times 2} = \frac{\cancel{6}}{2} \quad \frac{3}{10}$$

$$P(A|B_3) = \frac{4C_2}{5C_2} = \frac{4 \times 3 \times 2}{1 \times 2 \times 3} = \frac{24}{6} = \frac{6}{10}$$

$$P(A|B_4) = \frac{5C_2}{5C_2} = 1$$

$$= \frac{\frac{1}{4} \cdot 1}{\frac{1}{4}\left[\frac{1}{10} + \frac{3}{10} + \frac{6}{10} + 1\right]} = \frac{1}{\left(\frac{20}{10}\right)} = \frac{10}{20} = \frac{1}{2}.$$

3. There are 3 true coins and 1 false coin with head on both sides. A coin is choosen at random and tossed 4 times. If head occurs all the 4 times. what is the probability that the false coin has been choosen and used.

$P(T) = P$ (The coin is a true coin) $= \frac{3}{4}$

$P(F) = P$ (The coin is a false coin) $= \frac{1}{4}$

Let A (Event of getting all heads in 4 times)

The $p(A/T) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{16}$

$p(A/F) = 1$

By Baye's thm :

$$p(F/A) = \frac{P(F)\, P(A/F)}{P(T) \cdot P(A/T) + P(F) \cdot P(A/F)}$$

$$= \frac{\frac{1}{4} \cdot 1}{\frac{3}{4} \cdot \frac{1}{16} + \frac{1}{4} \cdot 1} = \frac{\frac{1}{4}}{\frac{3}{64} + \frac{1}{4}}$$

$$= \frac{\frac{1}{4}}{\frac{1}{4}\left(\frac{3}{16} + 1\right)}$$

$$= \frac{16}{19}$$