Standard Distribution (Discrete)

1. Binomial distribution
2. Poisson distribution
3. Geometric distribution

Binomial Distribution

Binomial distribution is derived from experiment known as Bernoulli trial.

A random experiment whose outcomes c be classified into two categories, usually calle 'success' and 'failure', is called a Bernoull trail.

A random variable X is said to binomial distribution with parameter n and p its pmf is given by

$$P[X=x] = nc_x \, p^x \, q^{n-x}, \quad x = 0, 1, 2, \cdots$$

X is called a binomial random variable.

where n - no. of trials
p - probability of success.
q - probability of failure
$p+q = 1$

1. Find the MGF of the binomial distribution and hence find mean and variance.

$$P[X=x] = nc_x\, p^x q^{n-x}$$

$$MGF = M_X(t) = E\left[e^{tX}\right]$$

$$= \sum_{x=0}^{n} e^{tx}\, nc_x\, p^x q^{n-x}$$

$$= q^n + nc_1\, pe^t q^{n-1} + nc_2\, (pe^t)^2 q^{n-2} + \cdots + nc_n\, (pe^t)^n$$

$$= (pe^t+q)^n$$

$(x+y)^n = x^n + nc_1\, x^{n-1} y + nc_2\, x^{n-2} y^2 \cdots + nc_n\, y^n$

$$Mean = \left\{\frac{d}{dt}\left[M_X(t)\right]\right\}_{t=0}$$

$$= \left\{\frac{d}{dt}(pe^t+q)^n\right\}_{t=0}$$

$$= \left\{n\,(pe^t+q)^{n-1}(pe^t)\right\}_{t=0}$$

$$= n\,(p+q)^{n-1}\, p$$

$nc_r = 1c_1$

$$= np$$

$$E[X^2] = \left\{\frac{d^2}{dt^2}\left[M_X(t)\right]\right\}_{t=0}$$

$$= \left\{\frac{d}{dt}\, np\, e^t\, (pe^t+q)^{n-1}\right\}_{t=0}$$

$$= np\left[e^{t}(n-1)(pe^{t}+q)^{n-2}(pe^{t}) + (pe^{t}+q)\right]$$

$$= np\left[(n-1)(p+q)^{n-2}p + (p+q)\right]$$

$$= np\left[(n-1)p+1\right] = np\left[np-p+1\right]$$

$$= np\left[np+q\right] = n^2p^2+npq$$

$$\text{Variance} = E[X^2] - [E(X)]^2$$

$$= n^2p^2 + npq - n^2p^2$$

$$= npq$$

2. For a Binomial distribution with mean 6 & standard deviation $\sqrt{2}$, find the first two terms

Gn. Mean $= np = 6 \rightarrow (1)$

Variance $= npq = 2 \rightarrow (2)$

$(2)/(1) \Rightarrow \dfrac{npq}{np} = \dfrac{2}{6} = \dfrac{1}{3}$

$\Rightarrow \boxed{q = \frac{1}{3}}$ $\qquad p = 1-q$ $\qquad \boxed{p = \frac{2}{3}}$

$np = 6$

$n\left(\frac{2}{3}\right) = 6$

$\boxed{n = 9}$

$P(X=x) = nC_x\, p^x q^{n-x}, \quad x = 0$

$$P[X=0] = 9C_0 \left(\frac{2}{3}\right)^0 \left(\frac{1}{3}\right)^9$$

$$= \left(\frac{1}{3}\right)^9$$

$$P[X=1] = 9C_1 \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^8$$

$$= 18\left(\frac{1}{3}\right)^9$$

⑧ In a large consignment of electric bulb 10% are defective. A random sample of 20 is taken for inspection. Find the prob. that
(i) All are good bulbs (ii) Atmost there are 3 defective bulbs (iii) Exactly there are 3 defective bu

Let x denote the no. of defective bulbs.
Let p – the prob. that an electric bulb is defective $= \frac{1}{10}$

$q = 1 - p = \frac{9}{10}$ and $n = 20$.

Binomial distribution $P[x = x] = nc_x \, p^x \, q^{n-x}$

$x = 0, 1, 2, \ldots$

(i) All are good bulbs:

$= P[x = 0] = 20c_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^{20} = \left(\frac{9}{(10)}\right)^{20}$

$= 0.1216$

(ii) Almost there are 3 defective bulbs.

$P[x \leq 3] = P[x = 0] + P[x = 1] + P[x = 2] + P[x =$

$= 20c_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^{20} + 20c_1 \left(\frac{1}{10}\right) \left(\frac{9}{10}\right)^{19}$

$+ 20c_2 \left(\frac{1}{10}\right)^2 \left(\frac{9}{10}\right)^{18} + 20c_3 \left(\frac{1}{10}\right)^3 \left(\frac{9}{10}\right)^{17}$

$= \left(\frac{9}{10}\right)^{20} + \cancel{20} \frac{9^{19}}{10^{20}} \; 2 \left(\frac{9}{10}\right)^{19} + \frac{190}{100} \left(\frac{9}{10}\right)^{18} +$

$= 0.2666$

(iii) Exactly 3 are defective

$P[x = 3] = 20c_3 \left(\frac{1}{10}\right)^3 \left(\frac{9}{10}\right)^{17}$

$= 0.19$

1) Out of 800 families with 4 children each, how many families would be expected to have (i) 2 boys and 2 girls ; (ii) atleast 1 boy (iii) atmost 2 girls (iv) children of both genders. Assume equal probabilities for boys and girls.

Considering each child is a trial, $n = 4$. Assuming that birth of a boy is a success, $p = \frac{1}{2}$ and $q = \frac{1}{2}$.

Let x denote the no. of success (boys). Binomial distribution $P[X = x] = nC_x \, p^x q^{n-x}$, $x = 0, 1, 2, 3, 4$

i) $P[\text{at 2 boys and 2 girls}]$

$$= P[X = 2] = 4C_2 \left(\tfrac{1}{2}\right)^2 \left(\tfrac{1}{2}\right)^2 = 6 \left(\tfrac{1}{2}\right)^4$$

$$= \frac{6}{16} = \frac{3}{8}$$

∴ No. of families having 2 boys & 2 girls

$$= 800 \times \frac{3}{8} = 300$$

(ii) $P[\text{atleast 1 boy}] = P[X \geq 1] = 1 - P[X < 1]$

$$= 1 - P[X = 0] = 1 - 4C_0 \left(\tfrac{1}{2}\right)^0 \left(\tfrac{1}{2}\right)^4$$

$$= 1 - \frac{1}{2^4} = 1 - \frac{1}{16} = \frac{15}{16}$$

∴ No. of families having atleast 1 boy

$$= 800 \times \frac{15}{16} = 750$$

(iii) $P[\text{atmost } 2 \text{ girls}] = P[\text{atleast } 2 \text{ boys}]$

$$= P[x \geq 2] = 1 - P[x < 2]$$

$$= 1 - \{P[x=0] + P[x=1]\}$$

$$= 1 - \left\{ 4C_0 \left(\tfrac{1}{2}\right)^0 \left(\tfrac{1}{2}\right)^4 + 4C_1 \left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right)^3 \right\}$$

$$= 1 - \left\{ \left(\tfrac{1}{2}\right)^4 + 4\left(\tfrac{1}{2}\right)^4 \right\}$$

$$= 1 - \frac{1}{16}(5) = \frac{11}{16}$$

$\therefore$ No. of familis having atmost 2 girls

$$= 800 \times \frac{11}{16} = 550$$

(iv) $P[\text{children of both genders}]$

$$= 1 - P[\text{children of the same gender}]$$

$$= 1 - \{P[\text{all are boys}] + P[\text{all are girls}]\}$$

$$= 1 - \{P[x=4] + P[x=0]\}$$

$$= 1 - \left\{ 4C_4 \left(\tfrac{1}{2}\right)^4 \left(\tfrac{1}{2}\right)^0 + 4C_0 \left(\tfrac{1}{2}\right)^0 \left(\tfrac{1}{2}\right)^4 \right\}$$

$$= 1 - \left\{ \left(\tfrac{1}{2}\right)^4 + \left(\tfrac{1}{2}\right)^4 \right\}$$

$$= 1 - \frac{2}{2^4} = 1 - \frac{1}{2^3} = 1 - \frac{1}{8} = \frac{7}{8}$$

$\therefore$ No. of familie having children of both genders

$$= 800 \times \frac{7}{8} = 700$$