**TOPIC : 1.9 – Poisson Distribution**

## Poisson Distribution.

A random variable $X$ is said to follow Poisson distribution if it assumes only non-negative values and its pmf is given by

$$P[X=x] = \frac{e^{-\lambda}\lambda^{x}}{x!}, \quad x = 0, 1, 2, \dots \quad \lambda > 0$$

$\lambda$ is known as the parameter of the Poisson distribution. ($\lambda = np$).

1) Find the MGF of the Poisson distribution and hence find mean and variance.

The pmf of the Poisson distribution

$$P[X=x] = \frac{e^{-\lambda}\lambda^{x}}{x!}, \quad x = 0, 1, 2, \dots$$

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$$\text{MGF } M_X(t) = E[e^{tX}] = \sum_{x=0}^{\infty} e^{tx} \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{(\lambda e^{t})^{x}}{x!}$$

$$= e^{-\lambda}\left[1 + \frac{(\lambda e^{t})}{1!} + \frac{(\lambda e^{t})^{2}}{2!} + \dots\right]$$

$$= e^{-\lambda} e^{\lambda e^{t}} = e^{\lambda(e^{t}-1)}$$

$$\text{Mean} = E[x] = \left\{\frac{d}{dt}\left(M_x(t)\right)\right\}_{t=0}$$

$$= \left[\frac{d}{dt} e^{\lambda(e^t-1)}\right]_{t=0} = \left\{e^{\lambda(e^t-1)} \lambda e^t\right\}_{t=}$$

$$\boxed{\text{Mean} = \lambda}$$

$$E[x^2] = \left\{\frac{d^2}{dt^2} M_x(t)\right\}_{t=0}$$

$$= \lambda \left\{\frac{d}{dt} e^t e^{\lambda(e^t-1)}\right\}_{t=0}$$

$$= \lambda\left[e^t e^{\lambda(e^t-1)} \lambda e^t + e^{\lambda(e^t-1)} e^t\right]_{t=}$$

$$= \lambda[\lambda+1] = \lambda^2 + \lambda$$

$$Var(x) = E[x^2] - [E(x)]^2$$

$$= \lambda^2 + \lambda - \lambda^2$$

$$\boxed{Var(x) = \lambda}$$

1. very week the average no. of wrong-number ione calls received by a certain mail order house seven. what is the Prob. that they will receive 2 wrong calls tomorrow?

The average no. of wrong-number phone calls received in a week $\} = 7$

Average number of wrong-number calls per day $= \frac{7}{7} = 1 = \lambda$.

Let $x$ denote the no. of wrong-number phone calls per day. The pmf of Poisson $P[x=x] = \frac{e^{-\lambda}\lambda^x}{x!}$

$$\therefore P[x=2] = \frac{e^{-1}(1)^2}{2!} = \frac{e^{-1}}{2} =$$

④ The number of monthly breakdown of a computer is a random variable having a Poiss dis. with mean = 1.8. Find the prob. that this computer will function for a month
(i) without a breakdown (ii) with only one break
and (iii) with atleast one breakdown.

Given Mean $= \lambda = 1.8$

Let $x$ denote the no. of breakdowns of a computer in a month.

The pmf of Poisson dis. $P[x=x] = \dfrac{e^{-\lambda}\lambda^{x}}{x!}$, $x = 0,1,2,\dots$

(i) P[with a breakdown]

$= P[x=0] = \dfrac{e^{-1.8}(1.8)^0}{0!} = e^{-1.8} = 0.1653$

P[with only one breakdown]

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$= \dfrac{e^{-1.8}(1.8)^1}{1!} = 0.2975$

) P[with atleast 1 breakdown] $= P[x \geq 1]$

$= 1 - P[x<1] = 1 - P[x=0] = 1 - 0.1653$

$= 0.8347$