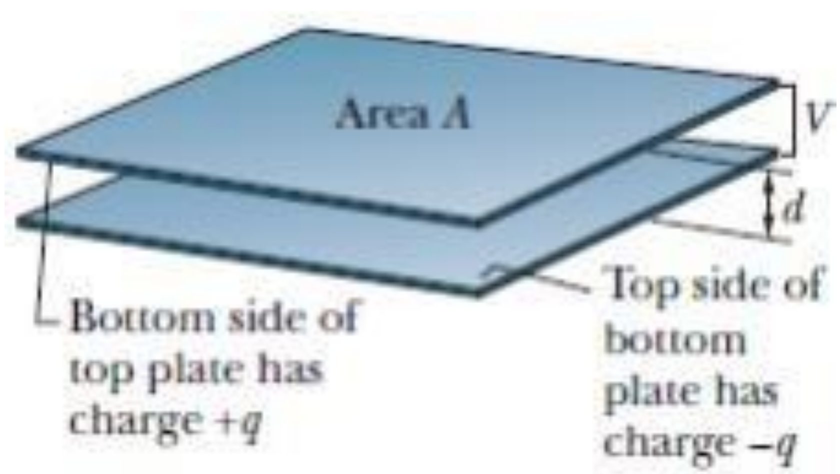
**Capacitors and Capacitance:** Parallel Plate; Cylindrical and Spherical capacitors; Capacitors in Series and Parallel; Energy Stored in an Electric Field; Dielectrics and Gauss' Law

**Capacitor:**

A capacitor is a passive electronic component that stores energy in the form of an electrostatic field. In its simplest form, a capacitor consists of two conducting plates separated by an insulating material called the dielectric.

The capacitance is directly proportional to the surface areas of the plates, and is inversely proportional to the separation between the plates. Capacitance also depends on the dielectric constant of the substance separating the plates.




This conventional arrangement, called a parallel-plate capacitor, consisting of two parallel conducting plates of area A separated by a distance d.

The symbol we use to represent a capacitor (⊣⊢) is based on the structure of a parallel-plate capacitor but is used for capacitors of all geometries.

We assume for the time being that no material medium (such as glass or plastic) is present in the region between the plates.

When a capacitor is charged, its plates have charges of equal magnitudes but opposite signs: +q and -q. However, we refer to the charge of a capacitor as being q, the absolute value of these charges on the plates. (Note that q is not the net charge on the capacitor, which is zero.)

Because the plates are conductors, they are equipotential surfaces; all points on a plate are at the same electric potential. Moreover, there is a potential difference between the two plates. For historical reasons, we represent the absolute value of this potential difference with V rather than with the $\Delta V$ we used in previous notation.

The charge q and the potential difference V for a capacitor are proportional to each other; that is,

$$q = CV.$$

The proportionality constant C is called the capacitance of the capacitor. Its value depends only on the geometry of the plates and not on their charge or potential difference. The capacitance is a measure of how much charge must be put on the plates to produce a certain potential difference between them: The greater the capacitance, the more charge is required.

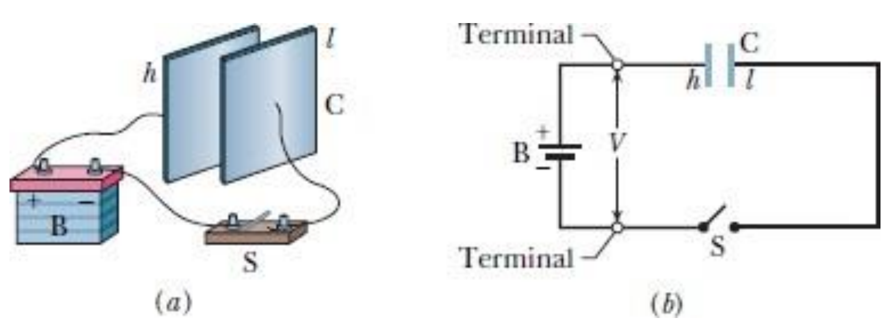
The SI unit, capacitance is the coulomb per volt. This unit occurs so often that it is given a special name, the farad (F):

$$1 \text{ farad} = 1 \text{ F} = 1 \text{ coulomb per volt} = 1 \text{ C/V}.$$

As you will see, the farad is a very large unit. Submultiples of the farad, such as the microfarad ($1\ \mu F = 10^{-6}$ F) and the picofarad ($1 \text{ pF} = 10^{-12}$ F), are more convenient units in practice

**Charging a Capacitor**

One way to charge a capacitor is to place it in an electric circuit with a battery. An electric circuit is a path through which charge can flow. A battery is a device that maintains a certain potential difference between its terminals.



In Fig.a, a battery B, a switch S, an uncharged capacitor C, and interconnecting wires form a circuit.

The same circuit is shown in the schematic diagram of Fig. b, in which the symbols for a battery, a switch, and a capacitor represent those devices. The battery maintains potential difference V between its terminals. The terminal of higher potential is labeled + and is often called the positive terminal; the terminal of lower potential is labeled - and is often called the negative terminal.

The circuit shown in Figs. *a* and *b* is said to be incomplete because switch S is open; that is, the switch does not electrically connect the wires attached to it. When the switch is closed, electrically connecting those wires, the circuit is complete and charge can then flow through the switch and the wires.

As we discussed, the charge that can flow through a conductor, such as a wire, is that of electrons.

When the circuit of Fig. (a,b) is completed, electrons are driven through the wires by an electric field that the battery sets up in the wires. The field drives electrons from capacitor plate *h* to the positive terminal of the battery; thus, plate *h*, losing electrons, becomes positively charged.

The field drives just as many electrons from the negative terminal of the battery to capacitor plate *l*; thus, plate *l*, gaining electrons, becomes negatively charged.

Initially, when the plates are uncharged, the potential difference between them is zero. As the plates become oppositely charged, that potential difference increases until it equals the potential difference V between the terminals of the battery.

Then plate h and the positive terminal of the battery are at the same potential, and there is no longer an electric field in the wire between them.

Similarly, plate *l* and the negative terminal reach the same potential, and there is then no electric field in the wire between them.

Thus, with the field zero, there is no further drive of electrons. The capacitor is then said to be fully charged, with a potential difference V and charge q that are related by Eq.

$$Q = C V$$

**Calculating the Capacitance**

To calculate the capacitance of a capacitor once we know its geometry. Because we shall consider a number of different geometries, it seems wise to develop a general plan to simplify the work.

In brief our plan is as follows:

(1) Assume a charge q on the plates

(2) Calculate the electric field between the plates in terms of this charge, using Gauss' law

$$\Phi = \oint \vec{E} \cdot d\vec{A}$$

(3) Calculate the potential difference V between the plates from Eq. ( $V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}.$ )

(4) Calculate C from Eq.( q = CV).

**Calculating the Electric Field**

To relate the electric field between the plates of a capacitor to the charge q on either plate, we shall use Gauss' law:

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q. \qquad (1)$$

Here q is the charge enclosed by a Gaussian surface and $\oint \vec{E} \cdot d\vec{A} = q.$ is the net electric flux through that surface. In all cases that we shall consider, the Gaussian surface will be such that whenever there is an electric flux through it, $\vec{E}$ will have a uniform magnitude E and the vectors $\vec{E}$ and $d\vec{A}$ will be parallel. The above equation, then reduces to

$$q = \varepsilon_0 E A \qquad (2)$$

**Calculating the Potential Difference**

The potential difference between the plates of a capacitor is related to the field by

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}. \qquad (3)$$

in which the integral is to be evaluated along any path that starts on one plate and ends on the other.

We shall always choose a path that follows an electric field line, from the negative plate to the positive plate.

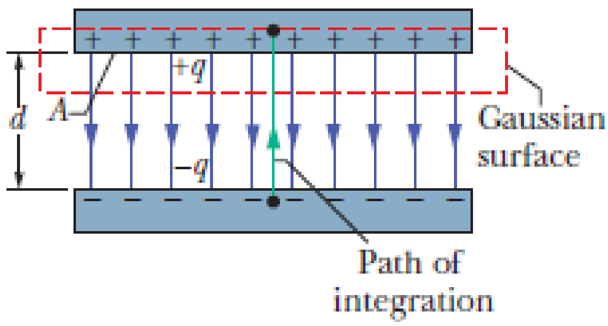
For this path, the vectors $\vec{E}$ and $d\vec{s}$ will have opposite directions; so the dot product will be equal to $\vec{E} \cdot d\vec{s}$.Thus, the right side of above Eq. will then be positive. Letting V represent the difference $V_f$-$V_i$, we can then recast Eq. as

$$V = \int_-^+ E\, ds \qquad (4)$$

in which the - and + remind us that our path of integration starts on the negative plate and ends on the positive plate.

**A Parallel-Plate Capacitor**

We assume, as Fig. 25-5 suggests, that the plates of our parallel-plate capacitor are so large and so close together that we can neglect the fringing of the electric field at the edges of the plates, taking $\vec{E}$ to be constant throughout the region between the plates.




We draw a Gaussian surface that encloses just the charge q on the positive plate, as in above Fig.. From Eq. 2 we can then write

$$q = \varepsilon_0 EA \tag{5}$$

Equation 4 yield

$$V = \int_{-}^{+} E\,ds = E\int_{0}^{d} ds = Ed. \tag{6}$$

In Eq. 6, E can be placed outside the integral because it is a constant; the second integral then is simply the plate separation d.

put q and V into the relation q = CV , we get

$$C = \frac{\varepsilon_0 A}{d} \tag{7}$$

$$\varepsilon_0 = 8.85 \times 10^{-12}\ \text{F/m} = 8.85\ \text{pF/m}.$$

$$\varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2$$