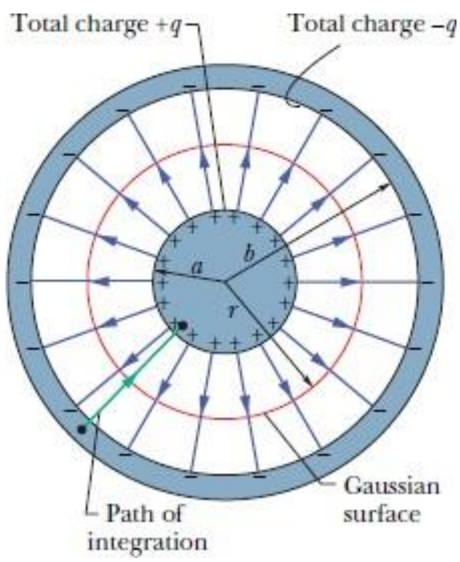
A Co-axial Cylindrical Capacitor

Figure shows, in cross section, a cylindrical capacitor of length *L* formed by two coaxial cylinders of radii *a* and *b*. We assume that *L*>>(*b* so that we can neglect the fringing of the electric field that occurs at the ends of the cylinders. Each plate contains a charge of magnitude *q*.



As a Gaussian surface, we choose a cylinder of length L and radius r, closed by end caps and placed as is shown in Fig. It is coaxial with the cylinders and encloses the central cylinder and thus also the charge q on that cylinder. Equation 2 then relates that charge and the field magnitude E as

$$q = \varepsilon_0 EA = \varepsilon_0 E(2\pi rL)$$

In which 2πrL is the area of the curved part of the Gaussian surface. There is no flux through the end caps. Solving for E yields

$$E = \frac{q}{2\pi\varepsilon_0 L r}. \quad (8)$$

Put these values in eq 4

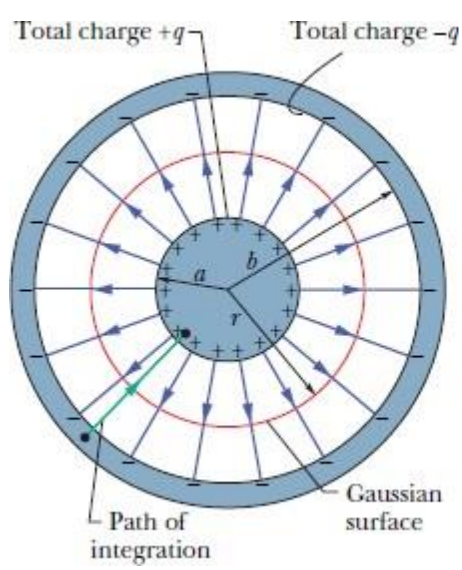
$$V = \int_{-}^{+} E\, ds = -\frac{q}{2\pi\varepsilon_0 L}\int_b^a \frac{dr}{r} = \frac{q}{2\pi\varepsilon_0 L}\ln\left(\frac{b}{a}\right) \quad (9)$$

where we have used the fact that here ds = -dr (we integrated radially inward).
From the relation C = q/V, we then have

$$C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)} \quad (10)$$

We see that the capacitance of a cylindrical capacitor, like that of a parallel-plate capacitor, depends only on geometrical factors, in this case the length L and the two radii b and a.

A Spherical Capacitor



This also Figure can also serve as a central cross section of a capacitor that consists of two concentric spherical shells, of radii a and b. As a Gaussian surface we draw a sphere of radius r concentric with the two shells; then Eq. 2 yields

$$q = \varepsilon_0 E A = \varepsilon_0 E(4\pi r^2),$$

in which $4\pi r^2$ is the area of the spherical Gaussian surface. We solve this equation for E, obtaining

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \quad (11)$$

Which we recognize as the expression for the electric field due to a uniform spherical charge distribution (Eq. 11).

If we substitute this expression into Eq. 4, we find

$$V = \int_{-}^{+} E\, ds = -\frac{q}{4\pi\varepsilon_0}\int_{b}^{a}\frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right) = \frac{q}{4\pi\varepsilon_0}\frac{b-a}{ab} \qquad (12)$$

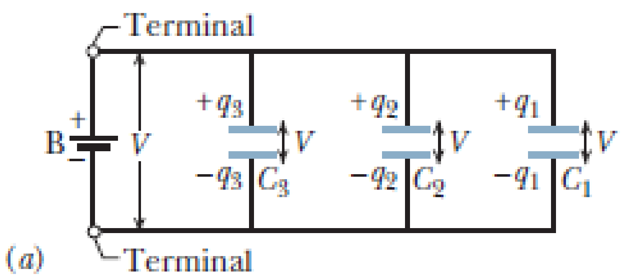
where again we have substituted -dr for ds. If we now substitute Eq. 12 into q = CV, we find

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a} \qquad (13)$$

Capacitors in Parallel

Figure. (a) Shows an electric circuit in which three capacitors are connected in parallel to battery B.

Each capacitor has the same potential difference V, which produces charge on the capacitor. (In Fig. a, the applied potential V is maintained by the battery.) In general, When we analyze a circuit of capacitors in parallel, we can simplify it with this mental replacement:



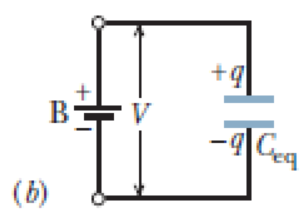


Figure b shows the equivalent capacitor (with equivalent capacitance C_{eq}) that has replaced the three capacitors (with actual capacitances C_1,C_2, and C_3) of Fig. a.

To derive an expression for Ceq in Fig. b, we first use Eq. q = CV to find the charge on each actual capacitor: The total charge on the parallel combination

$$q_1 = C_1V, \quad q_2 = C_2V, \quad \text{and} \quad q_3 = C_3V.$$

$$q = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)V$$

The equivalent capacitance, with the same total charge *q* and applied potential difference *V* as the combination, is then

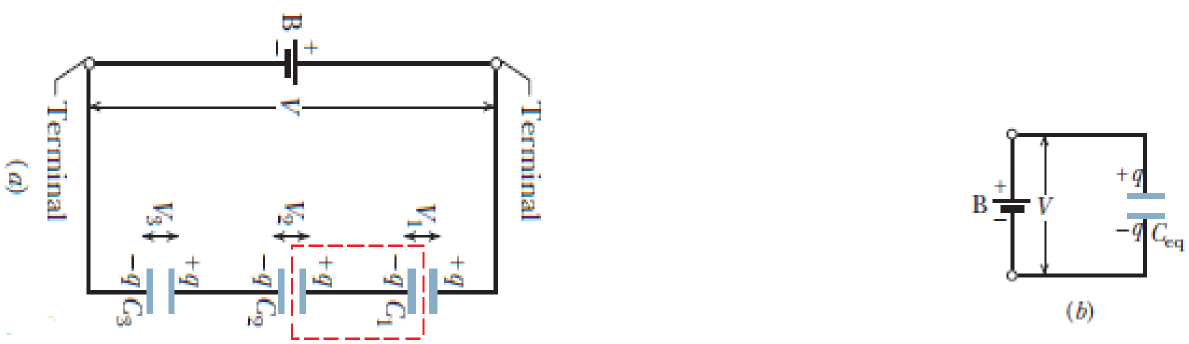
$$C_{eq} = \frac{q}{V} = C_1 + C_2 + C_3,$$

For *n* number of capacitors,

$$C_{eq} = \sum_{j=1}^{n} C_j$$

Capacitors in Series

When the battery is first connected to the series of capacitors, it produces charge -q on the bottom plate of capacitor 3. That charge then repels negative charge from the top plate of capacitor 3 (leaving it with charge +q). The repelled negative charge moves to the bottom plate of capacitor 2 (giving it charge - q). That charge on the bottom plate of capacitor 2 then repels negative charge from the top plate of capacitor 2 (leaving it with charge +q) to the bottom plate of capacitor 1 (giving it charge - q). Finally, the charge on the bottom plate of capacitor 1 helps move negative charge from the top plate of capacitor 1 to the battery, leaving that top plate with charge +q.



To derive an expression for *C*eq in Fig. 25-9*b*, we first use q = CV to find the potential difference of each actual capacitor:

$$V_1 = \frac{q}{C_1}, \quad V_2 = \frac{q}{C_2}, \quad \text{and} \quad V_3 = \frac{q}{C_3}.$$

The total potential difference V due to the battery is the sum of these three potential differences. Thus,

$$V = V_1 + V_2 + V_3 = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right).$$

$$C_{eq} = \frac{q}{V} = \frac{1}{1/C_1 + 1/C_2 + 1/C_3},$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

For *n* number *of* capacitors as

$$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$$

The equivalent capacitance of a series of capacitances is always less than the least capacitance in the series