UNIT-I
SOLUTION OF EQUATIONS AND EIGEN VALUE PROBLEMS
PART-A

1. **State the order of convergence and convergence condition for Newtons Raphson method.**

   Solution:

   The order of convergence is 2.

   Condition for convergence is $\left|f(x)f''(x)\right| < \left|f'(x)\right|^2$

2. **Write the iterative formula of Newton Raphson method.**

   Solution:

   $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

3. **Using Newton-Raphson method, find the iteration formula to compute $\sqrt{N}$ where N is a positive number and hence find $\sqrt{5}$.**

   Solution:

   If $x = \sqrt{N},\ \ then\ x^2 - N = 0$ is the equation to be solved.

   $let\ \ f(x) = x^2 - N, \quad f'(x) = 2x$

By N.R formula, if x_n is the n[th] iterate $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}.$

$$= x_n - \frac{x_n^{\ 2} - N}{2x_n}.$$

$$= \frac{2x_n^{\ 2} - x_n^{\ 2} + N}{2x_n}$$

$$= \frac{x_n^{\ 2} + N}{2x_n}$$

$$x_{n+1} = \frac{1}{2}\left[x_n + \frac{N}{x_n}\right]$$

To Find $\sqrt{5}$

Put N=5. Also x=$\sqrt{5}$ lies between 2 and 3.

$$x_0 = 2$$

$$x_{n+1} = \frac{1}{2}\left[x_n + \frac{N}{x_n}\right]$$

Let $x_1 = \frac{1}{2}\left[x_0 + \frac{5}{x_0}\right] = 2.25$

$$x_2 = \frac{1}{2}\left[x_1 + \frac{5}{x_1}\right] = 2.2361$$

$$x_3 = \frac{1}{2}\left[x_2 + \frac{5}{x_2}\right] = 2.2361$$

Hence the approximate value of $\sqrt{5} = 2.2361$.

4. **Find an iterative formula to find the reciprocal of a given number N.**
Solution:

$$Let \;\; x = \frac{1}{N} \qquad Therefore \;\; N = \frac{1}{x}$$

$$f(x) = \frac{1}{x} - N = 0 \;\; ; \quad f'(x) = -\frac{1}{x^2}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

$$= x_n - \frac{\left(\frac{1}{x_n} - N\right)}{\left(-\frac{1}{x_n{}^2}\right)}$$

$$= x_n + x_n{}^2\left(\frac{1}{x_n} - N\right)$$

$$= 2x_n - Nx_n{}^2 = x_n(2 - Nx_n).$$

5. **Give two direct and indirect methods to solve a system of equations**.
Solution:
Direct method:
   (i) Gauss elimination method
   (ii) Gauss Jordan method.

Indirect method:

   (i) Gauss Jacobi method
   (ii) Gauss Seidel method

6. **Solve the following system of equations, using Gauss-Jordan elimination method 2x+y=3; x-2y=-1.**

Solution: The given is equivalent to

$$\lfloor A,B \rfloor = \left(\begin{matrix} 2 & 1 \\ 1 & -2 \end{matrix}\middle|\begin{matrix} 3 \\ -1 \end{matrix}\right)$$

$$= \left(\begin{matrix} 2 & 1 \\ 0 & -5 \end{matrix}\middle|\begin{matrix} 3 \\ -5 \end{matrix}\right) \quad R_2 \leftrightarrow 2\,R_2 - R_1$$

$$= \left(\begin{matrix} 2 & 1 \\ 0 & 1 \end{matrix}\middle|\begin{matrix} 3 \\ 1 \end{matrix}\right) \quad R_2 \leftrightarrow \frac{R_2}{-5}$$

$$= \left(\begin{matrix} 2 & 0 \\ 0 & 1 \end{matrix}\middle|\begin{matrix} 2 \\ 1 \end{matrix}\right) \quad R_2 \leftrightarrow R_1 - R_2$$

$$= \left(\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}\middle|\begin{matrix} 1 \\ 1 \end{matrix}\right) \quad R_1 \leftrightarrow \frac{R_1}{2}$$

Hence x=1, y=1.

7. **To which forms are the augmented matrix transformed in the Gauss Jordon and Gauss Elimination method.**

Solution:

Gauss Elimination **:** Co-efficient matrix is transformed into upper triangular matrix.

Gauss Jordan **:** Co-efficient matrix is transformed into diagonal matrix.

8. **Write a sufficient condition for Gauss-Jacobi(or Gauss-Seidal) method to converge.**

Solution:

The process of iteration by Gauss seidal method will converge if in each equation of the system, the absolute value of the largest co-efficient is greater than the sum of the absolute value of the remaining co-efficient.

9. **Compare Gauss-Jacobi and Gauss seidal method**

Solution:

S. No.	**Gauss Jacobi method**	**Gauss –Seidal method**
1.	Indirect method	Indirect method.
2.	Convergence rate is slow	The rate of convergence of Gauss seidal method is roughly twice that of Gauss Jacobi method.
3.	Condition for convergence is the co -efficient matrix is diagonally dominant.	Condition for convergence is the co -efficient matrix is diagonally dominant.

UNIT-II
INTEPOLATION AND APPROXIMATION
PART-A

1. What do you mean by interpolation. **[A.U. 2006,2011]**

Solution: The process of finding the value of a function inside the given range is called interpolation.

2. **State Lagrange's interpolation formula. [April 2000, N/D 211]**

Solution: Let y=f(x) be a function which takes the values $y_0, y_1\ y_2, y_3 \ldots\ldots\ldots\ldots\ldots y_n$ corresponding to x= $x_0, x_1\ x_2, x_3 \ldots\ldots\ldots\ldots\ldots x_n$. Then Lagrange's interpolation formula is

$$y = f(x) = \frac{(x-x_1)(x-x_2)\ldots\ldots\ldots\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)\ldots\ldots\ldots\ldots(x_0-x_n)} y_0$$
$$+ \frac{(x-x_0)(x-x_2)\ldots\ldots\ldots\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)\ldots\ldots\ldots\ldots(x_1-x_n)} y_1$$
$$+ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$+ \frac{(x-x_1)(x-x_2)\ldots\ldots\ldots\ldots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\ldots\ldots\ldots\ldots(x_n-x_{n-1})} y_n .$$

3. Give the inverse of Lagranges interpolation formula.[N/D 2007]

Solution:

$$x = \frac{(y-y_1)(y-y_2)\ldots\ldots\ldots\ldots(y-y_n)}{(y_0-y_1)(y_0-y_2)\ldots\ldots\ldots\ldots(y_0-y_n)} x_0$$
$$+ \frac{(y-y_0)(y-y_2)\ldots\ldots\ldots\ldots(y-y_n)}{(y_1-y_0)(y_1-y_2)\ldots\ldots\ldots\ldots(y_1-y_n)} x_1$$
$$+ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$+ \frac{(y-y_1)(y-y_2)\ldots\ldots\ldots\ldots(y-y_{n-1})}{(y_n-y_0)(y_n-y_1)\ldots\ldots\ldots\ldots(y_n-y_{n-1})} x_n$$

4. Find the parabola of the form $y=ax^2+bx+c$ passing through the points (0,0), (1,1) & (2,20).[M/J 2007]

Solution: We use Lagrange's interpolation formula

$$y = f(x) = \frac{(x-x_1)(x-x_2).}{(x_0-x_1)(x_0-x_2).} y_0 + \frac{(x-x_0)(x-x_2).}{(x_1-x_0)(x_1-x_2).} y_1 + \frac{(x-x_0)(x-x_1).}{(x_2-x_0)(x_2-x_1).} y_2$$

$$= \frac{(x-1)(x-2).}{(0-1)(0-2).} 0 + \frac{(x-0)(x-2).}{(1-0)(1-2).} 1 + \frac{(x-0)(x-1).}{(2-0)(2-1).} 20$$

y = $9x^2-8x$.

5. **State the any two properties of divided differences.** **[A.U.2012]**

Solution:

(i) The divided differences are symmetrical in all their arguments. i.e., the value of any difference is independent of the order of the arguments.

(ii) The diveded differences of the sum or difference of two functions is equal to the sum or difference of the corresponding separate divided differences.

6. **Give the Newton's divided difference interpolation formula.** **[A.U.2010,2012]**

Solution:

$$f(x)= f(x_0)+(x-x_0)f(x_0,x_1)+(x-x_0)(x-x_1)f(x_0,x_1,x_2)+(x-x_0)(x-x_1)(x-x_2)f(x_0,x_1,x_2,x_3)+\ \ldots..$$

7. **What is a cubic spline ?** **[A.U 2010,2012]**

Solution:

A cubic polynomial which has continuous slope and curvature is called a cubic spline.

8. **Derive Newton's forward difference formula by using operator method.**

Solution:

$$\begin{aligned} P_n(x) &= P_n(x_0+uh) \\ &= E^u P_n(x_0) \\ &= E^u y_0 \\ &= (1+\Delta)^u y_0 \\ &= [1+\frac{u}{1!}\Delta+\frac{u(u-1)}{2!}\Delta^2+\frac{u(u-1)(u-2)}{3!}\Delta^2+\cdots\frac{u(u-1)(u-2).....u-(n-1)}{n!}\Delta^n]y_0 \\ &= [y_0+\frac{u}{1!}\Delta y_0+\frac{u(u-1)}{2!}\Delta^2 y_0+\frac{u(u-1)(u-2)}{3!}\Delta^{2y_0}+\cdots\frac{u(u-1)(u-2)...u-(n-1)}{n!}\Delta^n y_0] \end{aligned}$$

9. **Derive Newton's backward difference formula by using operator method.**

Solution:

$$\begin{aligned} P_n(x) &= P_n(x_n+vh) \\ &= E^v P_n(x_n) \\ &= E^v y_n \\ &= (1+\nabla)^v y_n \\ &= [1+\frac{u}{1!}\nabla+\frac{v(v+1)}{2!}\nabla^2+\frac{v(v+1)(v+2)}{3!}\nabla^2+\cdots\frac{v(v+1)(v+2).....v+(n-1)}{n!}\nabla^n]y_n \\ &= [y_n+\frac{v}{1!}\nabla y_n+\frac{v(v+1)}{2!}\nabla^2 y_n+\frac{v(v+1)(v+2)}{3!}\nabla^2 y_n+\cdots\frac{v(v+1)(v+2)....v+(n-1)}{n!}\nabla^n y_{y_n}] \end{aligned}$$

UNIT-III

NUMERICAL DIFFERENTIATION AND NUMERICAL INTEGRATION

PART-A

1. **Write down the expressions for** $\frac{dy}{dx}$ *and* $\frac{d^2y}{dx^2}$ *at* $x = x_0$ **by Newton's forward difference formula. (or)[M/J 2008]**

   State the formula to find the first and second order derivative using the forward differences.

   Solution:

$$\left.\frac{dy}{dx}\right|_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + ...\right]$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=x_0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 - \frac{11}{12}\Delta^4 y_0 + ...\right].$$

2. **Write down the expressions for** $\frac{dy}{dx}$ *and* $\frac{d^2y}{dx^2}$ *at* $x = x_n$ **by Newton's backward difference formula. [A/M 2004, N/D2005]**

   Solution:

$$\left.\frac{dy}{dx}\right|_{x=x_n} = \frac{1}{h}\left[\nabla y_0 + \frac{1}{2}\nabla^2 y_0 + \frac{1}{3}\nabla^3 y_0 + \frac{1}{4}\nabla^4 y_0 + ...\right]$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=x_n} = \frac{1}{h^2}\left[\nabla^2 y_0 + \nabla^3 y_0 + \frac{11}{12}\nabla^4 y_0 + ...\right]$$

3. **Create a forward difference table for the following data and state the degree of polynomial for the same.**

x	0	1	2	3
y	-1	0	3	8

Solution: The forward difference table is as follows

x:	y	Δy	Δ^2y	Δ^3y
0	-1			
		1		
1	0		2	
		3		0
2	3		2	
		5		
3	8			

Since Δ^2y is having constant terms, it will have a polynomial of degree 2.

4. **Find $\frac{dy}{dx}$ at x=1 from the following table.**

x:	1	2	3	4
y:	1	8	27	64

Solution:

The forward difference table is as follows

x:	y	Δy	Δ^2y	Δ^3y
1	1			
		7		
2	8		12	
		19		6
3	27		18	
		37		
4	64			

$$\left.\frac{dy}{dx}\right|_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \dots\dots\dots\right]$$

Here h=1, x_0=1, Δy_0=7, $\Delta^2 y_0$=12 and $\Delta^3 y_0$=6.

Therefore $\left.\frac{dy}{dx}\right|_{x=1} = \frac{1}{1}\left[7 - \frac{12}{2} + \frac{6}{3}\right] = 3.$

5. **Evaluate $\int_{1/2}^{1} \frac{1}{x} dx$ by trapezoidal rule dividing the range into 4 equal parts.[April 1996, M/J 2006]**

Solution: Here $h = \frac{1-\frac{1}{2}}{4} = 0.125$: $y = \frac{1}{x}$

x:	½=0.5	0.625	0.75	0.875	1
y:	2	1.6	1.3333	1.1429	1

By Trapezoidal rule,

$\int_a^b f(x)dx = \frac{h}{2}${(sum of the first and last ordinates)+2(sum of remaining ordinates}

$\int_{1/2}^{1} 1/x dx = \frac{0.125}{2}${(2+1)+2(1.6+1.3333+1.1429)}.

= 0.6970.

6. **State the Trapezoidal rule to evaluate $\int_{x_0}^{x_n} f(x)dx$.**

Solution:

$$\int_{x_0}^{x_n} f(x)dx = \frac{h}{2}\left[(y_0 + y_n) + 2(y_1 + y_2 + \cdots y_{n-1})\right]$$

(i.e) $\int_{x_0}^{x_n} f(x)dx = \frac{h}{2}${(sum of the first and last ordinates)+2(sum of remaining ordinates)}

7. **Write down the Simpson,s 1/3-Rule in numerical integration. [N/D 2007, N/D 2009].**
Solution: Simpson's one-third rule is given by

$$\int_{x_0}^{x_0+nh} f(x)dx = \frac{h}{3}[(y_0 + y_n) + 2(y_2 + y_4 + \cdots y_{n-2}) + 4(y_1 + y_3 + \cdots y_{n-1})]$$

(i.e) $\int_{x_0}^{x_0+nh} f(x)dx = \frac{h}{3}$\{(sum of the first and last ordinates)+2(sum of remaining odd ordinates+4(sum of even ordinates)\}

8. **Comp[are Simpson's 1/3rd rule with Trapezoidal rule.**
Solution:

S. No.	Trapezoidal rule	Simpson's 1/3rd rule
1.	Any number of intervals	Number of intervals must be even
2.	Least accuracy	More accuracy
3.	Here y is a linear function of x	Here y is a polynomial of degree two.

9. **State Gaussian two point quadrature formula.**
Solution:
Two point Gaussian quadrature formula is $\int_{-1}^{1} f(x)dx = f\left[-\frac{1}{\sqrt{3}}\right] + f\left[\frac{1}{\sqrt{3}}\right]$.

This formula is exact for polynomial up to degree 3.

10. **State three point Gaussian quadrature formulae.**
Solution:

Three point Gaussian quadrature formula is $\int_{-1}^{1} f(x)dx = \frac{5}{9}\left[f\left[-\sqrt{\frac{3}{5}}\right] + f\left[\sqrt{\frac{3}{5}}\right]\right] f + \frac{8}{9} f[0]$.

This formula is exact for polynomial up to degree 5.

11. **Use two-point Gaussian quadrature formula to solve** $\int_{-1}^{1} \frac{dx}{1+x^2}$

Solution:
Two point Gaussian quadrature formula is $\int_{-1}^{1} f(x)dx = f\left[-\frac{1}{\sqrt{3}}\right] + f\left[\frac{1}{\sqrt{3}}\right]$.

Here f(x) = $\frac{1}{1+x^2}$.

$$f\left[\frac{-1}{\sqrt{3}}\right] = \frac{1}{1+\frac{1}{3}} = \frac{3}{4} \quad \& \quad f\left[\frac{1}{\sqrt{3}}\right] = \frac{1}{1+\frac{1}{3}} = \frac{3}{4}$$

$$\therefore \int_{-1}^{1} \frac{dx}{1+x^2} = \frac{3}{4} + \frac{3}{4} = \frac{3}{2} = 1.5$$

UNIT IV-PART A

INITIAL VALUE PROBLEMS FOR ORDINARY DIFFERENTIAL EQUATIONS

1. **State Taylor series algorithm for the first order differential equations.**

   Solution: To find the numerical solution of $\frac{dy}{dx} = f(x, y$ with the condition $y(x_0)=y_0$. We expand 'y(x)' at a general point 'x_m' in a Taylor series, getting.,

$$y_{m+1} = y_m + \frac{h}{1!}y'_m + \frac{h^2}{2!}y''_m + \frac{h^3}{3!}y'''_m + \cdots \frac{h^r}{r!}y^r_m$$

   Here y^r_m denotes the rth derivatives of 'y' with respect to 'x' at the point (x_m, y_m).

2. **Using Taylor's series find y(0.1) for $\frac{dy}{dx} = 1 - y, y(0) = 0$.**

   Solution: Here $x_0=0$, $y_0=0$, h=0.1

   y ' = 1-y $\quad$ $y_0' = 1 - y_0 = 1 - 0 = 1$

   y'' = -y' $\quad$ $y_0'' = -y_0' = -1$

   y'''= - y'' $\quad$ $y_0''' = -y_0'' = 1$

   $y(x) = y(0) + hy'(0) + \frac{h^2}{2!}y''(0) + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

   $y(0.1) = 0 + (0.1)(1) + \frac{(0.1)^2}{2!}(-1) + \frac{(0.1)^3}{3!}(1) + \ldots\ldots\ldots\ldots$

   $= 0.1 - 0.005 + 0.000167$

   $= 0.0952.$

3. **State Euler algorithm to solve y'=f(x,y), y(x$_0$)=y$_0$.**

   Solution: $y_{n+1} = y_n + hf[x_n, y_n]$ when n=0,1,2……

   This is Euler algorithm. It can also be written as

   y(x+h) = y(x) + hf(x,y).

4. **Using Euler's method, solve the following differential equation y'= -y subject to y(0)=1. Hence find y(0.01).**

   Solution: Given; f(x,y)= -y, $x_0=0$, $y_0=1$.

   By Euler algorithm,

   $y_1 = y_0 + hf(x_0, y_0)$

   $= 1 + h(-y_0) = 1 + (x - x_0)(-y_0)$

   $= 1 + (x-0)(-1) = 1 - x$

   $y(0.01) = 1 - 0.01 = 0.99$

5. **State modified Euler algorithm to solve y'=f(x,y), y(x$_0$)=y$_0$.**

   Solution; $y_{n+1} = y_n + hf[x_n + \frac{h}{2},\ y_n + \frac{h}{2}f(x_n, y_n)]$.

6. **Using Modified Euler's method, find y(0.1) if $\frac{dy}{dx} = x^2 + y^2, y(0) = 1$.**

   Solution;

   Given; f(x,y)= x^2+y^2, $x_0=0$, $y_0=1$, h=0.1, $x_1=0.1$.

   By modified Euler method

$$y_{n+1} = y_n + hf[x_n + \frac{h}{2},\ y_n + \frac{h}{2}f(x_n, y_n)].$$

$$y_1 = y_0 + hf[x_0 + \frac{h}{2},\ y_0 + \frac{h}{2}f(x_0, y_0)].$$

$f(x_0, y_0) = x_0^2 + y_0^2$ = 0+1= 1

$= 1 + (0.1)f[0 + \frac{0.1}{2},\ \ 1 + \frac{0.1}{2}(1)]$

$= 1 + (0.1)f[0.05,\ \ 1.05]$

$= 1 + (0.1)[0.05^2 + 1.05^2\ ]$

$= 1 + (0.1)[0.0025 + 1.1025]$

$= \ 1 + (0.1)[1.1105] = 1.1105$

Therefore y(0.1) =1.1105.

7. Write down the R-K formula of fourth order to solve $\frac{dy}{dx} = f(x, y)$, with $y(x_0)=y_0$.

Solution:

Let h denote the interval between equidistant values of x. If the initial values are (x_0, y_0), the first increment in y is computed from the formulas.

$K_1 = h\, f(x_0, y_0)$

$K_2 = h\, f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$

$K_3 = h\, f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$

$K_4 = h\, f(x_0 + h, y_0 + k_3)$ and

$\Delta\, y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

Then $x_1 = x_0 + h$, $y_1 = y_0 + \Delta\, y$

The increment in y in the second interval is computed in a similar manner using the same four decimals, using the values x_1, y_1 in the place of x_0, y_0 respectively.

8. How many prior values are required to predict the next value in Milne's method.

Solution: Four values.

9. Write down Milne's predictor-corrector formula for solving initial value problem in first order differential equation.

Solution:

Milne's predictor formula is $y_{n+1} = y_{n-3} + \frac{4h}{3}(2y'_{n-2} - y'_{n-1} + 2y'_n) + \frac{14h^5}{45}y^5(\varepsilon_1)$

where ε_1 lies between x_{n-3} and x_{n+1}

Milne's corrector formula is $y_{n+1} = y_{n-1} + \frac{h}{3}(y'_{n-1} + 4y'_n + y'_{n+1}) - \frac{h^5}{90}y^5(\varepsilon_2)$

where ε_2 lies between x_{n-1} and x_{n+1}

10. Write down Adams-Bashforth predictor method.

Solution: Adam's predictor corrector formulas are

$y_{k+1}, P = \ y_k + \frac{h}{24}\left(55y_k{}' - 59y'_{k-1} + 37y'_{k-2} - 9y'_{k-3}\right)$

$$y_{k+1}, C = \ y_k + \frac{h}{24}\left(9y'_{k+1} + 19y'_k - 5y'_{k-1} + y'_{k-2}\right)$$

UNIT V-PART A
BOUNDARY VALUE PROBLEMS IN ORDINARY AND DIFFERENTIAL EQUATIONS

1. **Write the central difference approximation for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.**

   Solution:

   $$\frac{dy}{dx} = y' = \frac{1}{h}(y_{i+1} - y_i)$$
   $$\frac{d^2y}{dx^2} = y'' = \frac{1}{h^2}(y_{i-1} - 2y_i + y_{i+1}).$$

2. **State the condition for the equation.**
   $Au_{xx}+Bu_{yy}+Cu_{yy}+Du_x+Eu_y+Fu = G$ where A, B, C, D, E, F, G are function of x and y to be (i) elliptic (ii) parabolic (iii) hyperbolic

   Solution:

   The given equation is said to be

   (i) Elliptic at a point (x, y) in the plane if $B^2 - 4AC < 0$
   (ii) Parabolic if $B^2 - 4AC = 0$
   (iii) Hyperbolic if $B^2 - 4AC > 0$

3. **Classify the PDE $u_{xx}=u_t$. (OR) What is the classification of one dimensional heat flow equation.**

   Solution:

   Here A=1, B=0 and C=0.
   Therefore $B^2 - 4AC = 0-4(1)(0)=0$. Hence $u_{xx}=u_t$ is Parabolic equation.

4. **State Bender-Schmidt's explicit scheme in general for and using suitable value of λ, write the scheme in simplified form.**

   Solution:

   General form: $u_{i,j+1} = \lambda\, u_{i+1,j} + (1-2\lambda)u_{i,j} + \lambda\, u_{i-1,j}$ where $\lambda = \frac{k}{ah^2}$.

   Simple form: If $\lambda = \frac{1}{2}$, then $k=\frac{ah^2}{2}$ then the above equation becomes

   $u_{i,j+1} = \frac{1}{2}[u_{i+1,j} + u_{i-1,j}]$

5. **Write down the Crank-Nicolson formula to solve $u_t = u_{xx}$. (OR)**

   Write down the implicit formula to solve one dimensional heat flow equation. $U_{xx} = \frac{1}{c^2}u_t$

   Solution:

   General form:

   $$\lambda\,(u_{i+1,j+1} + u_{i-1,j+1}) - 2(\lambda + 1)u_{i,j+1} = 2(\lambda - 1)u_{i,j} - \lambda(u_{i+1,j} + u_{i-1,j}) \text{ where } \lambda = \frac{k}{ah^2}.$$

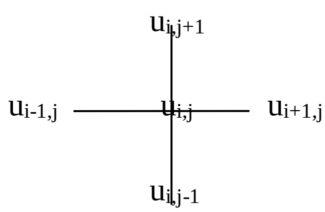
   Simple form: If $\lambda = 1$ then $k=ah^2$, then the above equation becomes

   $$u_{i,j+1} = \frac{1}{4}(u_{i+1,j+1} + u_{i-1,j+1} + u_{i+1,j} + u_{i-1,j}).$$

6. **What type of equations can be solved by using Crank-Nickolson's difference formula?**
   Solution: Crank-Nickolson's difference formula is used solve parabolic equations of the form. $U_{xx} = au_t$
7. **For what purpose Bender-Schmidt recurrence relation is used?**
   Solution:
   To solve one dimensional heat equation.
8. **Write down the general and simplest forms of the difference equation corresponding to the hyperbolic equation $u_{tt} = a^2 u_{xx}$.**
   Solution:
   General form: $u_{i,j+1} = 2(1-\lambda^2a^2)u_{i,j}+\lambda^2a^2(u_{i+1,j}+u_{i-1,j})-u_{i,j-1}$, where $\lambda=\frac{k}{h}$.
   Simple form: If $\lambda^2 = \frac{1}{a^2}$ coefficient of $u_{i,j}$ is = 0 and $k=\frac{h}{a}$, then the above equation becomes

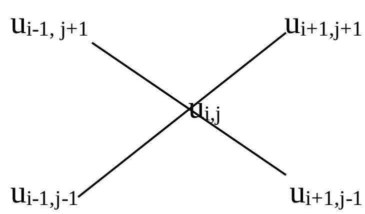
$$u_{i,j+1} = u_{i+1,j} + u_{i-1,j} - u_{i,j-1}$$

9. **Write down the standard five point formula to solve Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$**
   Solution:



$$u_{i,j} = \frac{1}{4}\left[u_{i-1,j} + u_{i+1,j} + u_{i,j-1} + u_{i,j+1}\right]$$

10. **Write the diagonal five-point formula to solve the Laplace equation $u_{xx}+ u_{yy} = 0$.**
   Solution:



$$u_{i,j} = \frac{1}{4}\left[u_{i-1,j+1} + u_{i-1,j-1} + u_{i+1,j-1} + u_{i+1,j+1}\right]$$

11. **Write Liebmann's iteration process.**
   Solution: $u_{i,j}{}^{(n+1)} = \frac{1}{4}\left[u_{i-1,j}{}^{(n+1)} + u_{i+1,j}{}^{(n)} + u_{i,j-1}{}^{(n)} + u_{i,j+1}{}^{(n+1)}\right]$.

12. **Write the difference scheme for solving the Poisson equation $\nabla^2 u=f(x,y)$.**
   Solution: $u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1} - 4u_{i,j} = h^2 f(ih, jh)$

PART-B

UNIT-I(Solution of equations and Eigen value Problems)

Newton-Raphson Method:

1. Find real positive root of $3x - \cos x - 1 = 0$ by N.R method correct to 6 decimal places.
2. Find a root of $x \log_{10} x - 1.2 = 0$ by N.R method correct to three decimal places.
3. Using N.R method, solve $x \log_{10} x = 12.34$ taking the initial value x_0 as 10.
4. Find the real root of x^3-2x-5=0 using N.R. method.

Gauss Gauss Jordan method:

5. Solve the system of equations by Gauss Jordan method.
   2x+3y-z=5; 4x+4y-3z=3; 2x-3y+2z=2.
6. Using Gauss-Jordan, solve the following system
   10x+y+z=12; 2x+10y+z=13; x+y+5z=7.

Gauss Jacobi and Gauss seidal method:

7. Solve the following system of equations by Gauss-Jacobi method & Gauss-Seidel method. $27x + 6y - z = 85,\ x + y + 54z = 110,\ 6x + 15y + 2z = 72$
8. Solve the following system of equations by Gauss-Seidel method.
   $4x + 2y + z = 14,\quad x + 5y - z = 10,\quad x + y + 8z = 20$

Inverse of a matrix Gauss-Jordan method:

9. Using Gauss-Jordan method, find the inverse of $A = \begin{pmatrix} 2 & 2 & 3 \\ 2 & 1 & 1 \\ 1 & 3 & 5 \end{pmatrix}$.
10. Using Gauss-Jordan method, find the inverse of $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$.

Eigen value of a matrix by power method:

11. Find the numerically larges eigen value of $A = \begin{pmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{pmatrix}$ by power method
12. Find the numerically largest Eigen value of $A = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$ by power method.
13. Find the dominant Eigen value and the corresponding Eigen vector of $A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$.

    Find also the least latent root and hence the third Eigen value also.

UNIT-II(Interpolation and approximation)-Part B

Lagrangian method:

1. Using Lagrange's interpolation formula, find y(10) from the following table

x:	5	6	9	11
y:	12	13	14	16

2. Find the polynomial f(x) by using Lagrange's interpolation formula and hence find the value of f(3), from the following table

x:	0	1	2	5
f(x):	2	3	12	147

Inverse Lagrangian method:

3. Find the corresponding to the annuity value 13.6 given the table:

Age x:	30	35	40	45	50
Annuity value y:	15.9	14.9	14.1	13.3	12.5

Newton's divided difference method:

4. Using Newton's divided difference formula, find the value of f(2), f(8) and f(15) given the following table

x:	4	5	7	10	11	13
f(x):	48	100	294	900	1210	2028

5. Determine f(x) as a polynomial in x for the following data, using Newton's divided difference formulae. Also find f(2).

x:	-4	-1	0	2	5
f(x):	1245	33	5	9	1335

Interpolating with a cubic spline:

6. Using cubic spline find y(1.5) & $y^1(1)$

x	1	2	3
y	-8	-1	18

7. Using cubic spline find y(0.5) & $y^1(1)$

x	0	1	2
y	-5	-4	3

Newton's forward and backward difference method:

8. Using Newton's forward interpolation formula, find the polynomial f(x) satisfying the following data. Hence evaluate at f(x) at x = 5.

x:	4	6	8	10
f(x)	1	3	8	10

9. From the following data, find θ at x=43 and x=84

x:	40	50	60	70	80	90
y:	184	204	226	250	276	304

10. From the given data, find the number of students whose wight is between 60 to to 70.

Weitht in Ibs	0-40	40-60	60-80	80-100	100-120
No. of students	250	120	100	70	50

UNIT-III (Numerical differentiation and integration)

Numerical differentiation:

1. Use the Newton divided difference formula to calculate f'(3) and f''(3) from the following table

x:	3.0	3.2	3.4	3.6	3.8	4.0
f(x)	-14	-10.032	-5.296	-0.256	6.672	14

2. Compute f '(0) and f''(4) from the following data:

x:	0	1	2	3	4
f(x)	1	2.718	7.381	20.086	54.598

3. Given the following data, find y'(6) and the maximum value of y.

x:	0	2	3	4	7	9
f(x)	4	26	58	112	466	922

4. Find the first three derivatives of f(x) at x=1.5 by Newtonn's forward interpolation from.

x:		1.5	2.0	2.5	3.0	3.5	4.0
f(x)		3.375	7.000	13.625	24.000	38.875	59.000

Numerical integration:

5. Using Trapezoidal rule, evaluate $\int_{-1}^{1} \frac{dx}{1+x^2}$ by talking eight equal intervals
6. Using Trapezoidal rule, evaluate $\int_{0}^{6} \frac{dx}{1+x^2}$ by talking six equal intervals
7. By dividing the range into ten equal parts, evaluate $\int_{0}^{\pi} \sin x\,dx$ by trapezoidal & Simpson's rule. Verify your answer with actual integration.
8. By dividing the range into ten equal parts, evaluate $\int_{0}^{\pi/2} \sin x\,dx$ by trapezoidal & Simpson's rule. Verify your answer with actual integration.

Rombergs Method

9. Evaluate $\int_{0}^{1} \frac{dx}{1+x}$ and correct to 3 decimal places using Romberg's method and hence find the value of $log_e 2$.
10. Evaluate $\int_{0}^{1} \frac{dx}{1+x^2}$ by using Romberg's method correct to 4 decimal places. Hence deduce an approximate value for π.

Gaussian three-point Quadrature formulae:

11. Use Gaussian three-Point formulae and evaluate $\int_{1}^{5} \frac{dx}{x}$.
12. Evaluate $\int_{0}^{2} \frac{x^2+2x+1}{1+(x+1)^1} dx$ by Gaussian three point formula.

Double Integrals:

13. Evaluate $\int_{0}^{1}\int_{0}^{1} \frac{1}{1+x+y} dxdy$ by trapezoidal rule.
14. Evaluate $\int_{1}^{2}\int_{1}^{2} \frac{1}{x+y} dxdy$ using T.R and Simpsons's rule. By taking h=0.25and k=0.25.
15. Evaluate $\int_{1}^{1.4}\int_{2}^{2.4} \frac{1}{xy} dxdy$ using T.R and Simpsons's rule. By taking h=k=0.1. Verify your answer with actual integration.

PART-B

UNIT-IV (Initial Value Problems for Ordinary Differential Equations)

Taylor series method:

1. By Taylor series method, find y(0.1), y(0.2) if $\frac{dy}{dx} = 3e^x + 2y,\ y(0) = 0$.
2. By Taylor series method, find y(0.1), y(0.2) if $\frac{dy}{dx} = x - y^2,\ y(0) = 1$.
3. Using Taylor series method, find y at x=1.1 by $\frac{dy}{dx} = x^2 + y^2,\ y(1) = 2$.
4. Using Taylor series, find y at x=0.1 if $\frac{dy}{dx} = x^2 y - 1,\ y(0) = 1$.

Euler method:

5. Solve $y^1 = \frac{y-x}{y+x}$, y(0)=1 at x=0.1 by taking h=0.02 by using Eulers method.

Modified Euler method:

6. Apply modified Euler's method to find y(0.2) and y(0.4) given $y^1=x^2+y^2$, y(0)=1 by taking h=0.2.

Runge-Kutta method:

7. Given $\frac{dy}{dx} = x^3 + y, y(0) = 2$ Compute y(0.2) , y(0.4) & y(0.6) by R.K method
8. By R.K method, solve $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}, y(0) = 1$for x=0.2 and x=0.4 with h=0.2.
9. Using R.K method of fourth order find y(0.1) and y(0.2) for the initial value problem $\frac{dy}{dx} = x + y^2, y(0) = 1$

Milne's method

10. Using Milne's method find y(4.4) given $5xy' + y^2 - 2 = 0$ y (4) = 1,y (4.1)= 1.0049, y(4.2) = 1.0097,y(4.3) = 1.0143
11. Using Rungekutta method of 4^{th} order , find the value of y at x=0.2, 0.4, 0.6 given $\frac{dy}{dx} = x^3 + y,\ y(0) = 2$. Also find the value of y at x=0.8 using Milne's predictor and corrector method.

Adams method:

12. Given that $y^1=y-x^2$; y(0)=1; y(0.2)=1.1218; y(0.4)=1.4682 and y(0.6)=1.7379, evaluate y(0.8) by Adam's method.
13. Find y(0.1), y(0.2), y(0.3) from $\frac{dy}{dx} = xy + y^2,\ y(0) = 1$by using R.K method and hence obtain y(0.4) using Adam's method.

UNIT-V (Boundary Value Problems in ODE and PDE)

One-D heat equation by explicit method (Bender Schmidt):

1. Solve $u_{xx}=32u_t, t\geq 0$ $0<x<1$, $u(0,t)=0$, $u(x,0)=0$, $u(1,t)=t$. $h=0.25$
2. Solve by Bender –Schmidt formula up to t=5 for the equation $u_{xx}=u_t$, subject to $u(0,t) = 0$, $u(5,t)=0$ and $u(x,0)=x^2(25-x^2)$, taking h=1.
3. Using Bender-Schmidt's method solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$, given $U(0,t)=0$, $u(1,t)=0$, $u(x,0) = \sin\pi x$, $0<x<1$ and $h=0.2$, Find the u upto t=0.1.
4. Solve $\frac{\partial u}{\partial t} = \frac{1}{2}\frac{\partial^2 u}{\partial x^2}$ with the condition $u(0,t)=0=U(4,t)$, $U(x,0)=x(4-x)$ Taking h=1 employing Bender-Schmidt recurrence equation. Continue through 10 time steps.

One-D heat by implicit method(Crank Nicholson):

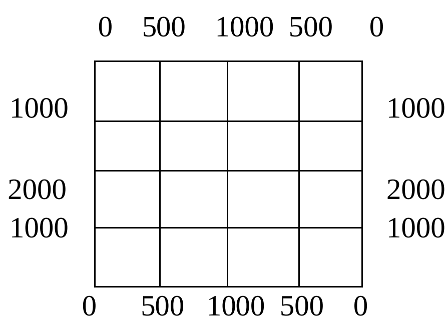
5. Using Crank-Nicholsons scheme, solve $u_{xx}=16u_t$, $0<x<1$, $t>0$ given $u(x,0)=0$, $u(0,t)=0$, $u(1,t)=100t$ compute u for one step in t- direction taking $h=\frac{1}{4}$.
6. Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ in $0<x<5$, $t\geq 0$ given that $u(x,0)=20$, $u(0,t)=0$, $u(5,t)=100$. Compute u for the time-step with h=1 by Crank-Nicholson method.

One-D wave equation:

7. Solve $\frac{1}{4}\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ with condition $u(0,t)=0=u(4,t)$, $u(x,0)=x(4-x)$ $u_t(x,0)=0$, taking h=1 up to t=4.
8. Solve using finite differences 16uxx=utt with the step length h=1 and up to t=1 given that $u(0,t)=u(5,t)=0$, $u(x,0)=x^2(5-x)$, in $u_t(x,0)=0$

Two-D Laplace equation

9. Solve $u_{xx}+u_{yy}=0$; $0\leq x, y\leq 1$ with $u(0,y)=10=u91,y)$ and $u(x,0)=20= u(x,1)$. Take h=0.25 and apply Liebmann method to 3 decimal accuracy.
10. By Iteration method, solve the Laplace equation $u_{xx}+u_{yy}=0$, over the square region, satisfying the boundary conditions.
    $u(0,y)=0, \quad 0\leq y\leq 3$
    $u(3,y)=9+y, \quad 0\leq y\leq 3$
    $u(x,0)=3x, \quad 0\leq x\leq 3$
    $u(x,3)=4x, \quad 0\leq x\leq 3$
11. Solve the elliptic equation $u_{xx}+u_{yy}=0$ for the following square mesh with boundary values as shown:



12. By iteration method, solve the elliptic equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ over a square region of side 4, satisfying the boundary conditions (i) u(0,y)=0, $0 \le y \le 4$ (ii) u(4,y) = 12+y, $0 \le y \le 4$, (iii) u(x,0) = 3x, $0 \le x \le 4$, (iv) u(x, 4) = x^2, $0 \le x \le 4$. By dividing the square into 16 square meshes of side 1.

Two-D Poisson equation:

13. Solve the Poisson equation $\nabla^2 = -10(x^2+y^2+10)$ over the square mesh with sides x=0, y=0, x=3 and y=3 with u=0 on the boundary and mesh length 1 unit.