SNS COLLEGE OF ENGINEERING

Kurumbapalayam(Po), Coimbatore – 641 107
Accredited by NAAC-UGC with 'A' Grade
Approved by AICTE & Affiliatd to Anna University,Chennai

Unit-I

PART A

1. State Baye's theorem.
2. Define discrete and Continuous random variable.
3. Write down the axioms of Probability.
4. A CRV X that can assume any value between x=2 and x=5 has a density function given by f(x) = k (1+x). Find k.
5. X and Y are independent random variables with variance 2 and 3. Find the variance of3X+4Y.
6. The mean of a Binomial distribution is 20 and S.D is 4. Determine the parameters of thedistribution.
7. Define Poisson distribution and write its mean and variance
8. State Memoryless property of Exponential Distribution
9. Find the value of 'K' for a continuous random variable X whose probability density function is given by $f(x) = Kx^2 e^{-x}; x \geq 0$.
10. Write the mean and variance of Binomial distribution

PART B

1. A random variable x has the following probability distribution

x	0	1	2	3	4	5	6	7
P(x)	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2$+K

   (i) Find the value of K
   (ii) Evaluate P[X<6] and P[X≥ **6**)
   (iii) If P[X≥ ***C***)>1/2 find minimum value of C
   (iv) Evaluate P[1.5<x<4.5/x>2]

2. A random variable X has the following probability distribution.

x	-2	1	0	1	2	3
P(x)	0.1	K	0.2	2K	0.3	3K

   a. Find K
   b. Evaluate P(x < 2) and P(-2 < x □ 2)
   c. Find the Cumulative distribution of x.
   d. Evaluate the mean of x.

3. The probability mass function of a discrete R. V X is given in the followingtable

X	0	1	2	3	4	5	6	7	8
P(X)	a	3a	5a	7a	9a	11a	13a	15a	17a

Find (i) the value of a , (ii) $P(X < 3)$, (iii) Mean of X, (iv) Variance ofX.

4. If the density function of a continuous random variable X is given by

$$f(x) = \begin{cases} ax, 0 \le x \le 1 \\ a, 1 \le x \le 2 \\ 3a - x, 2 \le x \le 3 \\ 0, elsewhere \end{cases}$$

(i) Find the value of ‘a”
(ii) Find the CDF of x
(iii) Compute P(x≤1.5) and p(x>1.5)

5. Find the MGF of Binomial distribution. Hence find its Mean and variance.
6. Find the MGF of Poisson distribution and hence find its mean and variance
7. Find the MGF of Exponential distribution and hence find its mean and Variance. Also provethe memory less property of the exponential distribution.
8. Find the MGF of Normal distribution & hence find its mean and variance
9. A bolt is manufactured by 3 machines A, B, and C. A turns out twice as many items as B andmachines B and C produce equal number of items. 2% of bolts produced by A and B are defective and 4% of bolts produced by C are defective. All bolts are put into 1 stock pile and1 is chosen from this pile. What is the probability that it is defective?
10. An urn contains 10 white and 3 black balls. Another urn contains 3 white and 5 black balls. Two balls are drawn at random from the first urn and placed in the second urn and then 1 ballis taken at random from the latter. What is the probability that it is a white ball?
11. Out of 800 families with 4 children each how many families would be expected to have
    i. 2 Boys and 2 Girls
    ii. At least 1 boy
    iii. At most 2 girls
    iv. Children of both gender,

Assume equal probabilities for boys and girls.

12. The number of monthly breakdowns of a computer is a random variable, having a Poisson distribution with mean equal to 1.8. find the probability that this computer will function for amonth.
    i. Without a breakdown
    ii. With only one breakdown
    iii. With at least one breakdown
13. The time (in hours) required to repair a machine is exponentially distributed with parameter λ=1/2 (i)What is the probability that the repairs time exceeds 2 hour?
    (ii) What is the conditional probability that the repair takes 10 hour given that its duration exceeds 9 hour?

Unit-II

PART A

1. The joint probability mass function of a two dimensional random variable (X,Y)is given by p(x,y)= k(2x+y), x =1,2 y=1,2, where K is constant. Find the value of k
2. Let X and Y have the joint p.m.f

Y/X	0	1	2
0	0.1	0.4	0.1
1	0.2	0.2	0

Find P(X+Y>1).

3. The joint pdf of a random variable (X,Y) is $f(x,y) = ke^{-(2x+3y)}; x > 0, y > 0.$ Find the value of k.
4. The joint pdf of random variable (X,Y) is given as $f(x,y) = \frac{1}{x}, 0 < x < y < 1$ Find the marginal pdf of Y.
5. The two regression equations of two random variables x & y are $4x - 5y + 33 = 0$ & $20x - 9y = 107$. Find the mean values of x and y.
6. The regression equations are 3x+2y = 26 and 6x +y = 31. Find the mean values of x &y
7. What is the angle between two regression lines?
8. Write the properties of regression lines.
9. If Y=-2X+ 3, find Cov(X, Y).

PART B

1. The joint probability mass function of (X Y), is given by p(x, y)=k(2x+3y) x = 0, 1, 2; y=1, 2, 3. Find k and all the marginal and conditional probability distributions. Also find the probability distribution of X+Y
2. The joint probability mass function of (X Y), is given by p(x, y)= $\frac{1}{72}$ (2x+3y) x = 0, 1, 2; y=1, 2, 3. Find k and all the marginal and conditional probability distributions.
3. The joint pdf of the random variable (X, Y) is given by $f(x,y) = Kxye^{-(x^2+y^2)}, x. > 0, y > 0.$ Find the value of K and also prove that X and Y are independent.
4. Given the joint pdf of X and Y $f(x,y) = \begin{cases} cx(x-y), 0 < x < 2, -x < y < x \\ 0 otherwise \end{cases}$,
   i. Evaluate c
   ii. Find Marginal pdf of X and Y.
   iii. Find the conditional density of Y/X
   iv. Two random variables X and Y have the joint p.d.f given by
5. $f(x,y) = \begin{cases} k(1 - x^2 y), 0 \le x, y \le 1 \\ 0 \, , \, Otherwise \end{cases}$.

   i) Find K

   ii) Obtain Marginal p.d.f of X and Y

   iii) Find the Correlation Coefficient between X and Y

6. The joint pdf of random variable if $f(x,y) = x + y, 0 \le x \le 1, 0 \le y \le 1$. Find the correlation coefficient between X & Y.
7. The joint probability density function of the two dimensional random variable (X,Y) is

$$f(x,y) = \begin{cases} 2 - x - y, 0 \le x \le 1,\ 0 \le y \le 1 \\ 0,\ otherwise \end{cases}$$. Find the correlation coefficient between X&Y.

8. Find the coefficient of correlation between X and Y from the data given below.

x	65	66	67	67	68	69	70	72
y	67	68	65	68	72	72	69	71

9. Find the coefficient of correlation between industrial production and export using the following data:

Production (X)	55	56	58	59	60	60	62
Export (Y)	35	38	37	39	44	43	44

10. The equations of two regression lines are 8x-10y+66 = 0 and 40x-18y-214 =0. Variance of x is 9. Find the mean values of x and y and correlation coefficient between x and y.
11. If X and Y are independent random variables with probability density function $f(x) = e^{-x}, x \ge 0$: $f(y) = e^{-y}, y \ge 0$ respectively. Show that the random variables $U = \frac{X}{X+Y}$ and V=X+Y are independent.
12. Two random variables X & Y have the following joint p.d.f $f(x,y) = \begin{cases} x + y\ , 0 \le x, y \le 1 \\ 0\ ,\ otherwise \end{cases}$. Find the probability density function of the random variable $U = XY$

Unit-III
PART A

1. Define random process.
2. Define a strictly stationary random process (SSS).
3. Define a wide sense stationary random process (WSS).
4. Define Markov process.
5. Define Markov chain and give an example.
6. Sate Chapman kolmokrov theorem.
7. Prove that a first order stationary process has a constant mean.
8. State the postulates of a Poisson process.
9. Write down any two applications of a Poisson process.
10. Prove that sum of two independent Poisson processes is again a Poisson process.
11. Consider the random process X(t) = cos(t+φ), where φ is a random variable with density function f(φ) = 1/π, -π/2<φ<π/2. Check whether the process is wide sense stationary or not.

12. If customers arrive at a bank according to a Poisson process with mean rate 2per minute. Find the probability that during a 1-minute arrival no customer arrives.

13. Let $A = \begin{pmatrix} 0 & 1 \\ 1/2 & 1/2 \end{pmatrix}$ be a stochastic matrix. Check whether it is regular.

PART B

14. Examine whether the random process X(t) = Acos(ωt+θ) is a wide sense stationary if A and ω are constants and θ is uniformly distributed random variable in (0, 2π).

15. Examine whether the random process X(t) = Asin(ωt+Y) where Y is uniformly distributed random variable in (0, 2π).Show that X(t) is wide sense stationary.

16. A random process X(t) defined by X(t) = A cost +B sint, -∞<t<∞, where A and B are independent random variables each of which takes a vale -2 with probability 1/3 and a value 1 with probability 2/3. Show that X(t) is wide – sense stationary.

17. The process $P\{X(t)=n\} = \begin{cases} \dfrac{(at)^{n-1}}{(1+at)^{n+1}}, n=1,2,... \\ \dfrac{at}{1+at}, n=0 \end{cases}$ Show that X(t) is stationary or not

18. Given that WSS random process X(t) = 10 cos (100t+θ) where θ is uniformly distributed random variable in (-π, π). Prove that the process {X(t)} is correlation ergodic.

19. The transition probability matrix of a Markov Chain $\{X_n\}$, n = 1, 2, 3, … having 3 states 1, 2 and 3 is $P = \begin{bmatrix} 0.1 & 0.5 & 0.5 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial distribution is P^0 = (0.7, 0.2, 0.1). Find i) $P(X_2 = 3)$ ii) $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$.

20. A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train but if he drives one day, then the next day he is likely to drive again he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drove to work if and only if a ‘6’ appeared. Find (i)the probability that he takes train on the third day (ii) the probability that he drives to work in the long run.

21. A housewife buys 3 kinds of cereals A, B, C, she never buys the same cereals in successive weeks. If she buys cereal A, the next week she buys cereal B. However if she buys B or C the next week she is3 times as likely to buy A as the other cereal. How often she buys each of the 3 cereals?

22. Define a Poisson process. Show that the sum of two independent Poisson process is also a Poisson Process.
23. Prove that the difference of two independent Poisson process is not a Poisson Process.
24. Suppose that customers arrive at a bank according to a Poisson process with a mean rate of 3per minute: find the probability that during a time interval of 2 min (i) exactly 4 customers arrive and (ii) more than 4-customers arrive. (iii) fewer than 4 customer in 2 minute interval
25. If customer arrive at a counter in accordance with a Poisson process with a mean rate of 2 per minute, find the probability that the interval between 2 consecutive arrivals is

(1) More than 1 minute

(2) Between 1 minute and 2 minute and 4 min. Or less.

Unit-4(Correlation and Spectral densities)

PART A

1. Prove that auto correlation function is an even function ofτ.
2. Prove that R(τ) is maximum at τ=0.
3. The autocorrelation function of a stationary process is $R(\tau) = 16 + \frac{9}{1+16\tau^2}$. Find the mean and variance of the process.
4. Find the mean and variance of ergodic process {X(t)} whose auto correlation is given by $R(\tau) = 25 + \frac{4}{1+6\tau^2}$.
5. Find the variance of the stationary process {X(t)} whose auto correlation function is given by $R_{xx}(\tau) = 2 + 4e^{-2|\tau|}$.
6. Find the mean, variance and root mean square value of the process whose auto correlation function is given by $R(\tau) = \frac{25\tau^2+36}{6.25\tau^2+4}$.
7. State any two properties of cross correlation function.
8. Prove that the spectral density of a real random process is an even function.
9. Find the auto correlation function whose spectral density is $S(\omega) = \begin{cases} \pi; |\omega| \le 1 \\ 0; otherwise \end{cases}$
10. Determine the auto correlation function of the random process with spectral density given by $S_{xy}(\omega) = \begin{cases} S_0 |\omega| < \omega_0 \\ 0 otherwise \end{cases}$
11. Define power spectral density function.

PART-B

Unit-4(Correlation and Spectral densities)

Relationship between Rxx(τ) and Sxx(ω)

1. The auto correlation function of the random binary transmission {X(t)} is given by $R(\tau) = 1 - \frac{|\tau|}{T} for |\tau| < T and R(\tau) = 0 for |\tau| < T$. Find the power spectrum of the process {X(t)}.

2. Find the power spectral density of the random process whose auto correlation function is $R(\tau) = \begin{cases} 1-|\tau|, for|\tau| \le 1 \\ 0, elsewhere \end{cases}$
3. Find the power spectral density of the random process whose auto correlation function is $R(\tau) = e^{-\alpha\tau^2}$
4. If the power spectral density of a WSS process is given by $S(\omega) = \begin{cases} \frac{b}{a}(a-|\omega|), & |\omega| \le a \\ 0, & |\omega| > a \end{cases}$. Find the autocorrelation function of the process.

Relationship between Rxy(τ) and Sxy(ω)

5. The cross – power spectrum of real random processes {X(t)} and {Y(t)} is given by $S_{xy}(\omega) = \begin{cases} a + bj\omega, & for|\omega| < 1 \\ 0, & elsewhere \end{cases}$. Find the cross correlation function.
6. If the cross power spectral density of X(t) and Y(t) is $S_{xy}(\omega) = \begin{cases} a + \frac{ib\omega}{\alpha}, -\alpha < \omega < \alpha, \alpha > 0 \\ 0, \quad elsewhere \end{cases}$ where 'a' and 'b' are constants. Find the cross correlation function.
7. If {X(t)} and {Y(t)} are two random processes with auto correlation function $R_{xx}(\tau)$ and $R_{yy}(\tau)$ respectively then prove that $|R_{xy}(\tau)| \le \sqrt{R_{xx}(0)R_{YY}(0)}$. Establish any two properties of auto correlation function $R_{xx}(\tau)$.
8. State and prove Winer khinchine theorem

Unit-5 (Linear system with Random inputs)

PART A

1. Define a system when it is called a linear system?
2. Define casual system.
3. Define a linear-time invariant system.
4. Check the following systems are linear or not (i) y(t)=tx(t) (ii) y(t) = x^2(t).
5. Check the following systems are time invariant or not (i) y(t) = tx(t) (ii) y(t) = x(t)-x(t-1).
6. Find the system transfer function, if a Linear time invariant system has an impulse function $H(t) = \begin{cases} \frac{1}{2c}; |t| \le c \\ 0; |t| \ge c \end{cases}$.
7. Write the relation between input and output of a linear time invariant system

Unit-5 (Linear system with Random inputs)

PART B

1. If the input to a time invariant stable, linear system is a WSS process, prove that the Output will also be a WSS process.
2. If {X(t)} is a WSS process and if $y(t) = \int_{-\infty}^{\infty} h(u)X(t-u)du$ then prove that
   (i) $R_{xy}(\tau) = R_{xx}(\tau) * h(-\tau)$
   (ii) $R_{yy}(\tau) = R_{xy}(\tau) * h(\tau)$ Where * denotes convolution.
   (iii) $S_{xy}(\omega) = S_{xx}(\omega)H * (\omega)$ Where $H * (\omega)$ is the complex conjugate of H(ω).
   (iv) $S_{yy}(\omega) = S_{xx}(\omega)|H(\omega)|^2$
3. Let X(t) be a WSS process which is the input to a linear time invariant system with unit impulse h(t) and output Y(t), then prove that $S_{xx}(\omega) = |H(\omega)|^2 S_{xx}(\omega)$ where $|H(\omega)|$ is Fourier transform of h(t).
4. A linear system is described by the impulse response $h(t) = \frac{1}{RC} e^{\frac{-t}{RC}} u(t)$. Assume an input process whose Auto correlation function is $B\delta(\tau)$. Find the autocorrelation mean and Auto correlation function of the output process.
5. Consider a system with transfer function $\frac{1}{1+j\omega}$. An input signal with autocorrelation function $m\delta(\tau) + m^2$is fed as input to the system. Find the mean and mean-square value of the output.
6. Assume a random process X(t) is given to a system with transfer function H(ω)=1 for $-\omega_0 < \omega < \omega_0$. If the autocorrelation function of the input process is $\frac{N_0}{2}\delta(t)$, find the auto correlation function of the output process.
7. If X(t) is the input voltage to a circuit and Y(t) is the output voltage. {X(t)} is a stationary random process with μ_x=0 and $R_{xx}(\tau) = e^{-2|\tau|}$. Find the mean μ_y and power spectrum $S_{yy}(\omega)$ of the output if the system transfer function is given by $H(\omega) = \frac{1}{\omega + 2i}$.
8. If X(t) is the input voltage to a circuitz (system) and Y(t) is the output voltage. {X(t)} is a stationary random process with μ_x=0 and $R_{xx}(\tau) = e^{-\alpha|\tau|}$. Find the mean μ_y , $S_{yy}(\omega)$ and $R_{yy}(\tau)$ if the power transfer function is given by $H(\omega) = \frac{R}{R + iL\omega}$.
9. A system has an impulse response $h(t) = e^{-\beta t}U(t)$, find the power spectral density of the output Y(t) corresponding to the input X(t).
10. A random process X(t) is the input to a linear system whose impulse function is $h(t) = 2e^{-t} : t \geq 0$. The auto correlation function of the process is $R_{xx}(\tau) = e^{-2|\tau|}$. Find the power spectral density of the output process Y(t).