UNIT - II (1)

TWO DIMENSIONAL RANDOM VARIABLES

Definition

Let S be the sample space associated with a random experiment E. Let $X = X(s)$ and $Y = Y(s)$ be two functions each assigning a real number to each outcome $s \in S$. Then (X, Y) is called a two dimensional random variable.

Discrete random variable

If the possible values of (X, Y) are finite or countably infinite, (X, Y) is called a two dimensional discrete r.v. The possible values of a two dimensional discrete r.v (X, Y) may be represented as (x_i, y_j) $i = 1, 2, \ldots m$ and $j = 1, 2, \ldots n$.

Continuous random variable

If (X, Y) can assume all values in a specifed region R in the xy-plane, then (X, Y) is called a two dimensional continuous r.v.

Joint Probability distribution of (X, Y)

Let (X, Y) be two dimensional r.v. Let

$P[X = x_i, Y = y_j] = P_{ij}$ P_{ij} is called the probability function of (X, Y) if the following conditions are satisfied.

(i) $P_{ij} > 0$ for all i and j

(ii) $\sum_j \sum_i P_{ij} = 1$

The set of triples (x_i, y_j, P_{ij}), $i = 1, 2, \dots m$ and $(j = 1, 2, \dots n)$ is called the joint probability distribution of (X, Y).

Joint Probability density function

If (X, Y) is a two dimensional continuous r.v. such that

$$P\left\{x - \frac{dx}{2} \le X \le x + \frac{dx}{2} \text{ and } y - \frac{dy}{2} \le Y \le y + \frac{dy}{2}\right\}$$

$$= f(x, y)\, dx\, dy.$$

Then $f(x, y)$ is called joint pdf of (X, Y) provided the following conditions are satisfied.

(i) $f(x, y) \ge 0$ for all $(x, y) \in R$ where R is the range space

(ii) $\iint_R f(x, y)\, dx\, dy = 1$

Joint Cumulative Distribution Function

If $f(x, y)$ is a two dimensional R.V., then

$F(x, y) = P[X \le x, Y \le y]$ is called c.d.f of (X, Y)

$$F(x,y) = \begin{cases} \sum\limits_{\substack{j \\ y_j \le y}} \sum\limits_{\substack{i \\ x_i \le x}} P_{ij} & \text{for discrete case} \\ \int_{-\infty}^{y} \int_{-\infty}^{x} f(x,y)\,dx\,dy & \text{for continuous case} \end{cases}$$

Marginal distributions

Discrete case

Let (x, y) be a two dimensional R.V. and $P_{ij} = P[x = x_i, y = y_j]$.

Then $P[x = x_i] = P_{i.} = \sum_j P_{ij}$ is called the marginal probability function of x

and If $P[y = y_j] = P_{.j} = \sum_i P_{ij}$ is called the marginal probability function of y.

continuous case

Let $f(x,y)$ be the joint pdf of a continuous two-dimensional R.V (x, y).

The marginal density function of X is defined by $g(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$.

The marginal density function of y is defined by $h(y) = \int_{-\infty}^{\infty} f(x,y)\,dx$.

Conditional Distributions

Discrete case

If $P_{ij} = P[X = x_i, Y = y_j]$ is the joint prob. fn. of two dimensional discrete r.v (X, Y) then the conditional prob. fn. X given $Y = y_j$ is defined by

$$P[X = x_i / Y = y_j] = \frac{P[X = x_i \cap Y = y_j]}{P[Y = y_j]} = \frac{P_{ij}}{P_{\cdot j}}$$

$|||^{ly}$ the conditional prob. fn. of Y gn. $X = x_i$ is defined by

$$P[\cancel{Y} = y_j / X = x_i] = \frac{P[Y = y_j \cap X = x_i]}{P[X = x_i]} = \frac{P_{ij}}{P_{i\cdot}}$$

Continuous case

If $f(x,y)$ is the joint pdf of a two dimensional r.v (X, Y), then the conditional pdf of X given $Y = y$ is defined by

$$f(x/y) = \frac{f(x,y)}{h(y)}$$

where $h(y)$ = marginal pdf of Y

the conditional density fn. of Y gn. $X = x$ is defined by

$$f(y/x) = \frac{f(x,y)}{g(x)}$$

where $g(x)$ = marginal pdf of X.

Independence of random variables

Two random variables X and Y are said to be independent if

$$P[X = x_i \cap Y = y_j] = P[X = x_i] \cdot P[Y = y_j]$$

i.e) $P_{ij} = (P_{i\cdot})(P_{\cdot j}) \quad \forall$ i and j

In case of continuous random variable, the random variables X & Y are independent if

$$f(x,y) = g(x)\, h(y)$$

where $f(x,y)$ = joint pdf of X and Y

$g(x)$ = Marginal density of X

$h(y)$ = Marginal density of Y.

Problems

① The joint distribution of X and Y is given by $f(x,y) = \frac{x+y}{21}$, $x = 1, 2, 3$; $y = 1, 2$. Find the marginal distributions.

$X \backslash Y$	1	2	$\sum_y f(x,y)$
1	$2/21$	$3/21$	$5/21$
2	$3/21$	$4/21$	$7/21$
3	$4/21$	$5/21$	$9/21$
$\sum_x f(x,y)$	$9/21$	$12/21$	1

Marginal distribution of X $\quad P[X=x] = \sum_y P(x,y)$

x	1	2	3
$f(x)$	$5/21$	$7/21$	$9/21$

Marginal distribution of Y $\quad P[Y=y] = \sum_x P(x,y)$

y	1	2
$f(y)$	$9/21$	$12/21$

② Let X and Y be two random variables having the joint probability function $f(x,y) = K(x+2y)$ where x and y can assume only the integer values 0, 1 & 2. Find the marginal and conditional distributions.

X \ Y	0	1	2	$\sum_y f(x,y)$
0	0	2K	4K	6K
1	K	3K	5K	9K
2	2K	4K	6K	12K
$\sum_x f(x,y)$	3K	9K	15K	27K

$$\sum_y \sum_x f(x,y) = 1$$

$$27K = 1$$

$$\boxed{K = 1/27}$$

Marginal distribution of X

X	0	1	2
P(X)	6/27	9/27	12/27

Marginal distribution of Y

Y	0	1	2
P(Y)	3/27	9/27	15/27

Conditional dis. of X gn. $Y = y$.

$$P[X = x / Y = y] = f(x,y) \frac{P[X = x \cap Y = y]}{P[Y = y]}$$

$\underline{Y = 0}$

$$P[X = 0 / Y = 0] = \frac{P[X = 0 \cap Y = 0]}{P[Y = 0]} = 0$$

$$P[X = 1 / Y = 0] = \frac{P[X = 1 \cap Y = 0]}{P[Y = 0]} = \frac{1/27}{3/27} = 1/3$$

$$P[X = 2 / Y = 0] = \frac{P[X = 2 \cap Y = 0]}{P[Y = 0]} = \frac{2/27}{3/27} = 2/3$$

$\underline{Y = 1}$

$$P[X = 0 / Y = 1] = \frac{P[X = 0 \cap Y = 1]}{P[Y = 1]} = \frac{2/27}{9/27} = 2/9$$

$$P[X = 1 / Y = 1] = \frac{P[X = 1 \cap Y = 1]}{P[Y = 1]} = \frac{3/27}{9/27} = 3/9$$

$$P[X = 2 / Y = 1] = \frac{P[X = 2 \cap Y = 1]}{P[Y = 1]} = \frac{4/27}{9/27} = 4/9$$

X \ Y	0	1	2
0	0	2/9	4/15
1	1/3	3/9	5/15
2	2/3	4/9	6/15

Conditional distribution of Y gn. $X = x$.

$X \backslash Y$	0	1	2
0	0	2/6	4/6
1	1/9	3/9	5/9
2	2/12	4/12	6/12

(3) The joint pmf of (x, y) is gn. by $p(x, y) = K[2x+3y]$, $x = 0, 1, 2$; $y = 1, 2, 3$. Find Find the marginal and conditional distributions.

$X \backslash Y$	1	2	3	$\sum_y f(x,y)$
0	3K	6K	9K	18K
1	5K	8K	11K	24K
2	7K	10K	13K	30K
$\sum_x f(x,y)$	15K	24K	33K	72K

$$\sum_x \sum_y f(x, y) = 1$$

$$72K = 1$$

$$\boxed{K = 1/72}$$

Marginal distribution of X

X	0	1	2
P(X)	$\frac{18}{72}$	$\frac{24}{72}$	$\frac{30}{72}$

Marginal distribution of Y

Y	1	2	3
P(Y)	$\frac{15}{72}$	$\frac{24}{72}$	$\frac{33}{72}$

conditional distribution of X gn. $Y = y_j$

X \ Y	1	2	3
0	$1/5$	$1/4$	$\frac{3}{11}$
1	$1/3$	$1/3$	$\frac{1}{3}$
2	$7/15$	$5/12$	$\frac{13}{33}$

Conditional distribution of Y gn. $X = x_i$.

X \ Y	1	2	3
0	$1/6$	$1/3$	$1/2$
1	$5/24$	$1/3$	$11/24$
2	$7/30$	$1/3$	$13/30$

④ The joint pdf of the two dimensional r.v (X, Y) is gn. by $f(x,y) = \begin{cases} 8xy, & 0 \le x \le y \le 1 \\ 0, & \text{elsewhere} \end{cases}$

Find the marginal and conditional density functions ~~an~~ also find $P[X < \frac{1}{2} \cap Y < \frac{1}{4}]$.

Marginal density function of X

$$g(x) = \int_{-\infty}^{\infty} f(x,y)\, dy = \int_{x}^{1} 8xy\, dy$$

$$= 8x \left(\frac{y^2}{2}\right)_x^1 = 4x\left[1 - x^2\right]$$

$$= 4x - 4x^3, \quad 0 < x < 1$$

Marginal density function of Y

$$h(y) = \int_{-\infty}^{\infty} f(x,y)\, dx = \int_{0}^{y} 8xy\, dx$$

$$= 8y\left(\frac{x^2}{2}\right)_0^y = 4y\,(y^2)$$

$$= 4y^3, \quad 0 < y < 1$$

Conditional density function of X given Y

$$f(x/y) = \frac{f(x,y)}{h(y)} = \frac{8xy}{4y^3}$$

$$= \frac{2x}{y^2}, \quad 0 < x < y < 1$$

Conditional density function of Y given X -

$$f(y/x) = \frac{f(x,y)}{g(x)} = \frac{8xy}{4x(1-x^2)}$$

$$= \frac{2y}{1-x^2} \quad , \quad 0 < x < y < 1$$

$$P\left[X < \tfrac{1}{2} \cap Y < \tfrac{1}{4}\right] = \int_{y=0}^{1/4} \int_{x=0}^{y} f(x,y)\, dx\, dy$$

$$= \int_{y=0}^{1/4} \int_{x=0}^{y} 8xy\, dx\, dy$$

$$= \int_{y=0}^{1/4} 8y \left(\frac{x^2}{2}\right)_0^y dy$$

$$= 4 \int_0^{1/4} y^3\, dy = 4\left(\frac{y^4}{4}\right)_0^{1/4}$$

$$= \left(\frac{1}{4}\right)^4 = \frac{1}{256}$$

5 The joint pdf of two random variables X and Y is

$$f(x,y) = \begin{cases} Kx(x-y) , & 0 < x < 2 \\ & |y| < x \\ 0 , & \text{otherwise} \end{cases}$$

Find K, marginal and conditional density of Y given X.

WKT $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y)\, dx\, dy = 1$

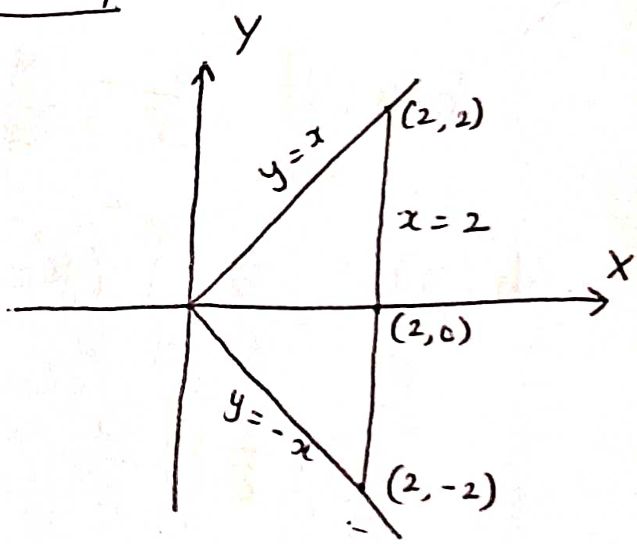
$$\int_{x=0}^{2}\int_{y=-x}^{x} K\,(x^2 - xy)\,dy\,dx = 1$$

$$K\int_{x=0}^{2}\left[x^2y - x\frac{y^2}{2}\right]_{-x}^{x} dx = 1$$

$$K\int_{x=0}^{2}\left[\left(x^3 - \frac{x^3}{2}\right) - \left(-x^3 - \frac{x^3}{2}\right)\right] dx = 1$$

$$K\int_{x=0}^{2} 2x^3\,dx = 1 \Rightarrow 2K\left(\frac{x^4}{4}\right)_0^2 = 1$$

$$\Rightarrow \frac{K}{2}(16) = 1 \Rightarrow \boxed{K = \frac{1}{8}}$$



Marginal density of X

$$g(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$$

$$= \int_{-x}^{x} \frac{1}{8}(x^2 - xy)\,dy$$

$$= \frac{1}{8}\left[x^2y - \frac{xy^2}{2}\right]_{-x}^{x}$$

$$= \frac{1}{8}\left[\left(x^3 - \frac{x^3}{2}\right) - \left(-x^3 - \frac{x^3}{2}\right)\right]$$

$$g(x) = \frac{2x^3}{8} = \frac{x^3}{4}, \quad 0 < x < 2$$

Marginal density of Y

$$h(y) = \int_{-\infty}^{\infty} f(x,y)\,dx$$

$$h(y) = \frac{1}{8}\int_{-y}^{2} (x^2 - xy)\, dx \qquad (-2 < y < 0)$$

$$= \frac{1}{8}\left[\frac{x^3}{3} - \frac{yx^2}{2}\right]_{-y}^{2}$$

$$= \frac{1}{8}\left[\left(\frac{8}{3} - 2y\right) - \left(\frac{-y^3}{3} - \frac{y^3}{2}\right)\right]$$

$$= \frac{1}{8}\left[\frac{8}{3} - 2y + \frac{y^3}{3} + \frac{y^3}{2}\right]$$

$$= \frac{1}{\cancel{8}}\left[\frac{\cancel{16} - 12y + \cancel{2}y^3 + 3y^3}{6}\right]$$

$$h(y) = \frac{1}{3} - \frac{y}{4} + \frac{5y^3}{48}, \quad -2 < y < 0$$

$$h(y) = \frac{1}{8}\int_{y}^{2} (x^2 - xy)\, dx \qquad (0 < y < 2)$$

$$= \frac{1}{8}\left[\frac{x^3}{3} - \frac{yx^2}{2}\right]_{y}^{2}$$

$$= \frac{1}{8}\left[\left(\frac{8}{3} - 2y\right) - \left(\frac{y^3}{3} - \frac{y^3}{2}\right)\right]$$

$$h(y) = \frac{1}{3} - \frac{y}{4} + \frac{y^3}{48}, \quad 0 < y < 2$$

Conditional density of Y gn. X

$$f(y/x) = \frac{f(x,y)}{g(x)} = \frac{\frac{x}{8}(x-y)}{\frac{x^3}{4}}$$

$$f(y/x) = \frac{x-y}{2x^2}, \quad -x < y < x, \quad 0 < x < 2$$

⑥ The random variables X and Y have joint pdf

$$f(x,y) = \begin{cases} x^2 + \frac{xy}{3}, & 0 \le x \le 1,\ 0 \le y \le 2 \\ 0, & \text{otherwise} \end{cases}$$

(i) Are X and Y independent?

(ii) Find the conditional pdf of X given Y.

(iii) ~~Find $P[X+Y \ge 1]$ and~~ $P[Y < \frac{1}{2} / X < \frac{1}{2}]$.

(i) Marginal density fn. of X

$$g(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_0^2 \left(x^2 + \frac{xy}{3}\right) dy$$

$$= \left[x^2 y + \frac{x}{3}\frac{y^2}{2}\right]_0^2 = \left[2x^2 + \frac{2x}{3}\right], \quad 0 \le x \le 1$$

Marginal density fn. of Y

$$h(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = \int_0^1 \left(x^2 + \frac{xy}{3}\right) dx$$

$$= \left[\frac{x^3}{3} + \frac{y}{3}\left(\frac{x^2}{2}\right)\right]_0^1 = \left[\frac{1}{3} + \frac{y}{6}\right], \quad 0 \le y \le 2.$$

$$g(x)\,h(y) = \left[2x^2 + \frac{2x}{3}\right]\left[\frac{1}{3} + \frac{y}{6}\right]$$

$$\ne f(x,y)$$

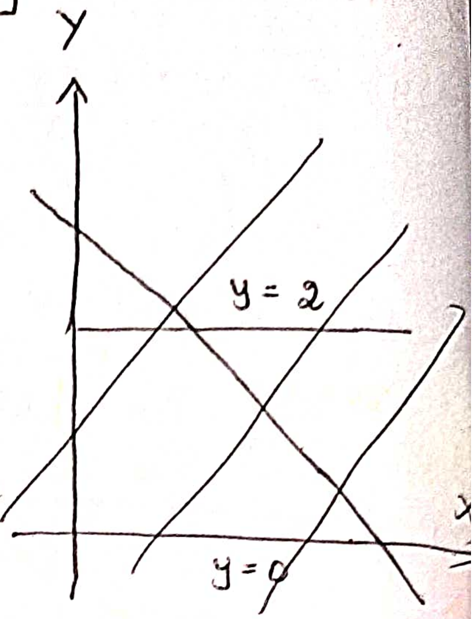
$\therefore$ X and Y are not independent.

(ii) Conditional pdf of X given Y

$$f(x/y) = \frac{f(x,y)}{h(y)} = \frac{x^2 + \frac{xy}{3}}{\frac{1}{3} + \frac{y}{6}} = \frac{\frac{3x^2 + xy}{3}}{\frac{2+y}{6}}$$

$$= \frac{6x^2 + 2xy}{2+y} \qquad \begin{array}{l} 0 \le x \le 1 \\ 0 \le y \le 2 \end{array}$$

(iii) $P[x+y \geq 3] = 1 - P[x+y < 3]$



(iii) $P[y < \frac{1}{2} / x < \frac{1}{2}]$

$$= \frac{P[x<\frac{1}{2} \cap y < \frac{1}{2}]}{P[x<\frac{1}{2}]} \rightarrow (1)$$

$$P[x<\tfrac{1}{2} \cap y<\tfrac{1}{2}] = \int_0^{1/2}\int_0^{1/2} \left(x^2 + \frac{xy}{3}\right) dx\, dy$$

$$= \int_0^{1/2} \left[\frac{x^3}{3} + \frac{y}{3}\frac{x^2}{2}\right]_0^{1/2} dy$$

$$= \int_0^{1/2} \left[\frac{1}{24} + \frac{y}{3}\left(\frac{1}{8}\right)\right] dy = \left[\frac{1}{24}y + \frac{1}{24}\frac{y^2}{2}\right]_0^{1/2}$$

$$= \left[\frac{1}{48} + \frac{1}{48}\left(\frac{1}{4}\right)\right] = \frac{1}{48} + \frac{1}{192} = \frac{5}{192}$$

$$P[x<\tfrac{1}{2}] = \int_{y=0}^{2}\int_{x=0}^{1/2} \left[x^2 + \frac{xy}{3}\right] dx\, dy$$

$$= \int_{y=0}^{2} \left[\frac{x^3}{3} + \frac{y}{3}\frac{x^2}{2}\right]_0^{1/2} dy$$

$$= \int_{y=0}^{2} \left[\frac{1}{24} + \frac{y}{6}\left(\frac{1}{4}\right)\right] dy = \int_{y=0}^{2} \left[\frac{1}{24} + \frac{y}{24}\right] dy$$

$$= \frac{1}{24}\left[y + \frac{y^2}{2}\right]_0^2 = \frac{1}{24}[2+2] = \frac{1}{6}$$

Sub. in (1),

$$P[y<\tfrac{1}{2} / x<\tfrac{1}{2}] = \frac{5/192}{1/6} = \left(\frac{5}{\cancel{192}_{32}}\right)\cancel{6}$$

$$= \frac{5}{32}$$

(7) The joint pdf of the two dimensional r.v (X, Y) is gn. by $f(x,y) = kxy\, e^{-(x^2+y^2)}$, $x>0$, $y>0$. Find the value of K and examine the independence of X and Y.

$2i\,(x^2+y^2)$

WKT $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$

$$\Rightarrow \int_0^{\infty}\int_0^{\infty} Kxy\, e^{-(x^2+y^2)}\,dx\,dy = 1$$

$$\Rightarrow K\int_0^{\infty} x e^{-x^2}\,dx \int_0^{\infty} y e^{-y^2}\,dy = 1$$

Put $x^2 = u$ | $y^2 = v$

$2x\,dx = du$ | $2y\,dy = dv$

$$\Rightarrow K\int_0^{\infty} e^{-u}\frac{du}{2}\int_0^{\infty} e^{-v}\frac{dv}{2} = 1$$

$$\Rightarrow \frac{K}{4}\left[\frac{e^{-u}}{-1}\right]_0^{\infty}\left[\frac{e^{-v}}{-1}\right]_0^{\infty} = 1$$

$$\Rightarrow \frac{K}{4}[1][1] = 1 \Rightarrow \boxed{K = 4}$$

Marginal density fn. of X

$$g(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_0^{\infty} 4xy\, e^{-(x^2+y^2)}\,dy$$

$$= 4x e^{-x^2}\int_0^{\infty} y e^{-y^2}\,dy = 4x e^{-x^2}\int_0^{\infty} e^{-u}\frac{du}{2}$$

$$= 2x e^{-x^2}\left[\frac{e^{-u}}{-1}\right]_0^{\infty} = 2x e^{-x^2},\ x>0$$

Marginal density fn. of y

$$h(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = \int_0^{\infty} 4xy\,e^{-(x^2+y^2)}\,dx$$

$$= 4y e^{-y^2} \int_0^{\infty} x e^{-x^2}\,dx = 4y e^{-y^2} \int_0^{\infty} e^{-u} \frac{du}{2}$$

$$= 2y e^{-y^2} \left[\frac{e^{-u}}{-1}\right]_0^{\infty} = 2y e^{-y^2}, \quad y > 0$$

$$g(x)\,h(y) = 2x e^{-x^2}\, 2y e^{-y^2}$$

$$= 4xy\, e^{-(x^2+y^2)}$$

$$= f(x,y)$$

$\therefore$ x and y are independent

⑧ If x and y are two random variables having joint pdf. $f(x,y) = \begin{cases} \frac{1}{8}(6-x-y), & 0<x<2,\ 2<y<4 \\ 0, & \text{otherwise} \end{cases}$

Find (i) $P[x<1 \cap y<3]$

(ii) $P[x+y<3]$ and $P[x<1 / y<3]$. ✓

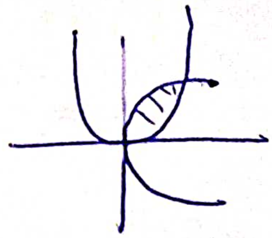
(i) $$P[x<1 \cap y<3] = \int_0^1 \int_2^3 f(x,y)\,dy\,dx$$

$$= \int_0^1 \int_2^3 \frac{1}{8}(6-x-y)\,dy\,dx = \frac{1}{8}\int_0^1 \left[6y - xy - \frac{y^2}{2}\right]_2^3 dx$$

$$= \frac{1}{8}\int_0^1 \left[\left(18 - 3x - \frac{9}{2}\right) - \left(12 - 2x - 2\right)\right] dx$$

$$= \frac{1}{8} \int_0^1 \left[8 - x - 9/2\right] dx$$

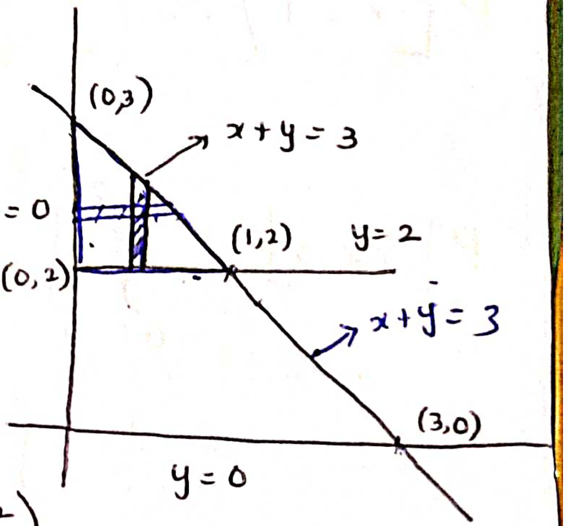
$$= \frac{1}{8} \left[8x - \frac{x^2}{2} - \frac{9}{2}x\right]_0^1$$



$$= \frac{1}{8} \left[8 - \frac{1}{2} - \frac{9}{2}\right] = \underline{\underline{\frac{3}{8}}}$$

(ii) $P\left[X + Y < 3\right]$

$$= \int_{x=0}^{1} \int_{y=2}^{3-x} \frac{1}{8}(6 - x - y)\, dy\, dx$$



$$= \frac{1}{8} \int_{x=0}^{1} \left[6y - xy - \frac{y^2}{2}\right]_2^{3-x} dx$$

$$= \frac{1}{8} \int_{x=0}^{1} \left[\left(6(3-x) - x(3-x) - \frac{(3-x)^2}{2}\right) - \left(12 - 2x - 2\right)\right] dx$$

$$= \frac{1}{8} \int_{x=0}^{1} \left[(6-x)(3-x) - \frac{(3-x)^2}{2} - (10 - 2x)\right] dx$$

$$= \frac{1}{8} \int_{x=0}^{1} \left[18 - 6x - 3x + x^2 - \frac{(3-x)^2}{2} - 10 + 2x\right] dx$$

$$= \frac{1}{8} \int_{x=0}^{1} \left[8 - 7x + x^2 - \frac{(3-x)^2}{2}\right] dx$$

$$= \frac{1}{8} \left[8x - \frac{7x^2}{2} + \frac{x^3}{3} + \frac{(3-x)^3}{6}\right]_0^1$$

$$= \frac{1}{8}\left[\left(8 - \frac{7}{2} + \frac{1}{3} + \frac{4}{3}\right) - \frac{9}{2}\right]$$

$$= \frac{1}{8}\left[8 + \frac{-21+2+8-27}{6}\right]$$

$$= \frac{1}{8}\left[\frac{48 - 48 + 10}{6}\right] = \frac{10}{48} = \frac{5}{24}$$

$$P[x<1/y<3] = \frac{P[x<1 \cap y<3]}{P[y<3]}$$

$$P[y<3] = \int_0^2\int_2^3 \frac{1}{8}(6-x-y)\,dy\,dx$$

$$= \frac{1}{8}\int_0^2 \left[6y - xy - \frac{y^2}{2}\right]_2^3 dx$$

$$= \frac{1}{8}\int_0^2 \left[\left(18 - 3x - \frac{9}{2}\right) - \left(12 - 2x - 2\right)\right] dx$$

$$= \frac{1}{8}\int_0^2 \left[8 - x - \frac{9}{2}\right] dx$$

$$= \frac{1}{8}\left[8x - \frac{x^2}{2} - \frac{9}{2}x\right]_0^2$$

$$= \frac{1}{8}\left[16 - 2 - 9\right] = \frac{1}{8}(5)$$

$$= 5/8$$

$$P[x<1/y<3] = \frac{3/8}{5/8} = 3/5$$

(9) The joint pdf of random variable X & Y is given by $f(x,y) = \begin{cases} \lambda x y^2 & ; 0 \le x \le y \le 1 \\ 0 & ; \text{otherwise} \end{cases}$

Find λ and the marginal density fn. of X.

WKT $$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$$

$$\int_{y=0}^{1}\int_{x=0}^{y} \lambda x y^2 \,dx\,dy = 1$$

$$\Rightarrow \lambda \int_{y=0}^{1} y^2 \left(\frac{x^2}{2}\right)_0^y dy = 1 \Rightarrow \frac{\lambda}{2}\int_0^1 y^4\,dy = 1$$

$$\Rightarrow \frac{\lambda}{2}\left[\frac{y^5}{5}\right]_0^1 = 1 \Rightarrow \frac{\lambda}{10} = 1 \Rightarrow \boxed{\lambda = 10}$$

Marginal density fn. of X

$$g(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_{y=x}^{1} 10xy^2\,dy$$

$$= 10x\left(\frac{y^3}{3}\right)_x^1 = \frac{10}{3}x(1-x^3)$$

$$g(x) = \frac{10}{3}[x - x^4], \quad 0 \le x \le 1$$