Newton Raphson method:

Given an approximate value of a root of an equation, a better and closer approximation to the root can be found by using an iterative process called Newton's method (or) Newton-Raphson method.

Let α_0 be an approximate value of a root of the equation $f(x)=0$

Let α be the exact-root nearer to α_0. Then $\alpha=\alpha_0+h$ where h is very small positive (or) negative.

$\therefore f(\alpha)=f(\alpha_0+h)=0$ since α is the exact root of $f(x)=0$.

By Taylor expansion,

$$f(\alpha)=f(\alpha_0+h)=f(\alpha_0)+hf'(\alpha_0)+\frac{h^2}{2!}f''(\alpha_0)+\cdots$$

(ie) If h is small, neglecting $h^2, h^3 \ldots$ etc., we get $f(\alpha_0)+hf'(\alpha_0)=0$.

$$\therefore h=\frac{-f(\alpha_0)}{f'(\alpha_0)} \quad \text{if } f'(\alpha_0)\neq 0.$$

$$\therefore \alpha=\alpha_0+h=\alpha_0-\frac{f(\alpha_0)}{f'(\alpha_0)} \text{ approximately}$$

Let this value be α_1.

$$\alpha_1=\alpha_0-\frac{f(\alpha_0)}{f'(\alpha_0)},$$

α_1 is a better approximate root than α_0.

Starting with this α_1 we get

$$\alpha_2 = \alpha_1 - \frac{f(\alpha_1)}{f'(\alpha_1)}$$ which is still better.

continuing like this, we iterate this process until $|\alpha_{r+1} - \alpha_r|$ is less than the quantity desired.

$$\therefore \alpha_{r+1} = \alpha_r - \frac{f(\alpha_r)}{f'(\alpha_r)}, \quad r = 0, 1, 2 \ldots$$

This is the iterative formula ~~to~~ of Newton-Raphson method.

Find the positive root of $f(x) = 2x^3 - 3x - 6 = 0$ by Newton-Raphson method correct to five decimal places.

Soln: Let $f(x) = 2x^3 - 3x - 6$, $f'(x) = 6x^2 - 3$

$$f(1) = 2 - 3 - 6 = -7 = -ve$$

$$\& \; f(2) = 16 - 6 - 6 = 4 = +ve$$

$\therefore$ a root lies between 1 & 2.

Take $\alpha_0 = 2$

$$\therefore \alpha_{i+1} = \alpha_i - \frac{f(\alpha_i)}{f'(\alpha_i)}$$

$$= \alpha_i - \left[\frac{2\alpha_i^3 - 3\alpha_i - 6}{6\alpha_i^2 - 3} \right]$$

$$= \frac{6\alpha_i^3 - \cancel{3\alpha_i} - 2\alpha_i^3 + \cancel{3\alpha_i} + 6}{6\alpha_i^2 - 3}$$

$$\therefore \alpha_{i+1} = \frac{4\alpha_i^3+6}{6\alpha_i^2-3}$$

$$\alpha_1 = \frac{4(2)^3+6}{6(2)^2-3} = \frac{38}{21}$$

$$\alpha_1 = 1.809524$$

$$\alpha_2 = \frac{4(1.809524)^3+6}{6(1.809524)^2-3} = 1.784200$$

$$\alpha_3 = \frac{4(1.784200)^3+6}{6(1.784200)^2-3} = 1.783769$$

$$\alpha_4 = \frac{4(1.783769)^3+6}{6(1.783769)^2-3} = 1.783769$$

The better approximate root is 1.783769 //