2) Find the real positive root of $3x - \cos x - 1 = 0$ by Newton's method correct to 4 decimal places.

Solu: Let $f(x) = 3x - \cos x - 1 \Rightarrow f'(x) = 3 + \sin x$

$f(0) = 0 - 1 - 1 = -2 = -ve$

$f(0.5) = -1.72758256 = -ve$.

$f(1) = 1.2159197694 = +ve$.

Take $x_0 = 1$

$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= x_0 - \frac{(3x_0 - \cos x_0 - 1)}{3 + \sin x_0}$

$$\alpha_{i+1} = \alpha_i - \frac{f(\alpha_i)}{f'(\alpha_i)}$$

$$= \alpha_i - \frac{(3\alpha_i - \cos\alpha_i - 1)}{3 + \sin\alpha_i}$$

$$= \frac{3\alpha_i + \alpha_i \sin\alpha_i + \cos\alpha_i - 1 - 3\alpha_i}{3 + \sin\alpha_i}$$

$$\therefore \alpha_{i+1} = \frac{\alpha_i \sin\alpha_i + \cos\alpha_i + 1}{3 + \sin\alpha_i}$$

$$\therefore \alpha_0 = \frac{\alpha_0 \sin\alpha_0 + \cos\alpha_0 + 1}{3 + \sin\alpha_0}$$

$$\alpha_1 = \frac{2.381773d91}{3.841470985} = 0.62001595\alpha$$

$$\alpha_2 = 0.607129066$$

$$\alpha_3 = 0.607101648$$

$$\alpha_4 = 0.607101648$$

∴ The root is 0.60710 correct to six decimals.

Find an iterative formula to find $\sqrt{N}$ (where N is a positive number) and hence find $\sqrt{5}$.

Find an iterative formula to find the reciprocal of a given number N and hence find the value of $\frac{1}{19}$.

Find the positive root of $x = \cos x$ using Newton's method.