Inverse interpolation:

So far, given a set of x & y we were finding the values of y corresponding to some $x = x_k$ (which is not given in the table). Here, we treat y as a function of x. Now the problem is, given some $y = y_r$, we should find the corresponding x. This process of finding x given y is called the inverse interpolation.

In such a case, we will take y as independent variable and x as dependent variable and use Lagrange's interpolation formula.

Taking y as independent variable,

$$x = \frac{(y-y_1)(y-y_2)\ldots(y-y_n)}{(y_0-y_1)(y_0-y_2)\ldots(y_0-y_n)}x_0 + \frac{(y-y_0)(y-y_2)\ldots(y-y_n)}{(y_1-y_0)(y_1-y_2)\ldots(y_1-y_n)}x_1 + \ldots + \frac{(y-y_0)(y-y_1)\ldots(y-y_{n-1})}{(y_n-y_0)(y_n-y_1)\ldots(y_n-y_{n-1})}x_n$$

This formula (1) is called formula of inverse interpolation.

(1) From the data given below, find the value of x, when $y = 13.5$

x :	93.0	96.2	100.0	104.2	108.7
y :	11.38	12.80	14.70	17.07	19.91

Soln:

By Lagrange's formula for inverse interpolation

$$x = \frac{(y-12.80)(y-14.70)(y-17.07)(y-19.91)}{(11.38-12.80)(11.38-14.70)(11.38-17.07)(11.38-19.91)} \times 93$$

$$+ \frac{(y-11.38)(y-14.70)(y-17.07)(y-19.91)}{(12.80-11.38)(12.80-14.70)(12.80-17.07)(12.80-19.91)} \times 96.2$$

$$+ \frac{(y-11.38)(y-12.80)(y-17.07)(y-19.91)}{(17.07-11.38)(17.07-12.80)(17.07-14.70)(17.07-19.91)} \times 104.2$$

$$+ \frac{(y-11.38)(y-12.80)(y-14.70)(y-17.07)}{(19.91-11.38)(19.91-12.80)(19.91-14.70)(19.91-17.07)} \times 108.7$$

Putting $y = 13.5$ on the RHS, & simplifying

$$x = -7.812629 + 68.3721132 + 43.595887 - 7.2733429 + 0.77008490$$

$$x = 97.6557503$$