1. From the following table

x	1 (x_0)	2 (x_1)	3 (x_n)	4
y	-8	-1	18	

Compute $y(1.5)$ & $y'(1)$. Using Cubic Spline.

Soln:

Here $h=1$, & $n=2$. Also assume $M_0=0$ & $M_2=0$

We have $M_{i-1}+4M_i+M_{i+1}=\frac{6}{h^2}[y_{i-1}-2y_i+y_{i+1}]$ for $i=1,2\ldots(n-1)$

From this $i=1$

$$M_0+4M_1+M_2=6[y_0-2y_1+y_2]$$

$$4M_1=6[-8-2(-1)+18]=72$$

$$M_1=18$$

W.K.T $S(x)=\frac{1}{6h}[(x_i-x)^3M_{i-1}+(x-x_{i-1})^3M_i]+\frac{1}{h}(x_i-x)[y_{i-1}-\frac{h^2}{6}M_{i-1}]+\frac{1}{h}(x-x_{i-1})[y_i-\frac{h^2}{6}M_i]$ — ①

From ①, for $1\le x\le 2$, putting $i=1$, we get

$$S(x)=\frac{1}{6h}[(x_1-x)^3M_0+(x-x_0)^3M_1]+\frac{1}{h}(x_1-x)[y_0-\frac{h^2}{6}M_0]+\frac{1}{h}(x-x_0)[y_1-\frac{h^2}{6}M_1]$$

$$=\frac{1}{6}\{18(x-1)^3\}+(2-x)(-8)+[(-(x-1)-(x-1)^3]$$

$$=\frac{1}{6}\{18(x-1)^3\}+(2-x)(-8)-4(x-1)$$

$$=3(x-1)^3+4x-12$$

$$=3x^3-9x^2+13x-15$$

$\Rightarrow y(1.5)=-\frac{45}{8}$ & $y'\approx S'(x)=9(x-1)^2+4 \Rightarrow y'(1)=4.$

2) Using Cubic spline, find $y(0.5)$ & $y'(1)$ given $M_0 = M_2 = 0$ and the table.

x	0	1	2
y	-5	-4	3

Solu: Here $h=1$, & $n=2$

We have,

$$M_{i-1} + 4M_i + M_{i+1} = \frac{6}{h^2}\{y_{i-1} - 2y_i + y_{i+1}\}$$

for $i = 1, 2 \ldots n-1$

Putting $i=1$ $M_0 + 4M_1 + M_2 = 6[y_0 - 2y_1 + y_2]$

$$= 6[-5 - 2(-4) + 3] = 36$$

$$4M_1 = 36 \Rightarrow M_1 = 9.$$

We will derive the cubic spline in [0,1]

W.K.T $S(x) = \frac{1}{6h}\{(x_i - x)^3 M_{i-1} + (x - x_{i-1})^3 M_i\}$
$+ \frac{1}{h}(x_i - x)\{y_{i-1} - \frac{h^2}{6}M_{i-1}\} + \frac{1}{h}(x - x_{i-1})\{y_i - \frac{h^2}{6}M_i\}$

for $i = 1, 2, 3 \ldots n$

put $S(x) = \frac{1}{6}\{(x_1 - x)^3 M_0 + (x - x_0)^3 M_1\} + \frac{1}{1}(x_1 - x)[y_0 - \frac{h^2}{6}M_0]$
$+ \frac{1}{1}(x - x_0)[y_1 - \frac{1}{6}M_1]$

$= \frac{1}{6}\{(1-x)^3 0 + (x-0)^3 9\}$
$+ [(1-x)][-5 - \frac{1}{6}(0)] + [(x-0)][-4 - 9/6]$

$= \frac{1}{6}[9x^3] - 5(1-x) - \frac{33}{6}x$

$= \frac{3}{2}x^3 - \frac{x}{2} - 5$, where $0 \le x \le 1$.

$S(0.5) = \frac{3}{2}(\frac{1}{8}) - \frac{1}{4} - 5 = -81/16$

$S'(x) = \frac{9}{2}x^2 - \frac{1}{2} \Rightarrow S'(1) \approx y'(1) = \frac{9}{2} - \frac{1}{2} = 4$,