Newton forward and Backward difference formula

1. Derive Newton's forward difference formula by using operator method:

Solu: $p_n(x) = p_n(x_0+uh) = E^u p_n(x_0)$

$= E^u y_0 = (1+\Delta)^u y_0$

$= 1 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots$

$\cdots \frac{u(u-1)(u-2)\cdots(u-n+1)}{n!}\Delta^n y_0$

where $u = \frac{x-x_0}{h}$.

2. Derive Newton's backward difference formula by using operator method.

Sol: $p_n(x) = p_n(x_n+vh) = E^v p_n(x_n)$

$= (1-\nabla)^{-v} y_n$

$= \left[1 + v\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n + \cdots\right.$

$\left. \cdots \frac{v(v+1)(v+2)\cdots(v+n-1)}{n!}\nabla^n y_n\right]$

where $v = \frac{x-x_n}{h}$

Problems:

1. From the following data, find θ at $x=43$ and $x=84$.

x	40	50	60	70	80	90
θ	184	204	226	250	276	304

Also express θ in terms of x.

1. ~~From the following data, find~~

Soln: Since six data are given, $p(x)$ is of degree 5. To find θ at $x=43$ use forward interpolation and to find θ at $x=84$, use backward interpolation formula.

$$u = \frac{x-x_0}{h} = \frac{43-40}{10} = 0.3$$

& $$v = \frac{84-90}{10} = \frac{x-x_n}{h} = -0.6$$

Table

x	θ	$\Delta\theta$	$\Delta^2\theta$	$\Delta^3\theta$	$\Delta^4\theta$
40	184				
		20			
50	204		2		
		22		0	
60	226		2		0
		24		0	
70	250		2		
		26		0	
80	276		2		
		28			
90	304				

Newton's forward formula is

$$\theta(x) = \theta_0 + u\Delta\theta_0 + \frac{u(u-1)}{2!}\Delta^2\theta_0 + \cdots$$

$$= \theta[40 + (0.3-3)10]$$

$$\theta(x=43) = 184 + (0.3)20 + \frac{(0.3)(-0.7)(2)}{2}$$

$$= 189.79$$

Newton's Backward formula is

$$\theta(x) = \theta_n + v\nabla\theta_n + \frac{v(v+1)}{2}\nabla^2\theta_n + \cdots$$

$$\theta(x=84) = \theta[90 + (-0.6)10]$$

$$= 304 + (-0.6)(28) + \frac{(-0.6)(10.4)}{2}(2) = 286.96$$

$$\theta = \theta_0 + u\Delta\theta_0 + \frac{u(u-1)}{2!}\Delta^2\theta_0 + \cdots$$

where $u = \frac{x-40}{10}$

$$= 184 + u(20) + \frac{u(u-1)}{2!}(2)$$

$$= 184 + \frac{20(x-40)}{10} + \frac{(x-40)(x-50)}{100}$$

$$= 184 + 2x - 80 + \frac{1}{100}[x^2 - 90x + 2000]$$

$$= 0.01x^2 + 1.1x + 124 \;//$$