2) Given the following data, find $y'(6)$ and the maximum value of y.

x	0	2	3	4	7	9
y	4	26	58	112	466	922

Soln:

Since the arguments are not equally spaced, we will use Newton's divided diff. formula (or even Lagrange's formula)

Divided diff Table

x	$y = f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
0	4	11		
2	26	32	7	
3	58	54	11	1
4	112	118	16	1
7	466	228	22	1
9	922			

By Newton divided diff. formula.

$$y = f(x) = f(x_0) + (x-x_0) f(x_0, x_1) + (x-x_0)(x-x_1) f(x_0, x_1, x_2) + \cdots$$

$$= 4 + (x-0)(11) + (x-0)(x-2)(7) + (x-0)(x-2)(x-3)(1)$$

$$= x^3 + 2x^2 + 3x + 4$$

$\therefore y'(x) = 3x^2 + 4x + 3 \Rightarrow y'(6) = 3(6)^2 + 4(6) + 3 = 135$

$y(x)$ is maximum if $y'(x) = 0$. $\therefore 3x^2 + 4x + 3 = 0$

But the roots are imaginary. $\therefore$ there is no extremum value in the range. In fact, it is an increasing curve.

1. Find the gradient of the road at the middle point of the elevation above a datum line of seven points of road which are given below

x	0	300	600	900	1200	1500	1800
y	135	149	157	183	201	205	193

Solu:

We require $\frac{dy}{dx}\Big|_{x=900}$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
0	135						
		14					
300	149		−6				
		8		24			
600	157		18		−50		
		26		−26		70	
900	183		−8		−20		−86
		18		−6		−16	
1200	201		−14		−2		
		4		−16			
1500	205		−12				
1800	193						

Since $x=900$ is the middle of the table we use Stirling's formula:

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left\{\frac{1}{2}(\Delta y_0 + \Delta y_{-1}) - \frac{1}{12}(\Delta^3 y_{-1} + \Delta^3 y_{-2}) + \frac{1}{60}(\Delta^5 y_{-2} + \Delta^5 y_{-3}) - \cdots\right\}$$

$$= \frac{1}{300}\left\{\frac{1}{2}(18+26) - \frac{1}{12}(-6-20) + \frac{1}{60}(70-16)\right\}$$

$$= 0.085222$$

Hence, the gradient of the road at the middle pt. is 0.085222.