3. Evaluate $\int_{-3}^{3} x^4\,dx$ by using (1) Trapezoidal rule (2) Simpson's rule. Verify your results by actual integration

Soln:

Here $y(x) = x^4$ interval length $(b-a) = 6$.
So, we divide 6 equal intervals with $h = \frac{6}{6} = 1$
we form below the table.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| x : | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| y : | 81 | 16 | 1 | 0 | 1 | 16 | 81 |

(i) By Trapezoidal rule.

$$\int_{-3}^{3} f(x)\,dx = \frac{h}{2}\{(\text{sum of the 1st \& last ordinates}) + 2(\text{sum of the remaining ordinates})\}$$

$$\approx \frac{1}{2}\{(81+81) + 2(16+1+0+1+16)\}$$

$$\approx 115$$

(ii) By Simpson's $\frac{1}{3}$rd rule (Since no. of ordinates is odd)

$$\int_{-3}^{3} y\,dx = \frac{1}{3}\{(81+81) + 2(1+1) + 4(16+0+16)\}$$

$$\approx 98.$$

(iii) Since $n=6$, we can also use Simpson's $3/8^{th}$ rule

$$\int_{-3}^{3} y\,dx = \frac{3}{8}\{(81+81) + 3(16+1+1+16) + 2(0)\} \approx 99$$

(iv) By actual integration $\int_{-3}^{3} x^4\,dx = 97.2$

From the results obtained by various methods, we see that Simpson's rule gives better than trapezoidal rule (it is true in general but not always).