Two and three pt. Gaussian quadrature formula

- State 2 pt. Gaussian quadrature formula,

Two pt. Gaussian quadrature formula is

$$\int_{-1}^{1} f(x)\,dx = f\left(-\frac{1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)$$

This formula is exact for polynomials upto degree 3.

- Apply Gaussian 2-pt. formula to evaluate

(i) $\int_{-1}^{1} \frac{1}{1+x^2}\,dx$ (ii) $\int_{0}^{1} \frac{dx}{1+x^2}$

Solu: (i) Here $f(x) = \frac{1}{1+x^2}$,

$$f\left(-\frac{1}{\sqrt{3}}\right) = f\left(\frac{1}{\sqrt{3}}\right) = \frac{1}{1+1/3} = 3/4 \;//$$

$$\therefore \int_{-1}^{1} \frac{1}{1+x^2}\,dx = \frac{3}{4} + \frac{3}{4} = \frac{3}{2} = 1.5$$

By actual integration

$$\int_{-1}^{1} \frac{1}{1+x^2}\,dx = [\tan^{-1}x]_0^1 = 2\tan^{-1}(1)$$
$$= 2\frac{\pi}{4} = \frac{\pi}{2} = 1.5708$$

Here the error due to two-pt. formula is 0.0708.

(ii) $\int_{0}^{1} \frac{dx}{1+x^2}$

Solu: Given interval is 0 to 1, to make them as −1 to 1.

$$\int_{0}^{1} \frac{dx}{1+x^2} = \frac{1}{2}\int_{-1}^{1} \frac{dx}{1+x^2}$$
$$= \frac{1}{2}\left[f\left(-\frac{1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)\right]$$
$$= \frac{1}{2}\left[\frac{3}{4} + \frac{3}{4}\right] = 0.75 \;//$$

2. If the range is not (-1, 1) then what is the idea to solve the Gaussian quadrature problems.

Soln: $x = \frac{b-a}{2}z + \frac{b+a}{2}$.

Ex:

1. Evaluate $I = \int_0^1 \frac{dx}{1+x}$ by Gaussian formula with two points.

Soln: Let $f(x) = \frac{1}{1+x}$ [Given range is not in the exact form]

Let $x = \frac{b-a}{2}z + \frac{b+a}{2}$. Here $a=0, b=1$

$x = \frac{1-0}{2}z + \frac{1+0}{2}$

$x = \frac{1}{2}z + \frac{1}{2}$ $\left| \begin{array}{l} x=0 \Rightarrow z=-1 \\ x=1 \Rightarrow z=1 \end{array} \right.$

$\therefore I = \int_{-1}^{1} \frac{\frac{1}{2}dz}{1+(\frac{1}{2}z+\frac{1}{2})} = \frac{1}{2}\int_{-1}^{1} \frac{dz}{\frac{3}{2}+z/2}$

$= \int_{-1}^{1} \frac{dz}{z+3} = f(-\frac{1}{\sqrt{3}}) + f(\frac{1}{\sqrt{3}})$

$= 0.41277 + 0.27954$

$= 0.69231$

2. Evaluate $\int_0^{\pi/3} \sin t\, dt$ by Gaussian 2 pt. formula.