Three-point Gaussian quadrature

State three point Gaussian quadrature formula.

Three point Gaussian quadrature formula is

$$\int_{-1}^{1} f(x)\,dx = \frac{5}{9}\left\{ f\left(-\sqrt{\tfrac{3}{5}}\right) + f\left(\sqrt{\tfrac{3}{5}}\right)\right\} + \frac{8}{9} f(0)$$

This formula is exact for polynomials up to degree 5.

1) Using Gaussian three-pt. formula evaluate $\int_{-1}^{1} (3x^2+5x^4)\,dx$. Also compare with exact value.

Soln:

Let $f(x) = 3x^2+5x^4$ [Given Range is exact form]

$\therefore f(0) = 0$

$$f\left(-\sqrt{\tfrac{3}{5}}\right) = f\left(+\sqrt{\tfrac{3}{5}}\right) = 3\left(\tfrac{3}{5}\right) + 5\left(\tfrac{3}{5}\right)^2 = \tfrac{9}{5} + \tfrac{9}{5} = \tfrac{18}{5}$$

$$\therefore \int_{-1}^{1} f(x)\,dx = \frac{5}{9}\left[\frac{18}{5} + \frac{18}{5}\right] + 0$$

$$= \frac{5}{9} \cdot \frac{36}{5} = 4 \quad \text{——} (1)$$

Exact value:

$$\int_{-1}^{1} (3x^2+5x^4)\,dx = 2\int_{0}^{1} (3x^2+5x^4)\,dx \quad [\because 3x^2+5x^4 \text{ is even fun.}]$$

$$= 4$$

We get exact value by using Gaussian 3 pt. formula.

2) Using 3 pt. Gaussian formula find $\int_{0.2}^{1.5} e^{t^2}\,dt$

3) Use Gaussian 3 pt. formula and evaluate $I = \int_{1}^{5} \frac{dz}{z}$.