Double integrals using trapezoidal & Simpson's rules

General form of Trapezoidal rule:

$$\iint f(x,y)\,dx\,dy = \frac{hk}{4}\{\text{Sum of } f \text{ at the four corners.} + 2(\text{Sum of the values of } f \text{ at the remaining nodes on the boundary}) + 4(\text{Sum of the values of } f \text{ at the interior nodes})\}$$

General form of Simpson's rule:

$$\iint f(x,y)\,dx\,dy = \frac{hk}{9}\{(\text{Sum of the values of } f \text{ at 4 corners}) + 2(\text{Sum of the values of } f \text{ at the odd positions on the boundary except the corners}) + 4(\text{Sum of the values of } f \text{ at the even positions on the boundary}) + 8\{\text{Sum of the values of } f \text{ at the odd positions on the odd row of the matrix except boun. row}) + 8(\text{Sum of the values of } f \text{ at even positions on the odd row of the matrix except boun. row}) + 8\{8(\text{Sum of the values of } f \text{ at odd position}) + 16(\text{Sum of the values of } f \text{ at even positions on the even row of the matrix})\}$$

1) Evaluate $\int_0^2 \int_0^2 f(x,y)\,dx\,dy$ by Trapezoidal rule for the following data.

y/x	0	0.5	1	1.5	2
0	2	3	4	5	5
1	3	4	6	9	11
2	4	6	8	11	14

Soln:

Here $h = 0.5$, $k = 1$

$$I = \frac{(0.5)(1)}{4}\{(2+5+11+4)+2(3+4+5+7+8+6)+4(4+6+9)\}$$

$$= (0.125)(203)$$

$$= 25.375.$$

2) Using Simpson's rule evaluate $\int_0^1 \int_0^1 \frac{1}{1+x+y}\,dx\,dy$ taking $h = k = 0.5$

Soln:

y/x	0	0.5	1
0	1	0.6667	0.5
0.5	0.6667	0.5	4
1	0.5	0.4	0.3333

Soln:

$$I = \frac{(0.5)(0.5)}{9}\{(1+0.5+0.5+0.3333)+4(0.6667+0.6667+0.4+0.4)+16(0.5)\}$$

$$= (0.02778)(18.8669)$$

$$= 0.5241 //$$

3. Evaluate $\int_1^3 \int_3^4 \frac{1}{(x+y)^2} dx\,dy$ taking $h=k=0.5$ by both trapezoidal rule & Simpson's rule.

4. Evaluate $\int_{1.1}^{1.4} \int_{2}^{2.4} \frac{1}{xy} dx\,dy$ using Trapezoidal rule and Simpson's rule. Verify your result by actual integration.

5. Evaluate $\int_{1.2}^{1.4} \int_{1}^{1.4} \frac{1}{x+y} dx\,dy$ by trapezoidal rule and Simpson's rule.

6. Evaluate $\int_1^2 \int_1^2 \frac{dx\,dy}{x+y}$ by using 1/3 rule. taking $\Delta x = \Delta y = 0.25$

Sol:

y/x	1.0	1.25	1.5	1.75	2.0
1.0	0.5	0.4444	0.4	0.3636	0.3333
1.25	0.4444	0.4	0.3636	0.3333	0.3077
1.5	0.4	0.3636	0.3333	0.3077	0.2857
1.75	0.3636	0.3333	0.3077	0.2857	0.2667
2.0	0.3333	0.3077	0.2857	0.2667	0.25

$I = \frac{(0.25)(0.25)}{9}\{(0.5+0.3333+0.25+0.3333)$
$+2(0.4+0.4+0.2857+0.2857)$
$+4(0.3636+0.4444+0.4444+0.3636+0.3077$
$+0.2667+0.2667+0.3077)+4(0.3333)$
$+8(0.3636+0.3077)+8(0.3636+0.3077)$
$+16(0.4+0.3333+0.2857+0.3333)\}$
$= 0.3392$ //