Modified Euler's method:

$$y_{n+1} = y_n + h\left[f\left(x_n + \frac{1}{2}h,\ y_n + \frac{1}{2}hf(x_n, y_n)\right)\right] \quad \text{(4)}$$

(or) $$y(x+h) = y(x) + h\left[f\left(x + \frac{1}{2}h,\ y + \frac{1}{2}hf(x,y)\right)\right] \quad \text{(5)}$$

Equn. (4) (or) (5) called as modified Euler's formula

Note:

In Euler method $y_{n+1} = y_n + \Delta y$

where $\Delta y = hf(x_0, y_0)$ where $f(x_0, y_0)$ = slope at (x_0, y_0)

In modified E.M $y_{n+1} = y_n + \Delta y$.

where $\Delta y = h$[average of the slopes at x_0 & x_1]

(or)

$= h$[average of the values of $\frac{dy}{dx}$ at the end of the interval x_0 to x_1].

Problems - Modified Euler's method

1. Solve the equation $\frac{dy}{dx} = 1 - y$, given $y(0) = 0$ using modified Euler's method and tabulate the solutions at $x = 0.1, 0.2$, & 0.3. Compare your result with the exact solutions.

Solu:

Given $f(x,y) = 1 - y$, $x_0 = 0$, $y_0 = 0$, $x_1 = 0.1$, $x_2 = 0.2$, $x_3 = 0.3$, $h = 0.1$

By Modified Euler method

$$y_{n+1} = y_n + h f\left[x_n + h/2,\ y_n + \frac{1}{2} h f(x_n, y_n)\right] \quad \text{—— (1)}$$

$$y_1 = y_0 + h f\left[x_0 + h/2,\ y_0 + h/2\, f(x_0, y_0)\right] \quad \text{—— (2)}$$

$$f(x_0, y_0) = 1 - y_0 = 1 - 0 = 1$$

(2) $\Rightarrow$ $y_1 = 0 + (0.1) f\left[0 + \frac{0.1}{2},\ 0 + \frac{0.1}{2}(1)\right]$

$= (0.1) f[0.05, 0.05)$

$= (0.1)(1 - 0.05)$

$= (0.1)(0.95)$

$= 0.095$

$$y_2 = y_1 + h f\left[x_1 + h/2,\ y_1 + h/2\, f(x_1, y_1)\right] \quad \text{—— (3)}$$

$f(x_1, y_1) = 1 - y_1$

$= 1 - 0.095$

$= 0.905$

(3) $\Rightarrow y_2 = (0.095) + (0.1) f\left[(0.1) + \frac{0.1}{2}, (0.095) + \frac{0.1}{2}(0.905)\right]$

$= 0.095 + (0.1) f(0.15, 0.140975)$

$= 0.095 + 0.1\,(1 - 0.140975)$

$= 0.095 + 0.085975$

$= 0.18098.$

$$y_3 = y_2 + h f\left[x_2 + \frac{h}{2},\ y_2 + h/2\, f(x_2, y_2)\right] \quad \text{—— (4)}$$

$f(x_2, y_2) = 1 - y_2 = 1 - 0.18098$

$= 0.81902$

$= 0.18098 + (0.1) f\left[0.2 + \frac{0.1}{2},\ 0.18098 + \frac{0.1}{2}(0.81902)\right]$

$= 0.18098 + (0.1) f[(0.25, 0.18098 + 0.040951)]$

$= 0.18098 + (0.1)[1 - 0.2219317]$

$= (0.18098) + 0.07778069$

$= 0.258787$

$y(0.1) = 0.095$

$y(0.2) = 0.18098$, $y(0.3) = 0.258787$

Exact solu:

$\frac{dy}{dx} = 1-y$, given $\frac{dy}{1-y} = dx$

$\therefore -\log(1-y) = x + c$

$\log(1-y) = -x - C$

$\therefore 1-y = e^{-x} \cdot A$

At $x=0, y=0 \Rightarrow A = 1$

$\therefore y = 1 - e^{-x}$ //

Using this exact solu.

$y(0.1) = 1 - e^{-0.1} = 0.9516258$

$y(0.2) = 1 - e^{-0.2} = 0.18126924$

$y(0.3) = 1 - e^{-0.3} = 0.2518779$

The values are tabulated,

x	M.E	Exact solu
0.1	0.95	0.9516
0.2	0.18098	0.18127
0.3	0.258787	0.2518