Apply the fourth order R.k method to find $y(0.2)$ given that $y' = x+y$, $y(0) = 1$.

Solu:

Since h is not mentioned in the question, we take $h = 0.1$

Given $y' = x+y$; $y(0) = 1$

$\therefore f(x,y) = x+y$, $x_0 = 0$, $y_0 = 1$

$x_1 = 0.1$, $x_2 = 0.2$

By fourth order R.k method, for the 1st interval.

$K_1 = hf(x_0, y_0)$
$= (0.1)(x_0+y_0)$
$= (0.1)(0+1)$
$= 0.1$ //

$k_2 = hf(x_0 + h/2, y_0 + k_1/2)$
$= (0.1) f(0.05, 1.05)$
$= (0.1)(0.05 + 1.05) = 0.11$

$k_3 = hf(x_0 + h/2, y_0 + k_2/2)$
$= (0.1)(0.05, 1.055)$
$= (0.1)(0.05 + 1.055) = 0.1105$

$k_4 = hf(x_0 + h, y_0 + k_3)$
$= (0.1) f(0.1, 1.1105)$
$= (0.1)(0.1 + 1.1105)$
$= 0.12105$ //

$\therefore \Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$= \frac{1}{6}(0.1 + 0.22 + 0.2210 + 0.12105)$

$= 0.11034167$

$y(0.1) = y_1 = y_0 + \Delta y \approx 1.110342$ //

Now Starting from (x_1, y_1) we get (x_2, y_2)

Again apply R.K algorithm replacing (x_0, y_0) by (x_1, y_1)

$k_1 = hf(x_1, y_1)$

$= (0.1)(x_1 + y_1)$

$= (0.1)(0.1 + 1.110342)$

$= 0.1210342$ //

$k_2 = hf(x_1 + h/2, y_1 + k_1/2)$

$= (0.1)(0.15 + 1.1708591)$

$= 0.1320859$ //

$k_3 = hf(x_1 + h/2, y_1 + k_2/2)$

$= (0.1)(0.15 + 1.1763848)$

$(0.1)(0.15 + 1.1763848)$

$= 0.13263848$

$k_4 = hf(x_1 + h, y_1 + k_3)$

$= (0.1) f(0.2, 1.24298048)$

$= 0.144298048$

$\Delta y = \frac{1}{6}(k_1 + 2k_2) + 2k_3 + k_4) = \frac{1}{6}(0.79478008)$

$y(0.2) = y_1 + \Delta y = 1.110342 + \frac{1}{6}(0.79478008)$

$y(0.2) \approx 1.2428$ //