Multistep Methods:

These methods required fewer evaluation of the function (pair four values of the function) to estimate the solution at a point and iterations are performed till sufficient accuracy is achieved. Estimation of error is possible & the methods are called predictor corrector methods. The methods of Milne & Adam Bash forth belong to this type.

Milne's predictor & Corrector Method.

1\. Write Milne's predictor corrector formula.

Milne's predictor formula is

$$y_{n+1,p} = y_{n-3} + \frac{4}{3}h\,(2y'_{n-2} - y'_{n-1} + 2y'_n)$$

Milne's corrector formula is

$$y_{n+1,c} = y_{n-1} + \frac{h}{3}\,(y'_{n-1} - 4y'_n + y'_{n+1})$$

Note:

A predictor formula is used to predict the value of y at x_{i+1} & A corrector formula is used to correct the error & to improve that value of y_{i+1}.

1) Using Milne's method find $y(4.4)$ given $5xy' + y^2 - 2 = 0$ given $y(4) = 1$, $y(4.1) = 1.0049$, $y(4.2) = 1.0097$ & $y(4.3) = 1.0143$.

Soln:

$y' = \frac{2-y^2}{5x}$, $x_0 = 4$, $x_1 = 4.1$, $x_2 = 4.2$, $x_3 = 4.3$

$x_4 = 4.4$, $y_0 = 1$, $y_1 = 1.0049$, $y_2 = 1.0097$, $y_3 = 1.0143$.

$y_1' = \frac{2-y_1^2}{5x_1} = \frac{2-(1.0049)^2}{5(4.1)} = 0.0493$.

$y_2' = \frac{2-y_2^2}{5x_2} = \frac{2-(1.0097)^2}{5(4.2)} = 0.0467$.

$y_3' = \frac{2-y_3^2}{5x_3} = \frac{2-(1.0143)^2}{5(4.3)} = 0.0452$.

By Milne's predictor formula

$y_{4,p} = y_0 + \frac{4h}{3}(2y_1' - y_2' + 2y_3')$

$= 1 + 4\frac{(0.1)}{3}[2(0.0493) - 0.0467 + 2(0.0452)]$

$= 1.01897$

$y_4' = \frac{2-y_4^2}{5(x_4)} = \frac{2-(1.01897)^2}{5(4.4)} = 0.0437$

Using $y_{4,c} = y_2 + \frac{h}{3}(y_2' + 4y_3' + y_4')$

$= 1.0097 + \frac{0.1}{3}[(0.0467) + 4(0.0452) + 0.0437]$

$y_{4,c} = 1.01874$ //