1) Find $y(0.1), y(0.2), y(0.3)$ from $\frac{dy}{dx} = xy+y^2$, $y(0)=1$ by using R.K method & hence obtain $y(0.4)$ using Adam's method.

Soln:

$f(x,y) = xy+y^2$, $x_0=0$, $y_0=1$, $x_1=0.1$, $x_2=0.2$, $x_3=0.3$, $x_4=0.4$

$k_1 = hf(x_0, y_0)$
$= (0.1) f(0,1) = (0.1) 1 = 0.1$

$k_2 = hf(0.05, y_0+k_1/2)$
$= (0.1) f(0.05, 1.05)$
$= (0.1)[(0.05)(1.05) + (1.05)^2] = 0.1155$

$k_3 = hf(0.05, y_0+k_2/2)$
$= (0.1) f(0.05, 1.0578)$
$= (0.1)[0.05(1.0578) + (1.0578)^2]$
$= 0.1172$

$k_4 = hf(x_0+h, y_0+k_3)$
$= (0.1) f(0.1, 1.1172)$
$= (0.1)[(0.1)(1.1172) + (1.1172)^2]$
$= 0.13598$

$y_1 = y_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$
$y(0.1) = 1.1169$

Again, start from y_1,
$k_1 = hf(x_1, y_1) = (0.1) f(0.1, 1.1169)$
$= 0.1359$.

$k_2 = hf[x_1 + h/2, y_1 + k_1/2]$
$= (0.1) f(0.15, 1.1849) = 0.1582$.

$$K_3 = hf(0.5, y_1 + k_{3/2})$$

$$= (0.1) f(0.15, 1.196)$$

$$= 0.16098$$

$$k_4 = (0.1) f(0.2, 1.2779)$$

$$= 0.1889.$$

$$(ie) \ y(0.2) = 1.2774$$

Start from (x_2, y_2) to get y_3

$$k_1 = hf(x_2, y_2)$$

$$= (0.1) f(0.2, 1.2774)$$

$$= 0.1884.$$

$$k_2 = hf(x_2 + h/2, y_2 + k_1/2)$$

$$= (0.1) f(0.25, 1.3178)$$

$$= 0.2225$$

$$k_3 = hf(x_2 + h/2, y_2 + k_2/2)$$

$$= (0.1) f(0.25, 1.3887)$$

$$= 0.2274.$$

$$k_4 = hf(x + h, y_2 + k_3) = (0.1) f(0.3, 1.5048)$$

$$= 0.2716$$

$$y_3 = [1.2774] + \frac{1}{6}[0.1884 + 2(0.2225) + 2(0.2274)$$
$$+ 0.2716]$$

$$= 1.5041.$$

Now we use Adam's predictor formula.

$$y_{4,p} = y_3 + \frac{h}{24}[55y_3' - 59y_2' + 37y_1' - 9y_0'] \quad ——①$$

$$y_0' = x_0 y_0 + y_0^2 = 1$$

$$y_1' = x_1 y_1 + y_1^2 = 1.3592$$

$$y_2' = x_2 y_2 + y_2^2 = 1.8872$$

$$y_3' = x_3 y_3 + y_3^2 = 2.7135$$

using ①,

$$y_{4,p} = 1.5041 + \frac{0.1}{3}\left[55(2.7135) - 59(1.8872) + 37(1.3592) - 9(1)\right]$$

$$= 1.8341$$

$$y_4' \cdot p = x_4 y_4 + y_4^2$$

$$= (0.4)\left[1.8341 + (1.8341)^2\right]$$

$$= 4.0976.$$

Adams Corrector formula is

$$y_{4,c} = y_3 + \frac{h}{24}\left[9y_4' + 19y_3' - 5y_2' + y_1'\right]$$

$$= 1.5041 + \frac{0.1}{24}\left[9(4.0976) + 19(2.7135) - 5(1.8872) + 1.3592\right]$$

$$= 1.8389 \; //$$

$$y(0.4) = 1.8389 \; //$$