



### UNIT-V

#### Boundary value problems in ordinary and partial differential equations

Finite difference of 2nd order O.D.E

Consider a second order O.D.E given by  $f(x, y, y', y'') = 0$  with boundary conditions specified at  $x=a$  &  $x=b$ .

Divide the interval  $[a, b]$  into 'n' sub-intervals, each of length  $h = \frac{b-a}{n}$ . Let  $x_i = x_0 + ih$ ,  $i=0, 1, 2, \dots, n$ , where  $x_0 = a$  &  $x_n = b$ .

We use the notations  $y_i = y(x_i)$ ,  $y'_i = y'(x_i)$  &  $y''_i = y''(x_i)$ .

The finite difference approximations to the derivatives are given by

$$y'_i = \frac{y_{i+1} - y_{i-1}}{2h} \text{ and}$$

$$y''_i = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$$

Substituting the above approximations for  $y'(x)$  &  $y''(x)$  and setting  $i=1, 2, 3, \dots, n-1$  we get a system of eqns. for the  $(n-1)$  unknowns. Solving the system, the values  $y_1, y_2, \dots, y_{n-1}$  are known, from the boundary conditions,  $y_0 = y(a)$  &  $y_n = y(b)$  are known.



Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

Eliminating  $y_2$  from (2) & (3), we get

$$256 y_3 - 833 y_1 = \frac{65}{4} \quad \text{--- (6)}$$

Eliminating  $y_3$  from (5) & (6) we get

$$y_1 = -0.03488$$

$$y_3 = (833 y_1 + 65/4) / 256$$

$$\text{or } y_3 = -0.05002, \Rightarrow y_2 = \frac{33 y_1 + 1/4}{16} = -0.05652 //$$

Tabulating, we have,

x	0	0.25	0.5	0.75	1
y	0	-0.03488	-0.05652	-0.05002	0

2) Solve  $y'' - \pi y = 0$ , given  $y(0) = -1$ ,  $y(1) = 2$  by finite difference method taking  $n=2$ .

Solu:

If  $n=2$ , then  $h = \frac{b-a}{n} = \frac{1-0}{2} = 1/2$  since range is

The nodal pts are  $x_0 = 0, x_1 = 0.5, x_2 = 1$

$$\text{Given } y'' - \pi y = 0 \quad \text{--- (1)}$$

Using central difference approximation, we have

$$y'' = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$$

$$\text{(1)} \Rightarrow \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} - \pi_i y_i = 0$$

$$\text{or } y_{i+1} - (2 + h^2 \pi_i) y_i + y_{i-1} = 0$$

Where  $c=1, h=1/2, \pi_i = 0.5, y_0 = -1, y_2 = 2$ .

$$\therefore y_2 - (2 + \frac{1}{8}) y_1 + y_0 = 0$$

$$2 - \frac{17}{8} y_1 - 1 = 0 \Rightarrow y_1 = \frac{8}{7} = 0.4706$$

Tabulating, we get

x	0	0.5	1
y	-1	0.4706	2