SNS COLLEGE OF ENGINEERING

Kurumbapalayam (Po), Coimbatore – 641 107

AN AUTONOMOUS INSTITUTION

Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

2) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0<x\le 1$, $t\ge 0$ with $u(x,0) = x(1-x)$, $0<x<1$ and $u(0,t) = u(1,t) = 0$, $\forall t>0$ using explicit method with $\Delta x = 0.2$ for 5 time steps.

Solu:

Given $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$

Here $\alpha^2 = 1$, $\Delta x = 0.2$ (ie) $h = 0.2$

k is not given.

Let us chose k such that

$\lambda = \frac{k\alpha^2}{h^2} = \frac{1}{2}$ [Explicit formula valid if $0<\lambda\le\frac{1}{2}$]

$\frac{k}{(0.2)^2} = \frac{1}{2}$ [So choose $\lambda = \frac{1}{2}$]

$k = \frac{0.04}{2} = 0.02$

$u_{i,j+1} = \lambda u_{i+1,j} + (1-2\lambda)u_{i,j} + \lambda u_{i-1,j}$

$= \frac{1}{2}[u_{i+1,j} + u_{i-1,j}]$ — (1)

Given $u(x,0) = x(1-x)$

$u(0.2,0) = (0.2)(1-0.2) = 0.16$

$u(0.4,0) = 0.24$

$u(0.6,0) = 0.24$

$u(0.8,0) = 0.16$

Using (1) the values of u are tabulated below.

j \ i	0	0.2	0.4	0.6	0.8	1.0
0	0	0.16	0.24	0.24	0.16	0
0.02	0	0.12	0.2	0.2	0.12	0
0.04	0	0.1	0.16	0.16	0.1	0
0.06	0	0.08	0.13	0.13	0.08	0
0.08	0	0.065	0.105	0.105	0.065	0
0.10	0	0.0525	0.085	0.085	0.0525	0 Given $u(l,t)=0$

Given $u(0,t)=0$.

5) Solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ given $u(0,t)=0$, $u(4,t)=0$, $u(x,0)=x(4-x)$ assuming $h=k=1$. Find the values of u upto $t=5$.

Soln:

If we want to use Bender-Schmidt formula, we should have $k=\frac{a}{2}h^2$.

Here $k=h=1$, $a=1$. These values do not satisfy the condition. Hence we cannot apply Bender-Schmidt formula. Hence we apply explicit formula.

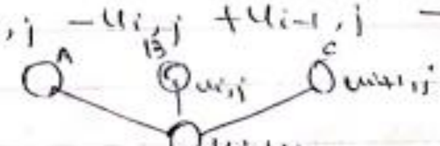
(ie) $u_{i,j+1} = \lambda u_{i+1,j} + (1-2\lambda)u_{i,j} + \lambda u_{i-1,j}$ —— (1)

Now $\lambda = \frac{k}{ah^2} = \frac{1}{1\times 1} = 1$.

Hence eqn. (1) reduces to

$u_{i,j+1} = u_{i+1,j} - u_{i,j} + u_{i-1,j}$ —— (2)

That is



Value of u at D = Value of u at A + Value of u at C − value of u at B.

Using (2), the values of u are tabulated below

x direction

j\i	0	1	2	3	4
0	0	3	4	3	0
1	0	1	2	1	0
2	0	1	0	1	0
3	0	−1	2	−1	0
4	0	3	−4	3	0
5	0	−7	10	−7	0

t - direction