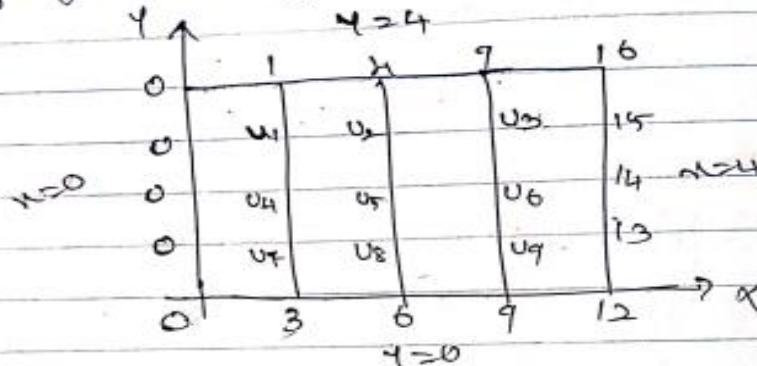




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2) Solve  $u_{xx} + u_{yy} = 0$  over the square mesh of side 4 units. Satisfying the following boundary conditions,  
(i)  $u(0, y) = 0$  for  $0 \leq y \leq 4$ .  
(ii)  $u(4, y) = 12 + y$ , for  $0 \leq y \leq 4$   
(iii)  $u(x, 0) = 3x$ , for  $0 \leq x \leq 4$   
(iv)  $u(x, 4) = x^2$  for  $0 \leq x \leq 4$

Solu. We divide the square mesh into 16 sub-squares of side 1 unit and calculate the numerical values of  $u$  on the boundary using given analytical expressions.



Let  $u_1, u_2, \dots, u_9$  be the values of  $u$  at the interior nine grid pts.

Finding Rough values

$$u_3 = \frac{1}{4} (4 + 6 + 0 + 14) = 6 \quad \text{SFPE}$$

$$u_1 = \frac{1}{4} (0 + 6 + 4 + 0) = 2.5 \quad \text{DFPE}$$

$$u_5 = \frac{1}{4} (16 + 6 + 14 + 4) = 10 \quad \text{DFPE}$$

$$u_7 = \frac{1}{4} (0 + 6 + 0 + 6) = 3 \quad \text{DFPE}$$

$$u_9 = \frac{1}{4} (6 + 14 + 6 + 12) = 9.5 \quad \text{DFPE}$$

We use SFPE to get the other value  $u$

$$u_2 = \frac{1}{4} (4 + 6 + 2.5 + 10) = 5.625$$



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$$U_4 = \frac{1}{4} (0 + 6 + 2.5 + 3) = 3.125$$

$$U_6 = \frac{1}{4} (6 + 14 + 10 + 9.5) = 9.875$$

$$U_8 = \frac{1}{4} (6 + 6 + 3 + 9.5) = 6.125$$

Now we proceed for iteration using always SFRP  
Iteration Scheme.

$$U_1 = \frac{1}{4} (1 + U_2 + U_4)$$

$$U_2 = \frac{1}{4} (4 + U_1 + U_3 + U_5)$$

$$U_3 = \frac{1}{4} (24 + U_2 + U_6)$$

$$U_4 = \frac{1}{4} (U_1 + U_5 + U_7)$$

$$U_5 = \frac{1}{4} (U_2 + U_4 + U_6 + U_8)$$

$$U_6 = \frac{1}{4} (14 + U_3 + U_5 + U_9)$$

$$U_7 = \frac{1}{4} [3 + U_4 + U_8]$$

$$U_8 = \frac{1}{4} [6 + U_5 + U_7 + U_9]$$

$$U_9 = \frac{1}{4} [22 + U_8 + U_6]$$

	$U_1$	$U_2$	$U_3$	$U_4$	$U_5$	$U_6$	$U_7$	$U_8$	$U_9$
0	2.5	5.625	10	3.125	6	9.875	3	6.125	9.5
1	2.4375	5.6094	9.8711	2.8594	6.1172	9.8721	2.9961	6.153	9.5063
2	2.3672	5.5888	9.8652	2.8698	6.1209	9.8731	3.0057	6.1582	9.5078
3	2.3646	5.5874	9.8151	2.8728	6.1229	9.8739	3.0078	6.1596	9.5083

Correct to two decimal places we have  $U_1 = 2.37$ ,

$U_2 = 5.59$ ,  $U_3 = 9.87$ ,  $U_4 = 2.88$ ,  $U_5 = 6.13$ ,  $U_6 = 9.88$ ,  $U_7 = 3.01$ ,  $U_8 = 6.16$ ,  $U_9 = 9.51$