



# **SNS COLLEGE OF ENGINEERING**

Kurumbapalayam (Po), Coimbatore – 641 107

**An Autonomous Institution**

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A' Grade  
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai



## **DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING**

### **UNIT – V**

**Stability Studies and Reactive Power Compensation**

**RUNGE – KUTTA METHOD**



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# FIRST ORDER RUNGE – KUTTA METHOD



Considering the differential equation

$$dy/dx=f(x,y)$$

With the initial condition  $y(x_0)=y_0$

By the Euler's method

$$y_{n+1}=y_n + h*f(x_n,y_n)$$

$$y_{n+1}=y_n + h*y'_n + (h^2/2!)y''_n + \dots \quad \text{(by Taylor's series)}$$



## SECOND ORDER RUNGE – KUTTA METHOD

$$Y_{n+1} = y_n + k$$

$$\text{Where } k = (1/2)(k_1 + k_2)$$

$$K_1 = h * f(x_n, y_n)$$

$$K_2 = h * f(x_n + h, y_n + k_1)$$



## Third Order Runge – Kutta method

►  $Y_{n+1} = y_n + k$

► Where  $k = (1/6)(k_1 + 4k_2 + k_3)$

$$K_1 = h * f(x_n, y_n)$$

$$K_2 = h * f(x_n + (h/2), y_n + (k_1/2))$$

$$K_3 = h * f(x_n + h, y_n + 2k_2 - k_1)$$



# Fourth Order Runge – Kutta method

►  $Y_{n+1} = y_n + k$

► Where  $k = (1/6)(k_1 + 2k_2 + 2k_3 + k_4)$

$$K_1 = h * f(x_n, y_n)$$

$$K_2 = h * f(x_n + (h/2), y_n + (k_1/2))$$

$$K_3 = h * f(x_n + (h/2), y_n + (k_2/2))$$

$$K_4 = h * f(x_n + h, y_n + k_3)$$



## EXAMPLE

► Solve the differential equation  $dy/dx=x+y$  , with the fourth order Runge-Kutta method ,where  $y(0)=1$  with  $x=0.2$  with  $h=0.1$ .

► Given data  $y(0)=1$  and  $h=0.1$

►  $dy/dx = x+y$

$$f(x,y)=dy/dx=x+y$$



## EXAMPLE

➤ 1<sup>st</sup> iteration

$$x_0=0, y_0=1, h=0.01, n=0$$

$$\begin{aligned} K1 &= h \cdot f(x_0, y_0) \\ &= 0.1 \cdot (0, 1) \\ &= 0.1 \cdot (0 + 1) \\ &= 0.1 \end{aligned}$$





## EXAMPLE

$$\begin{aligned} K2 &= h \cdot f(x_0 + (h/2), y_0 + (k1/2)) \\ &= 0.1 \cdot f(0 + (0.1/2), 1 + (0.1/2)) \\ &= 0.1 \cdot f(0.05, 1.05) \\ &= 0.1 \cdot (0.05 + 1.05) \\ &= 0.11 \\ K3 &= h \cdot f(x_0 + (h/2), y_0 + (k2/2)) \\ &= 0.1 \cdot f(0 + (0.1/2), 1 + (0.11/2)) \\ &= 0.1 \cdot f(0.05, 1.055) \\ &= 0.1 \cdot (0.05 + 1.055) \\ &= 0.1105 \end{aligned}$$



## EXAMPLE

$$\begin{aligned}K4 &= h \cdot f(x_0 + h, y_0 + k3) \\ &= 0.1 \cdot f(0 + 0.1, 1 + 0.1105) \\ &= 0.1 \cdot f(0.1, 1.1105) \\ &= 0.1 \cdot (0.1 + 1.1105) \\ &= 0.1211\end{aligned}$$

$$\begin{aligned}\text{So } y1 &= y_0 + k \\ &= y_0 + (1/6)(k1 + 2k2 + 2k3 + k4) \\ &= 1 + (1/6)(0.1 + 2(0.11) + 2(0.1105) + 0.1211) \\ &= 1.1103\end{aligned}$$

