1.12. TWO DIMENSIONAL (2D) TRANSFORMATION

During modeling of an object, it becomes necessary to transform the geometry many times. The transformation actually converts the geometry from one coordinate system to other coordinate system. The main types of 2D transformation which are often come across are as follows.

- Translation
- Scaling

- Reflection
- Rotation
- Shearing.

1.12.1. Translation

It is one of the most important and easily understood transformations in CAD. *Translation* is the movement of an object from one position to another position. It is accomplished by adding the distance through which the drawing is to be moved to the co-ordinates of each corner point. Figure 1.23 shows a square object. Let us now consider a point on the object, represented by P which is translated along x and y axes by added T_x and T_y to a new position P'.

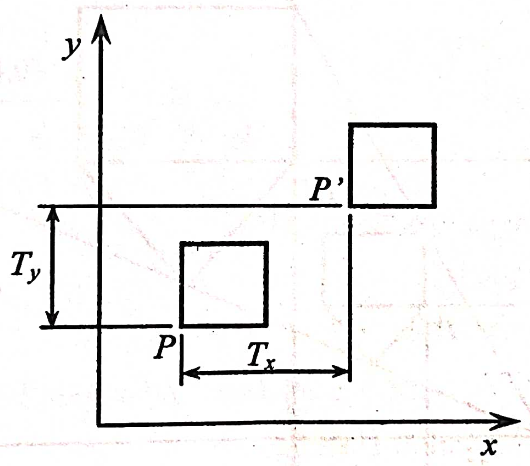

Figure 1.23 Translation

The new co-ordinate after transformation is given by the following equation.

$$P' = [X', Y']$$

$$X' = X + T_x$$

$$Y' = Y + T_y$$

$$\therefore \quad P' = [X + T_x, Y + T_y]$$

$$= [X \quad Y] + [T_x \quad T_y]$$

In matrix form, we can write the above equation as

$$[P'] = [X' \; Y' \; 1] = [X \; Y \; 1]\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ T_x & T_y & 1 \end{bmatrix}$$

$$P' = P \, . \, T$$

where $\quad T$ = Translation matrix.

It is normally the operation used in the CAD system as MOVE command.

1.12.2. Scaling

Scaling is the transformation applied to change the scale of an entity. It is done by increasing the distance between points of the drawing. It means that it can be done by multiplying the coordinates of the drawing by an enlargement or reduction factor called *scaling factor*. The size of the entity altered by the application of scaling factor is shown in Figure 1.24.

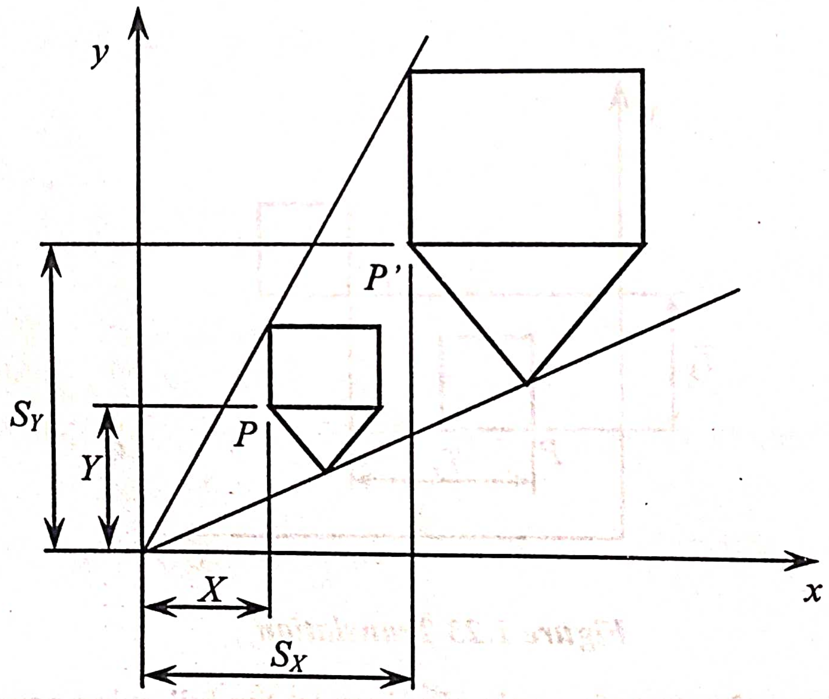

Figure 1.24 Scaling

The new co-ordinates after scaling are given by the following equations:

$$P' = [X', \; Y'] = [S_x \times X, \; S_y \times Y]$$

This equation can also represented in a matrix form as

$$[P'] = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}$$

$$[P'] = [S]\,[P]$$

where $$[S] = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} = \text{Scaling matrix}$$

For example, Figure 1.25(a) shows a triangle to be scaled before scaling. Figure 1.25(b) shows the same triangle after scaling. Here, all coordinates of the entity are multiplied by scaling matrix. Therefore, it is enlarged two times the original one.

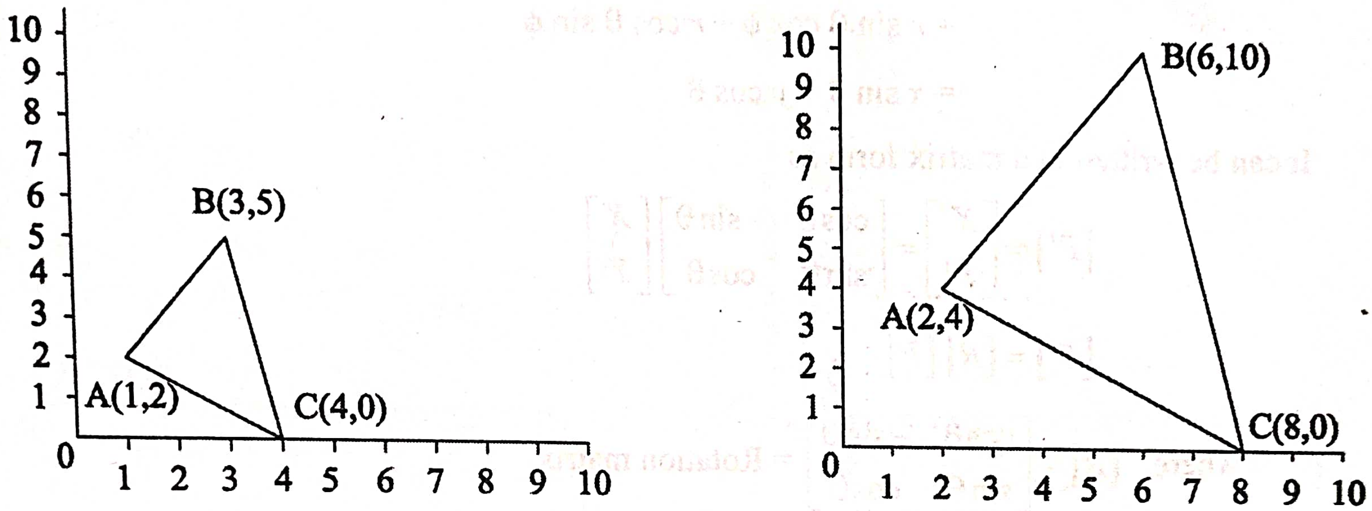

Figure 1.25 Scaling

1.12.3. Reflection

Refer Chapter **1.11.1 (iv)** in this book Page **1.37.**

1.12.4. Rotation

Rotation is another important geometric transformation in CAD. Here, the drawing is rotated about a fixed point. The final position and orientation of geometry is decided by the angle of rotation (θ) and the base point about which the rotation is to be done. Figure 1.26 shows a rotation transformation of an object about origin *O*. To develop the transformation matrix, consider a point *P* as the object in XY plane, being rotated in anticlockwise direction to the new position *P'* by an angle θ. The new position *P'* is given by

$$P' = [X', Y']$$

From Figure 1.26, the original position is specified by

$$X = r \cos \phi$$
$$Y = r \sin \phi$$

The new position P' is specified by

$$X' = r\cos(\phi + \theta)$$

$$= r\cos\theta\cos\phi - r\sin\theta\sin\phi$$

$$= x\cos\theta - y\sin\theta$$

$$Y' = r\sin(\phi + \theta)$$

$$= r\sin\theta\cos\phi + r\cos\theta\sin\phi$$

$$= x\sin\theta + y\cos\theta$$

It can be written in a matrix form as

$$[P'] = \begin{bmatrix} X' \\ Y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}$$

$$[P'] = [R].[P]$$

where $[R] = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ = Rotation matrix.

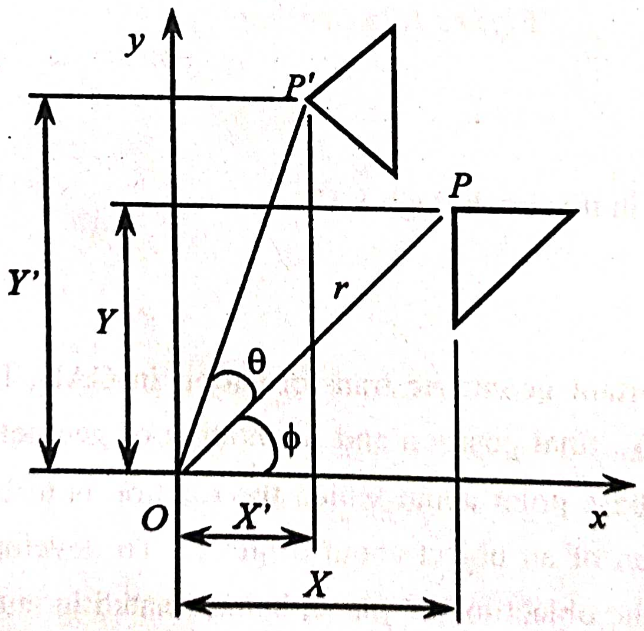

Figure 1.26 Rotation transformation

1.12.5. Shearing

A shearing transformation produces distortion of an object or an entire image. There are two types of shear mentioned below.

(a) X-shear

(b) Y-Shear

A X-shear transforms the point (X, Y) to (X', Y') by a shear factor S_{h_x}, where

$$X' = X - S_{h_x} Y$$

$$Y' = Y$$

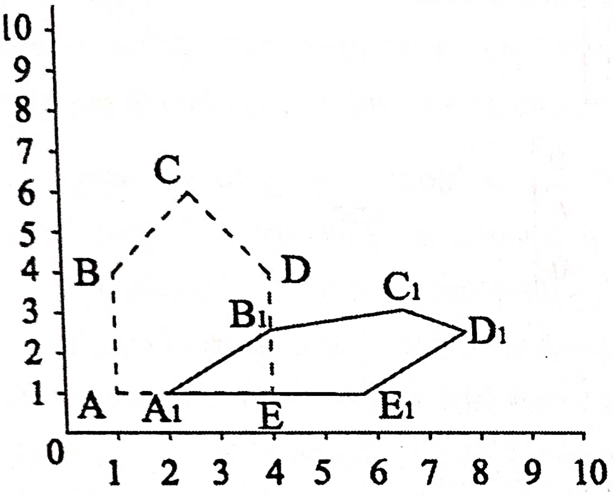

Figure 1.27 X-shear transformation

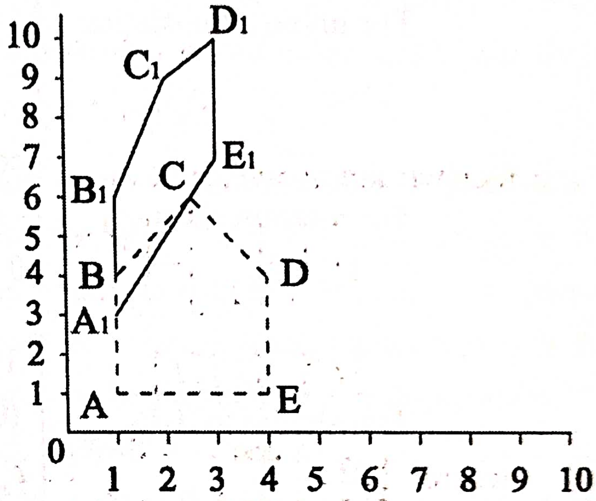

Figure 1.28 Y-shear transformation

Figure 1.27 shows X-shear applied to a base drawing represented by A-B-C-D-E. The entity A_1-B_1-C_1-D_1-E_1 represents the X-sheared drawing.

A Y-shear transforms the point (X, Y) to (X', Y') by a shear factor S_{h_y},

where $\quad X' = X$

$$Y' = S_{h_y} X + Y$$

Figure 1.28 shows Y-shear applied to a drawing. Figure 1.28 shows Y-shear applied to a base drawing represented by A-B-C-D-E. The entity A_1-B_1-C_1-D_1-E_1 represents the Y-sheared drawing.