Unit 2

GEOMETRIC MODELING

<u>GEOMETRIC MODELING</u>

- The mathematical description of the geometry of an object using a software is called as *geometric modeling*
- There are three basic methods
  - Wire – Frame Modeling
  - Surface modeling
  - Solid modeling

WIRE FRAME MODELING

- This is one of the most popular and commonly used method of geometric modeling.
- In construction of wire frame model, the edges of an object are presented as lines.
- Wire frame model is used for following representations
  - 2D Representation
  - Orthographic views representation

2D Wire Frame Model	3D Wire Frame Model
The co-ordinate system is 2D co-ordinate system i.e. *x* and *y* co-ordinates only	3D co-ordinate system is used for representing objects; *x, y* and *z* coordinates are used
3 Dimensional wire frame system generation is difficult	Both 2D and 3D wire frame generation is possible
Hidden lines may not complicate the figure	Difficult for the viewer to interpret the figure unless the hidden lines are removed
Curved surfaces are indicated by circles, arcs and ellipses	Curved surfaces are represented by spaced generators.

HERMITE CURVE

BEZIER CURVE

- Bezier curve was developed by *P. Bezier* at French car company "Renault Automobile Company".
- He used these curves to design automobile bodies.
- It provides the reasonable design flexibility and avoids large number of calculation.

$$P(u) = \sum_{i=0}^{n} P_i \, B_{i,n}\ (u), 0 \le u \le 1$$

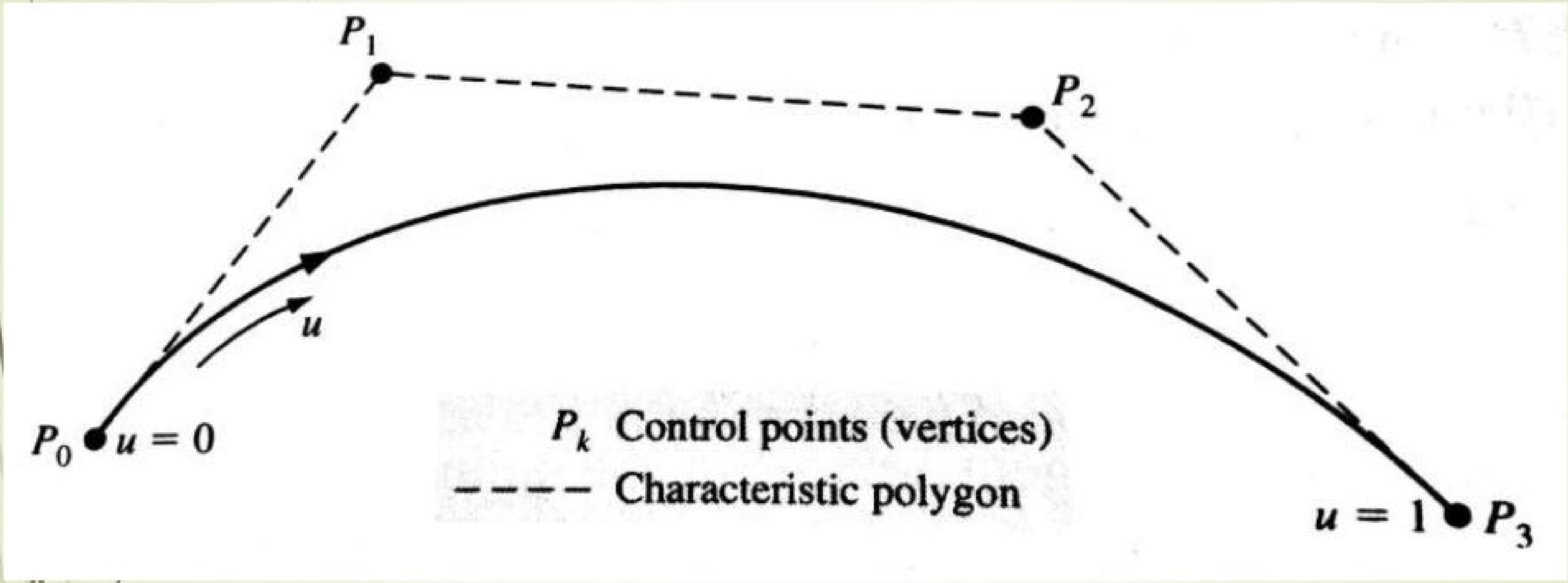
- $B_{i,n}(u)$ is the Bernstein function are given by

$$B_{i,n}\ (u) = C(n,i)\ u^i(1-u)^{n-i}$$

Where, $C\ (n,i) = \frac{n!}{i!(n-1)!}$

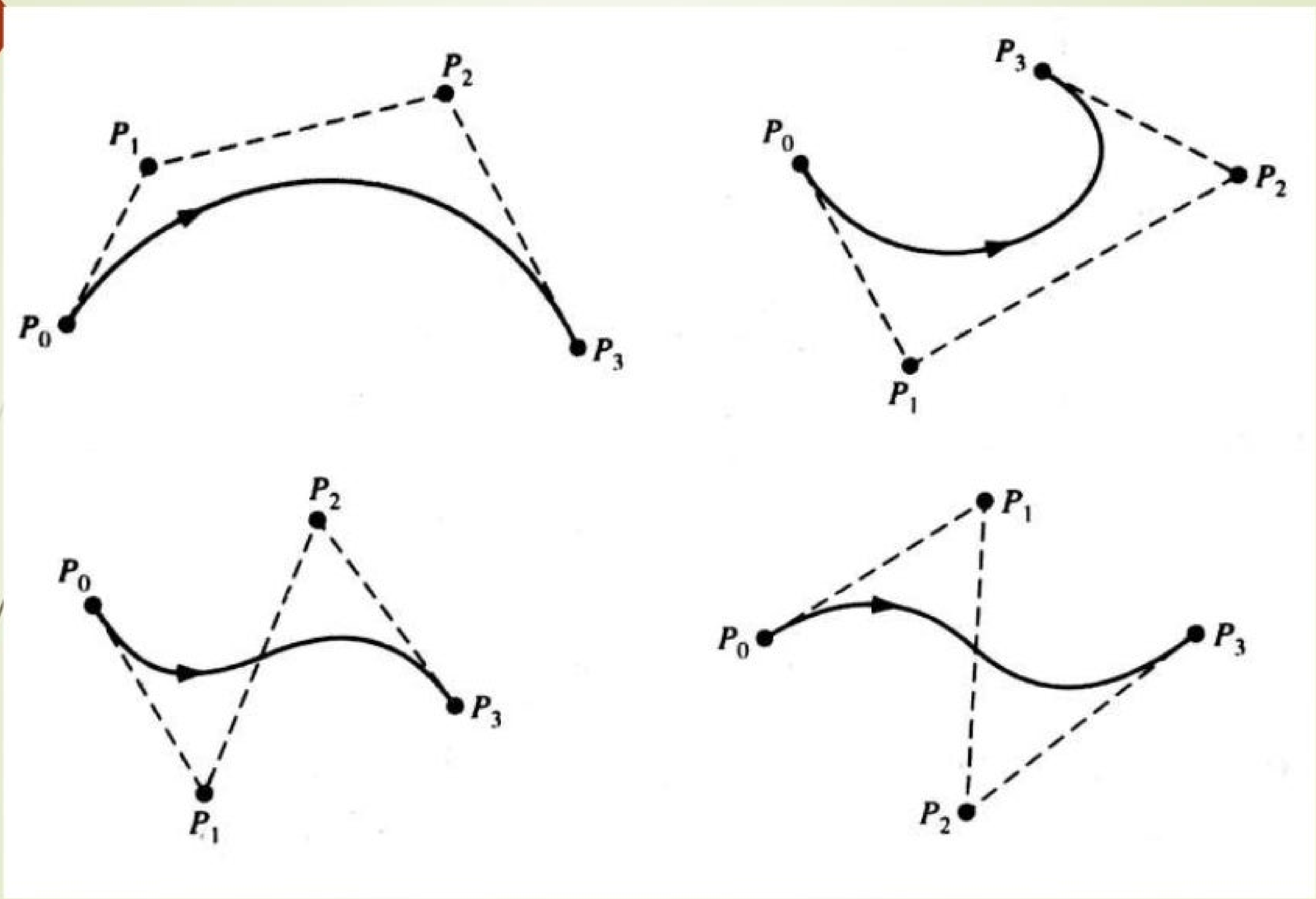
$$C(n, 0) = C(n, n) = 1.$$

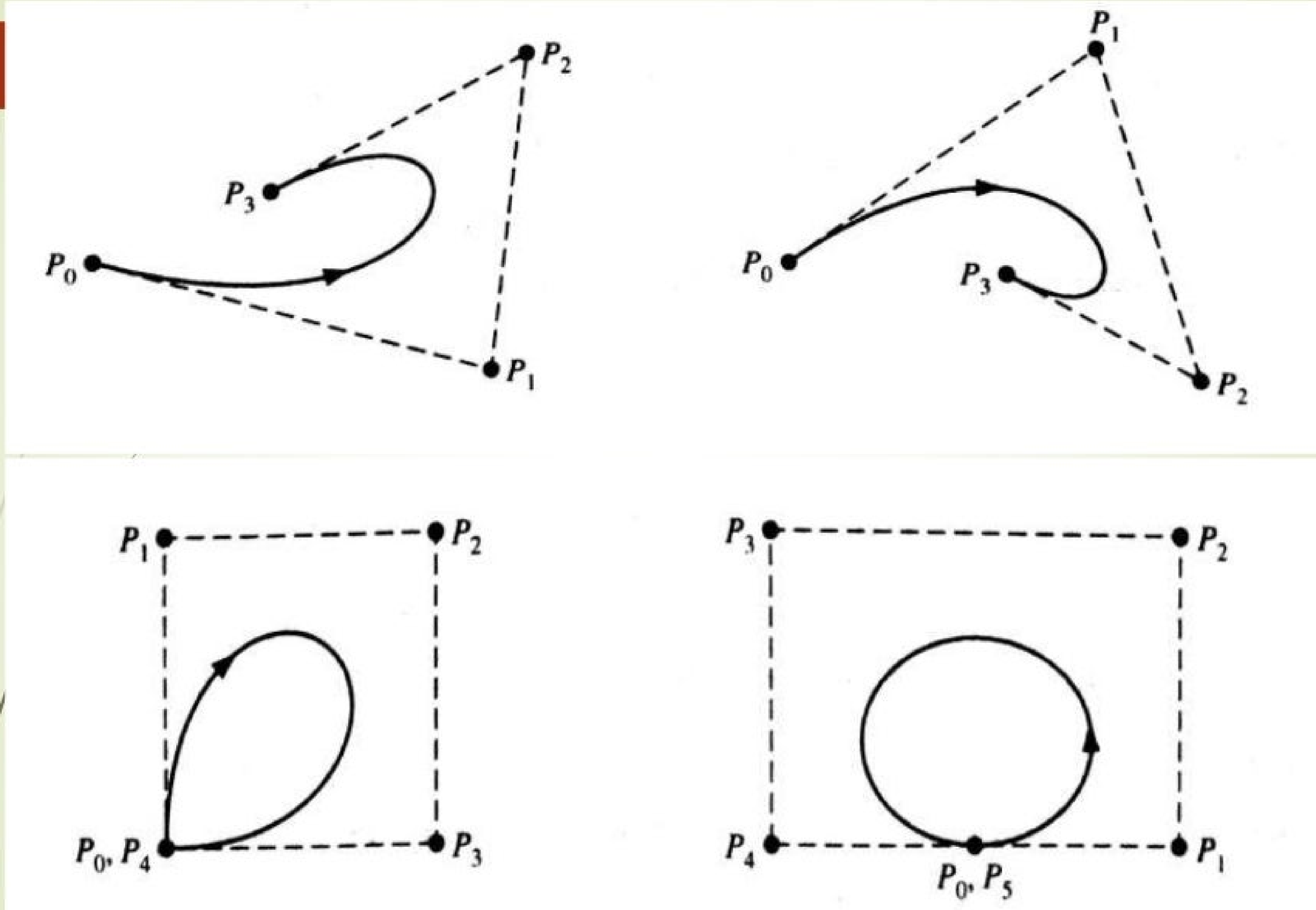
$$\mathbf{P}(u) = \mathbf{P}_0(1-u)^n + \mathbf{P}_1 C(n, 1)u(1-u)^{n-1} + \mathbf{P}_2 C(n, 2)u^2(1-u)^{n-2}$$
$$+ \cdots + \mathbf{P}_{n-1} C(n, n-1)u^{n-1}(1-u) + \mathbf{P}_n u^n, \qquad 0 \le u \le 1$$



P_1
P_2
u
P_0 $u = 0$
P_k Control points (vertices)
Characteristic polygon
$u = 1$ P_3

Bezier Curve For Various Control points

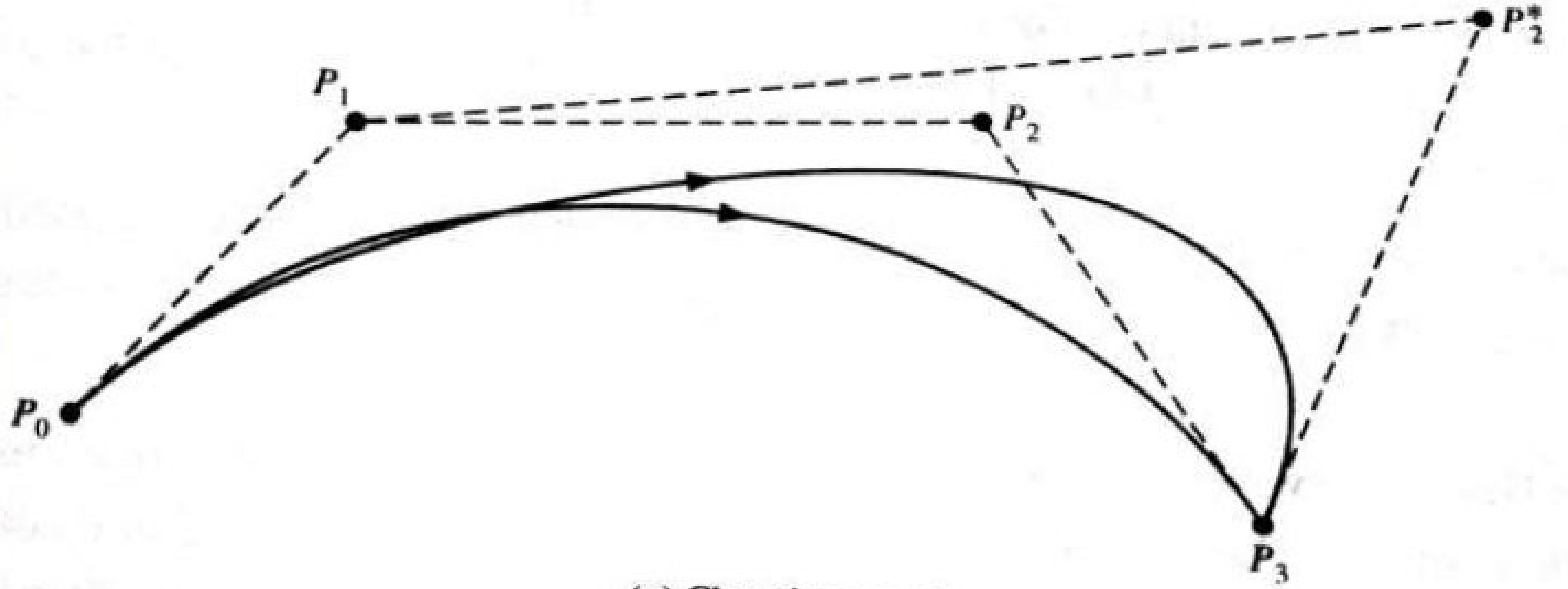



P_2
P_3
P_0
P_1
P_1
P_0
P_3
P_2
P_1
P_2
P_0, P_4
P_3
P_3
P_2
P_4
P_0, P_5
P_1

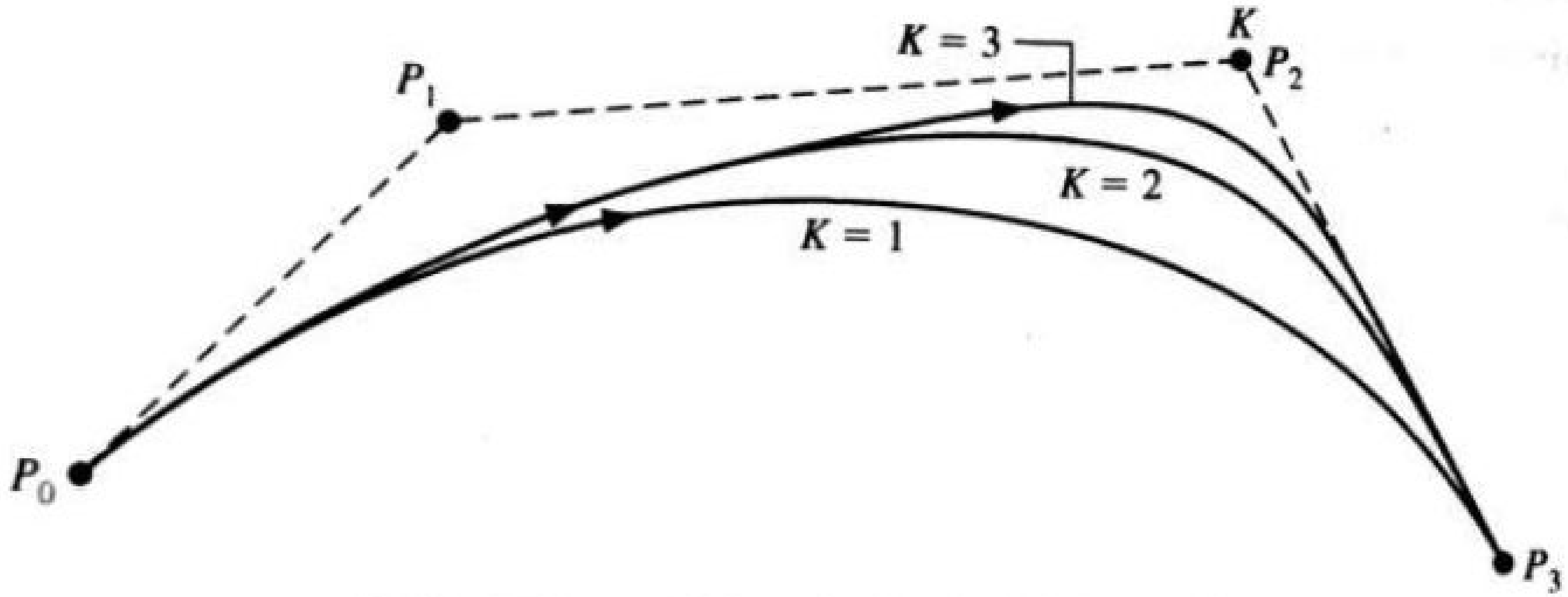
Cont…,

Characteristics of Bezier Curve

- A Bezier curve is defined on n+1 points $P_0, \ldots, P_n$ and is represented as a parametric polynomial curve of degree *n.*
- It always passes through the first and last control points.
- The Bezier curve is tangent to first and last segments of the characteristics polygon.
- The curve generally follows the shape of characteristics polygon.
- The degree of polynomial defining the curve segments is one less that the number defines the polygon points.
- Bezier curve exhibit a symmetry property.
- Each control point is weighted by its blending function for each *u* value.
- The curve lies entirely within the convex hull formed by four control points



(*a*) Changing a vertex



(*b*) Specifying multiple coincident points at a vertex

B-SPLINE CURVE

- It provide another effective method of generating curve defined polygons.
- These curves are widely used of approximation splines.

$$P(u) = \sum_{i=0}^{n} P_i \, B_{i,k} \ (u), 0 \leq u \leq \ u_{max}$$

$$B_{i,k}(u) = \frac{u - u_i}{u_{i+k-1}} B_{i,k-1}(u) + \frac{u_{i+k} - u}{u_{i+k} - u_{i+1}} B_{i+1,k-1}(u)$$

$$\text{where} \quad B_{i,1}(u) = \begin{cases} 1, & \text{if } u_i \leq u \leq u_{i+1} \\ 0, & \text{otherwise} \end{cases}$$

Characteristics of B-Spline Curve

- The local control of curve can be obtained by changing the position of control point or using multiple control points by placing several points at same location.
- A non-periodic B-spline curve passes through the first and last control points and it is tangent to first and last segment of control polygon.
- It allows us to vary the number of control points used to design a curve without changing the degree of polynomial.
- The degree of curve increases, it is more difficult to control and calculate accurately. Thus, a cubic B-spline curve is sufficient for many application.

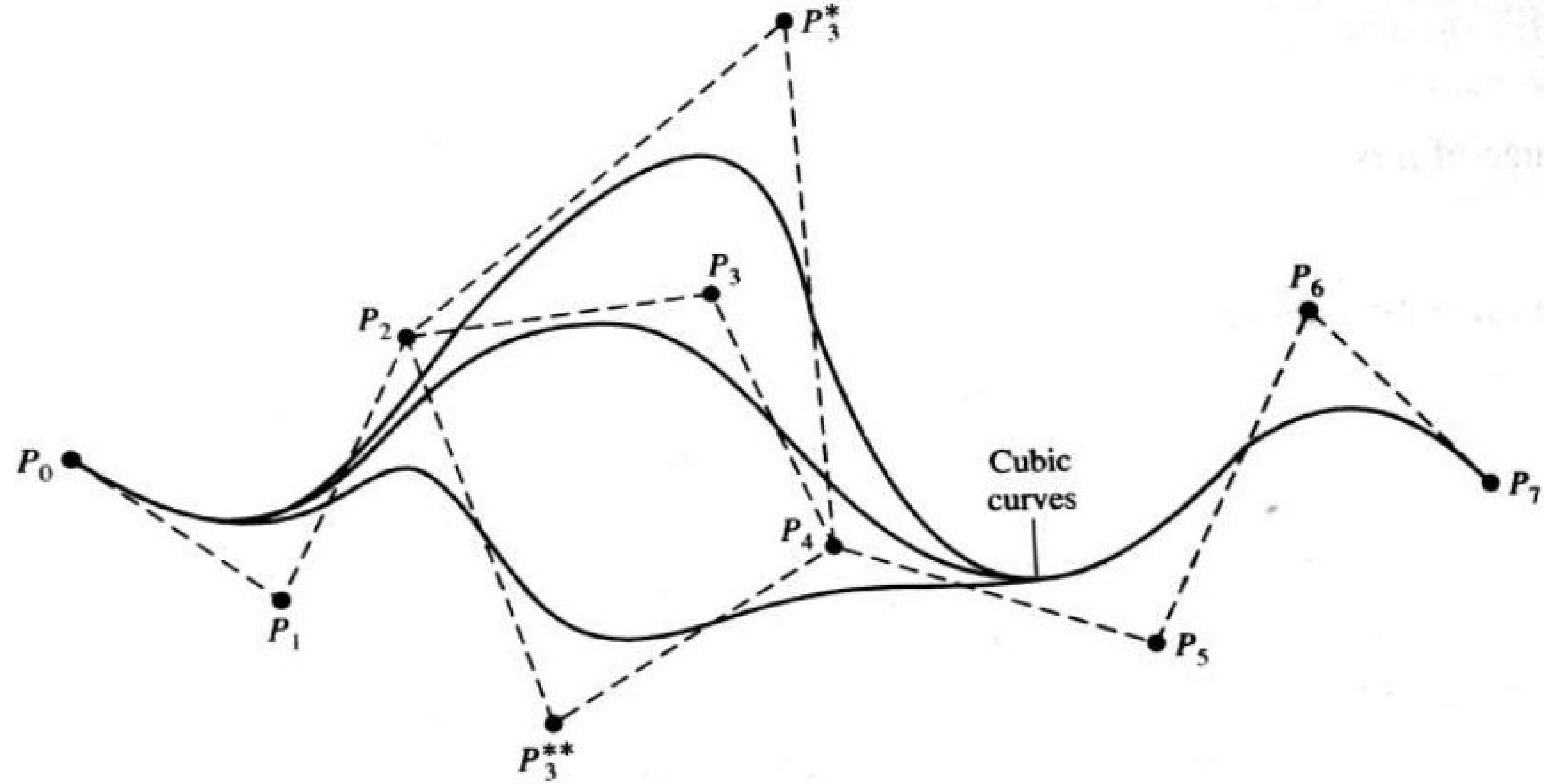


Figure 6.35 Local control of B-spline curves.

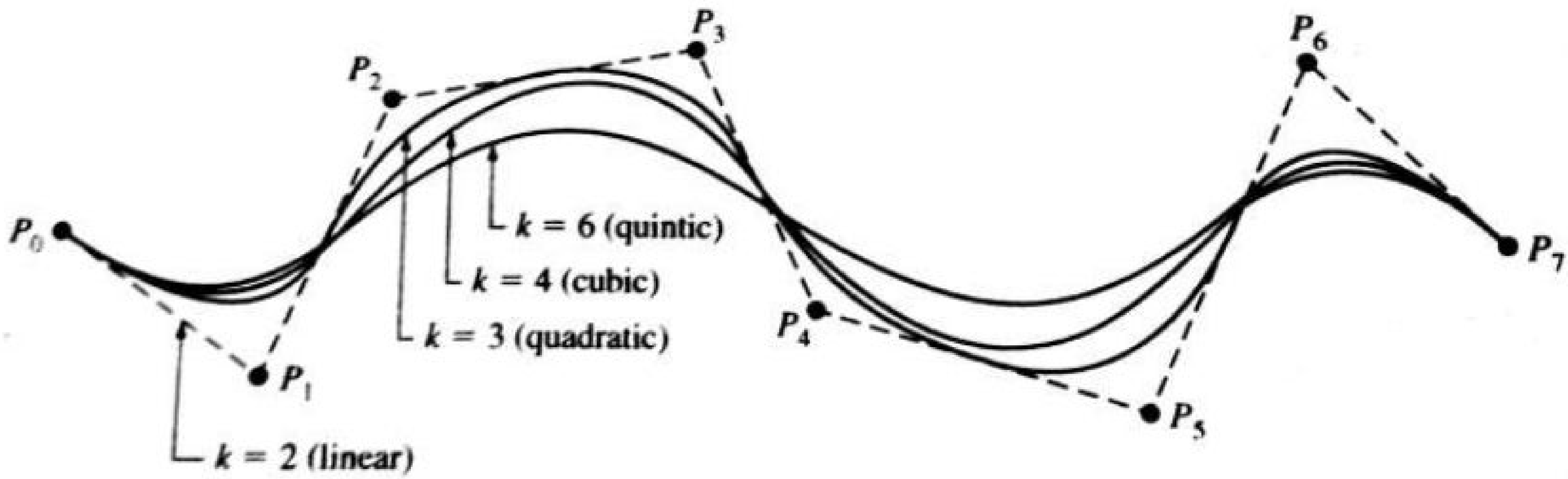


Figure 6.36 Effect of the degree of B-spline curve on its shape.

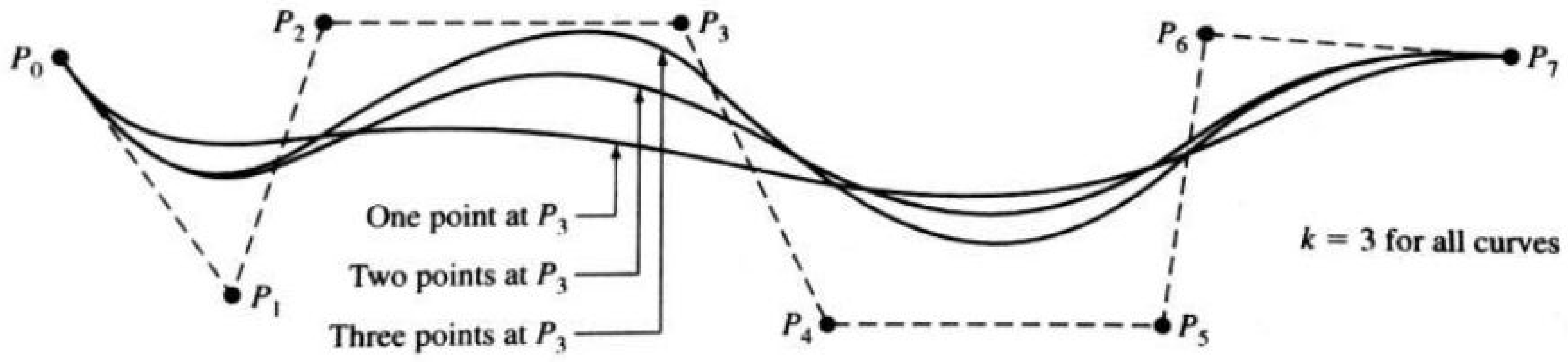
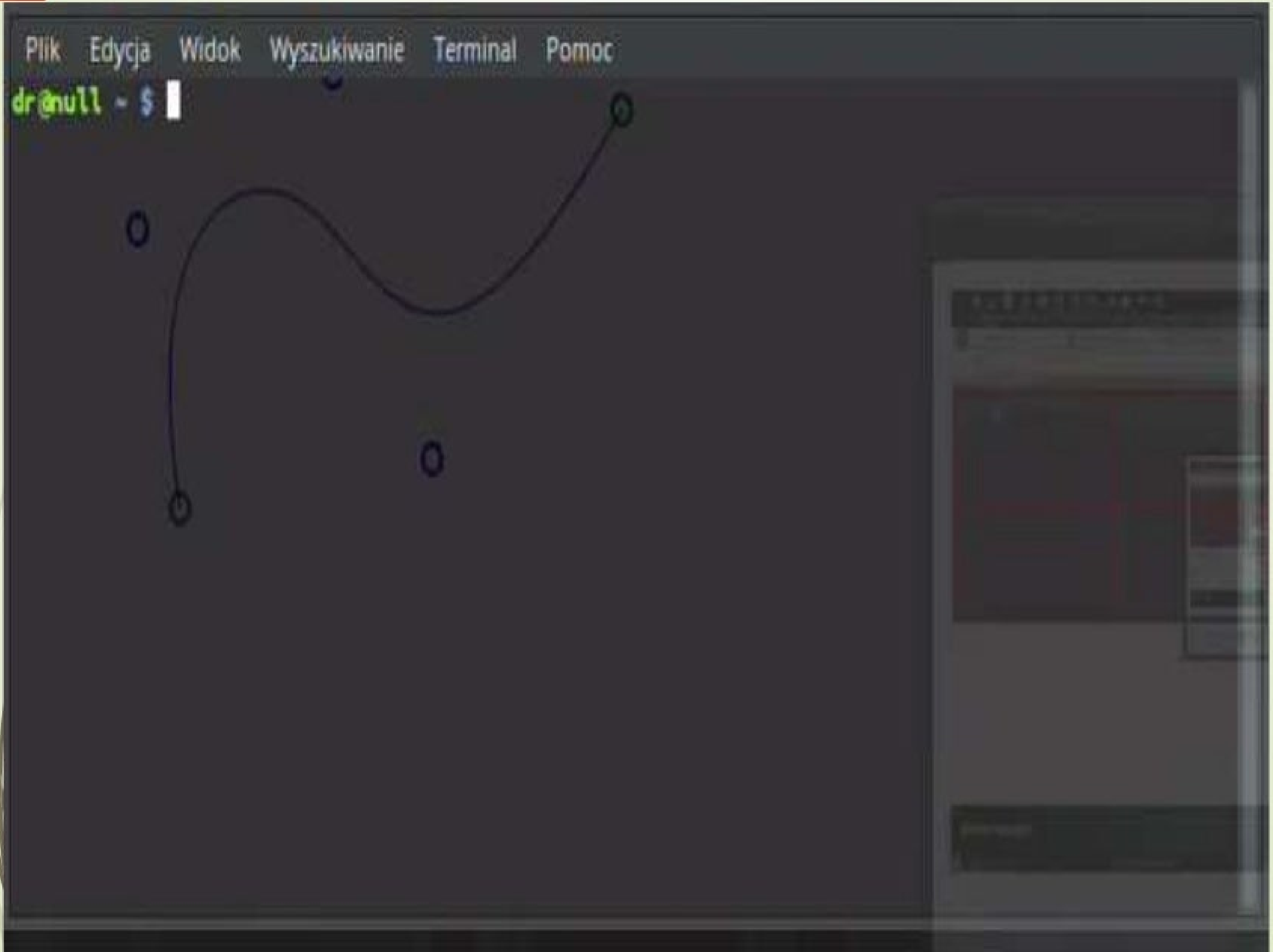


Figure 6.37 Effect of point multiplicity of B-spline curve on its shape.

Cont…,



RATIONAL CURVES

- A rational curve is defined by the algebraic ratio of two polynomials where as non-rational curve is defined by one polynomial.
- The most widely used rational curves are non-uniform rational b-splines (NURBS).
- A rational B – spline curve defined by

$$P(u) = \sum_{i=0}^{n} P_i \, B_{i,k} \, (u), 0 \leq u \leq \, u_{max}$$

- $B_{i,k}, (u)$ are the rational B – spline Basis function are given by

$$B_{i,k} \, (u) = \frac{w_i \, R_{i,k} \, (u)}{\sum_{i=0}^{n} w_i \, R_{i,k} \ \ (u)}$$

SURFACE MODELING

- The techniques of representation of objects (or) components by surface is called surface modeling.
- Objects can be clearly interpreted by the user.
- Main draw back here is that, no data is available about the interior of solid.
- Application is modeling car bodies, ships, aerospace structure, dies, etc.