CO-ORDINATE SYSTEMS

In general, there are two types of co-ordinate system

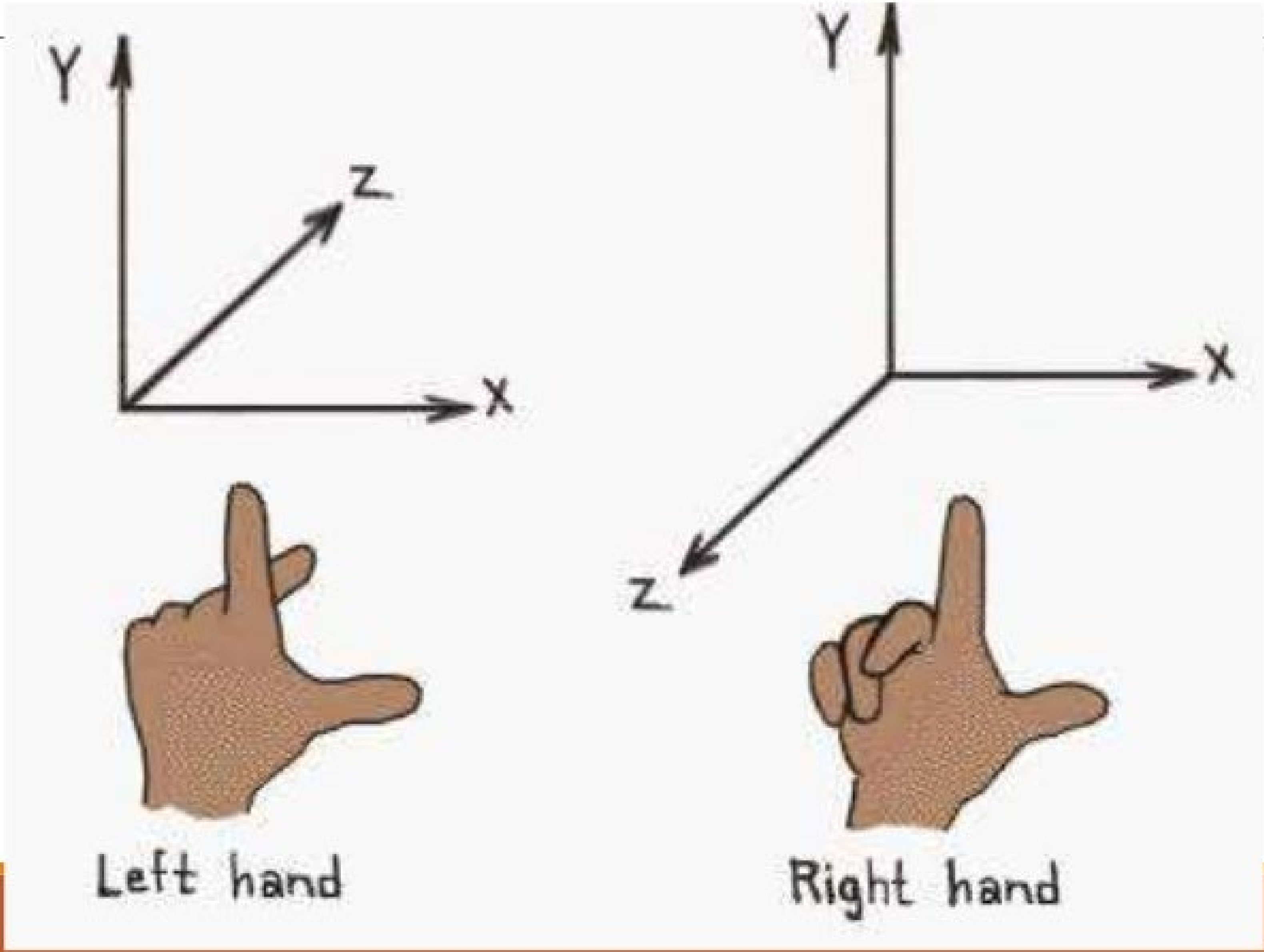
1. Cartesian – Linear distances X,Y and Z

2. Polar – Angles such as θ, α, φ

- Left hand and right hand co-ordinate systems
- Multiple co-ordinate systems

    - World co-ordinate system
    - Object co-ordinate system
    - Hierarchical co-ordinate system
    - View point co-ordinate system
    - Model view co-ordinate system
    - Screen co-ordinate system
    - View port co-ordinate system

Left and right handed coordinate system



2D & 3D (GEOMETRIC)TRANSFORMATION

- A **geometric transformation** is an operation that modifies its shape, size, position, orientation etc. with respect to its current configuration operating on the vertices.
- Some of the important transformations are
  - Translation
  - Scaling
  - Rotation
  - Reflection
  - Shear

2D & 3D TRASLATION

2D FORMULA

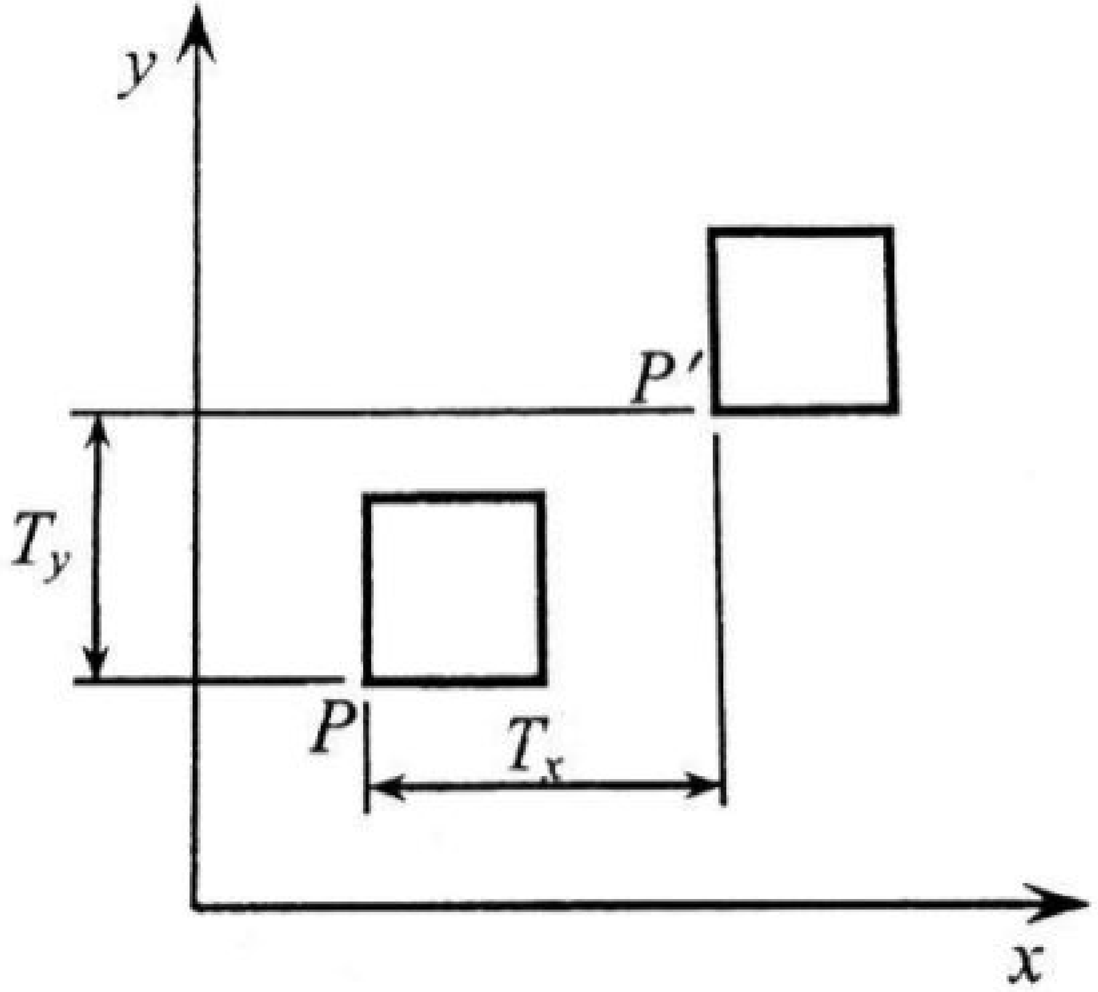
$$x' = x + t_x, \quad y' = y + t_y$$

$$\mathbf{P} = \begin{bmatrix} x \\ y \end{bmatrix}, \quad \mathbf{P}' = \begin{bmatrix} x' \\ y' \end{bmatrix}, \quad \mathbf{T} = \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

$$\mathbf{P}' = \mathbf{P}.\mathbf{T}$$

3D FORMULA

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



The new co-ordinate after transformation is given by the following equation.

$$P' = [X', Y']$$

$$X' = X + T_x$$

$$Y' = Y + T_y$$

$$\therefore \quad P' = [X + T_x, Y + T_y] = [X \;\; Y] + [T_x \;\; T_y]$$

In matrix form, we can write the above equation as

$$[P'] = [X' \; Y' \; 1] = [X \; Y \; 1]\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ T_x & T_y & 1 \end{bmatrix}$$

$$P' = P \, . \, T$$

where T = Translation matrix.

It is normally the operation used in the CAD system as MOVE command.

2D & 3D SCALING

2D FORMULA

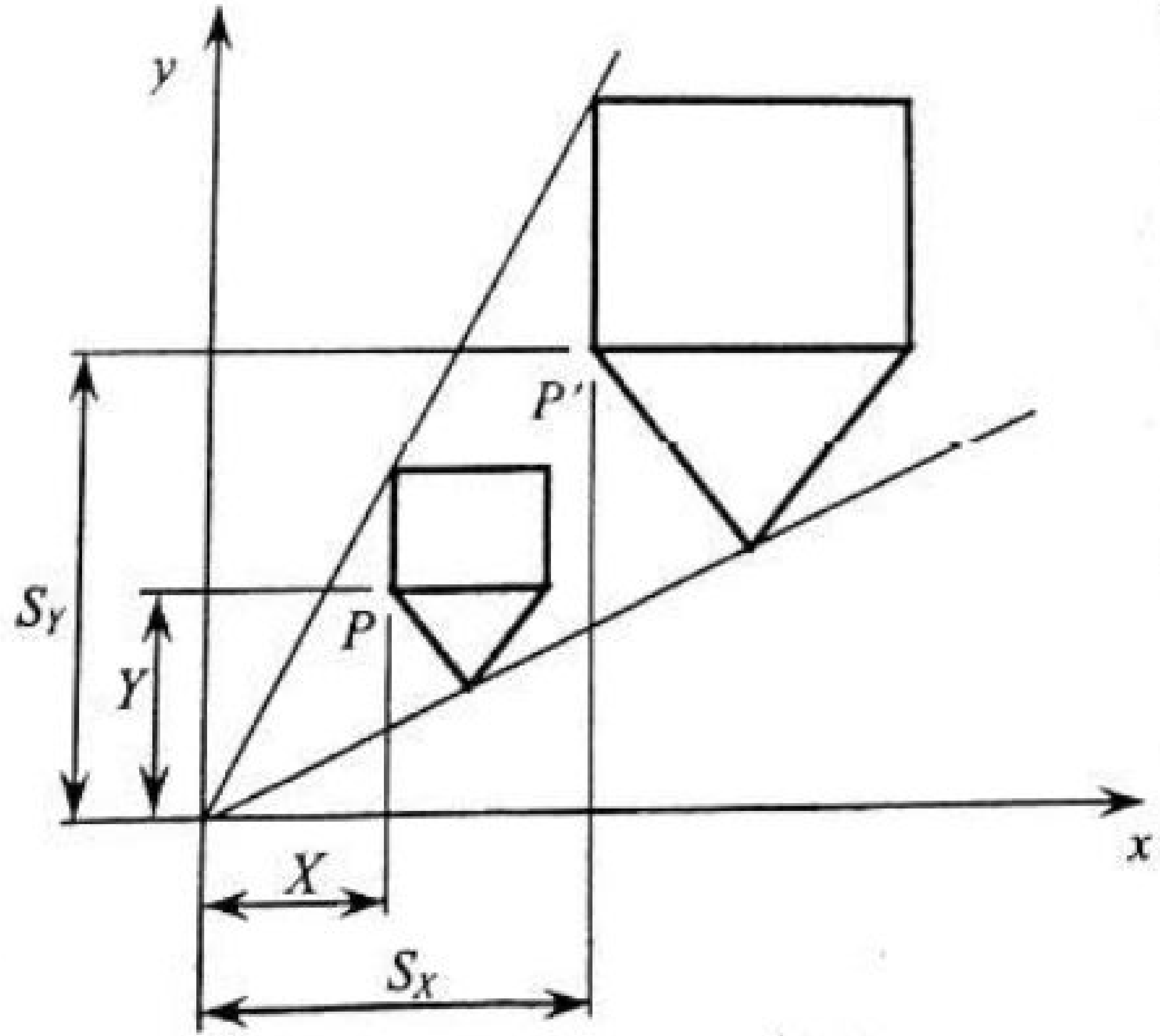
$$x' = x \cdot s_x, \quad y' = y \cdot s_y$$

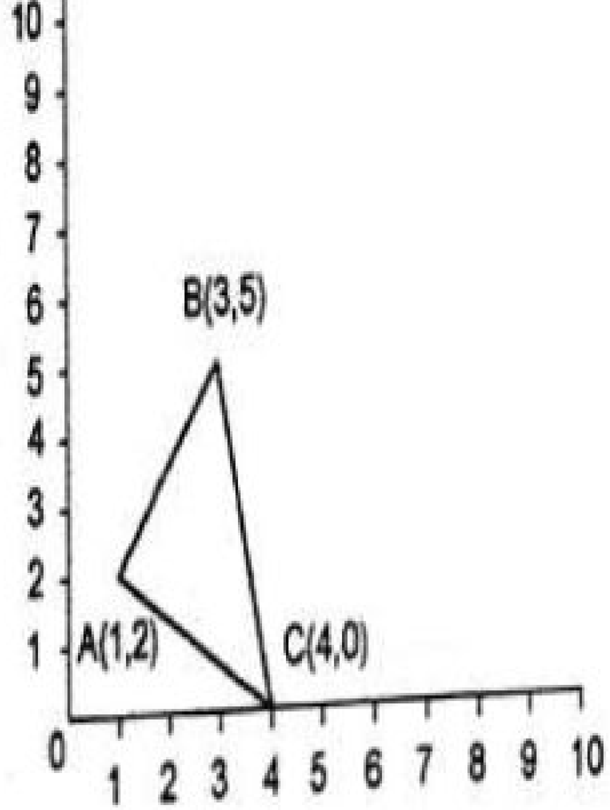
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

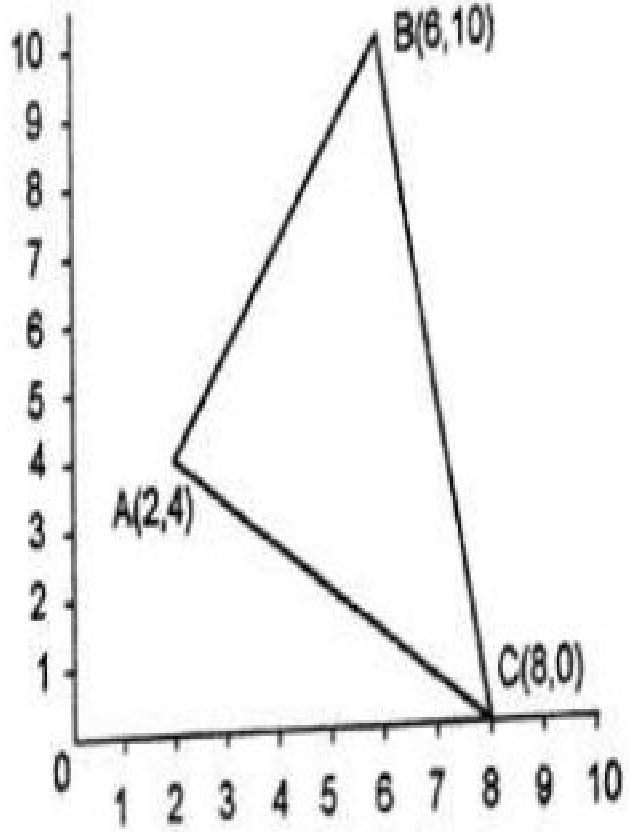
$$\mathbf{P}' = \mathbf{S} \cdot \mathbf{P}$$

3D FORMULA

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$


y
x
P'
P
S_Y
Y
X
S_X


B(3,5)
A(1,2)
C(4,0)
0 1 2 3 4 5 6 7 8 9 10


B(6,10)
A(2,4)
C(8,0)
0 1 2 3 4 5 6 7 8 9 10

2D & 3D ROTATION

2D FORMULA

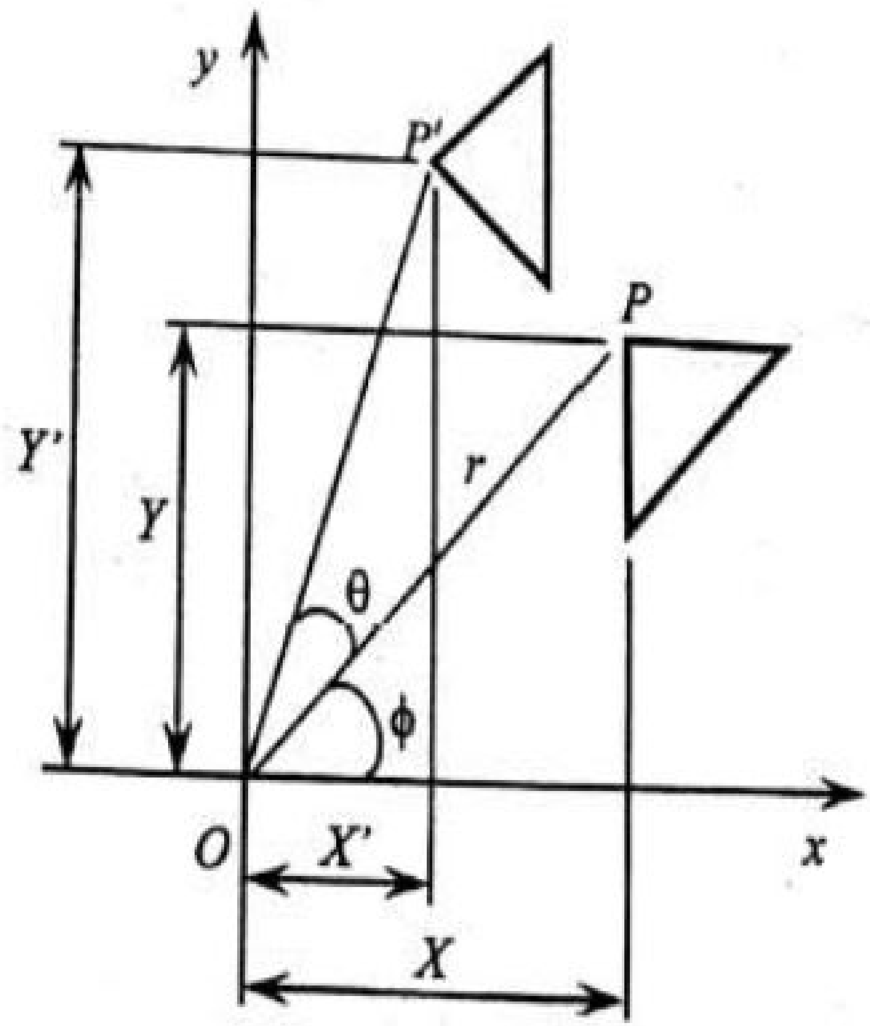
$$[P'] = \begin{bmatrix} X' \\ Y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}$$

3D FORMULA

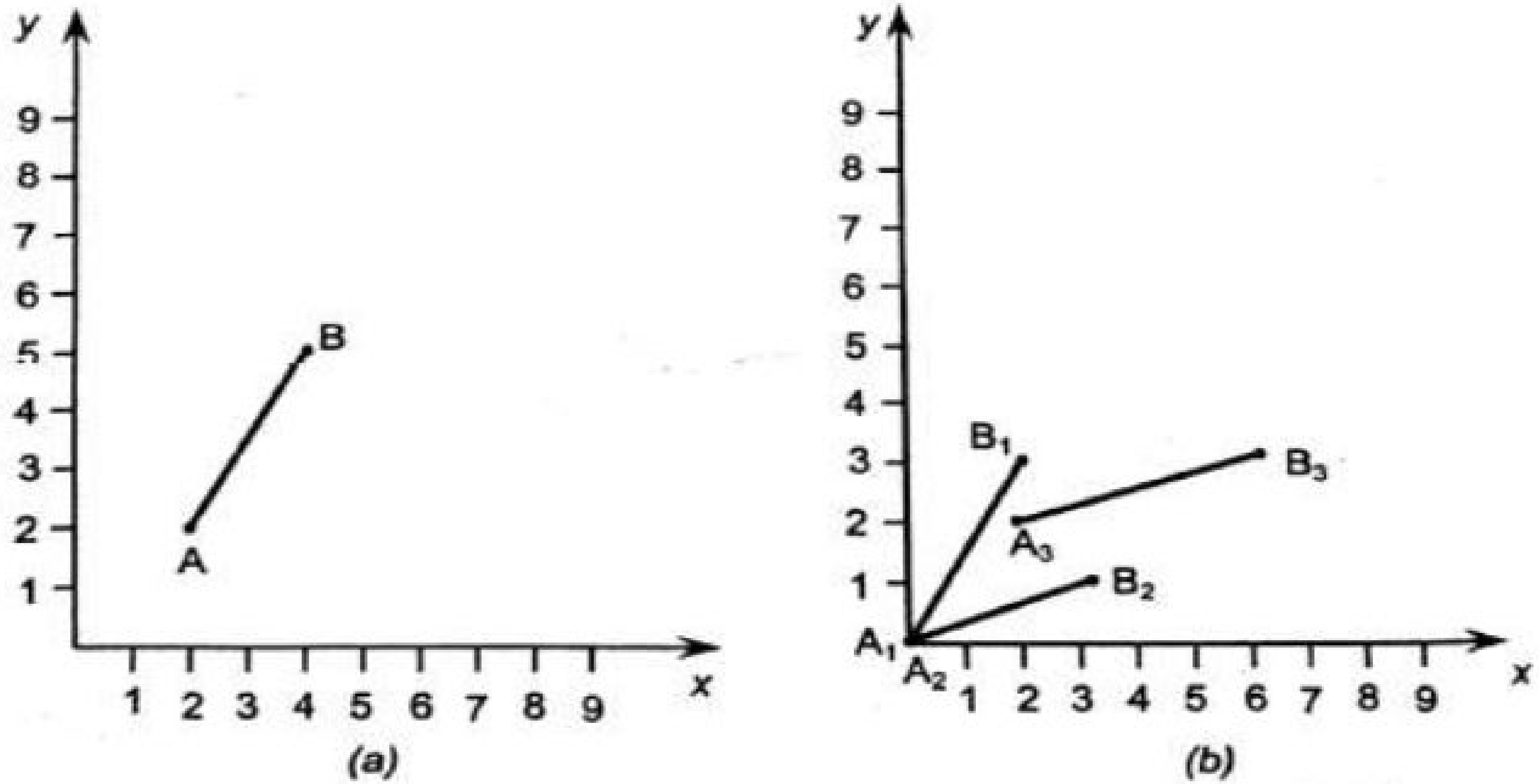
$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$


y
P'
P
Y'
Y
r
θ
ϕ
O
X'
X
x

CONCATENATION OR COMBINED TRANSFORMATION



HOMOGENEOUS TRANSFORMATION

- The three dimensional representation of two dimensional plane is called as homogeneous representation and the transformation using homogeneous representation is called homogeneous transformation.

HOMOGENEOUS TRANSFORMATION

(i) Translation:

The homogeneous coordinates for translation are given by

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ T_x & T_y & 1 \end{bmatrix}$$

∴ It can be again written as

$$\begin{bmatrix} X_1 & Y_1 & 1 \end{bmatrix} = \begin{bmatrix} X & Y & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ T_x & T_y & 1 \end{bmatrix}$$

(ii) Rotation:

The homogeneous coordinates for rotation are given by

$$R = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

∴ It can be again written as

$$\begin{bmatrix} X_1 & Y_1 & 1 \end{bmatrix} = \begin{bmatrix} X & Y & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(iii) Scaling:

The homogeneous coordinates for scaling are given by

$$S = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

∴ It can be again written as

$$[X_1 \ Y_1 \ 1] = [X \ Y \ 1] \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(iv) Shearing:

$$X\text{-shear} \quad [X_1 \ Y_1 \ 1] = [X \ Y \ 1] \begin{bmatrix} 1 & 0 & 0 \\ S_{hx} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Y\text{-shear} \quad [X_1 \ Y_1 \ 1] = [X \ Y \ 1] \begin{bmatrix} 1 & S_{hy} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

LINE DRAWING

- Straight line segments are used a great deal in computer generated pictures. The following criteria have been stipulated for line drawing displays:

    i. Lines should appear straight
    ii. Lines should terminate accurately
    iii. Lines should have constant density
    iv. Line density should be independent of length and angle
    v. Line should be drawn rapidly

DDA ALGORITHM

- A popular method uses an algorithm known as "*Digital Differential Analyzer*" which generates lines from their differential equations.
- For a line segment joining two points P1 and P2, a parametric representation is given by

$$P(u) = P1+(P2-P1)u$$

- In terms of x(u) and y(u)

$$x(u) = x1 + (x2-x1)u$$

$$y(u) = y1 + (y2-y1)u$$

BRESENHAM'S LINE ALGORITHM

- The basic principle of Bresenham's line algorithms is to select the optimum raster location to represents a straight line.
- To achieve optimum raster scan, the algorithm always increments either *x* or *y* by one unit depending on the slope line.
- The increment in the other variable is determined by examining the distance between the actual line location and the nearest pixel.
- This distance is called *decision variable* or *error.*

CLIPPING

If any part of the geometry is not inside the window, it is made invisible by the graphic software is called clipping.

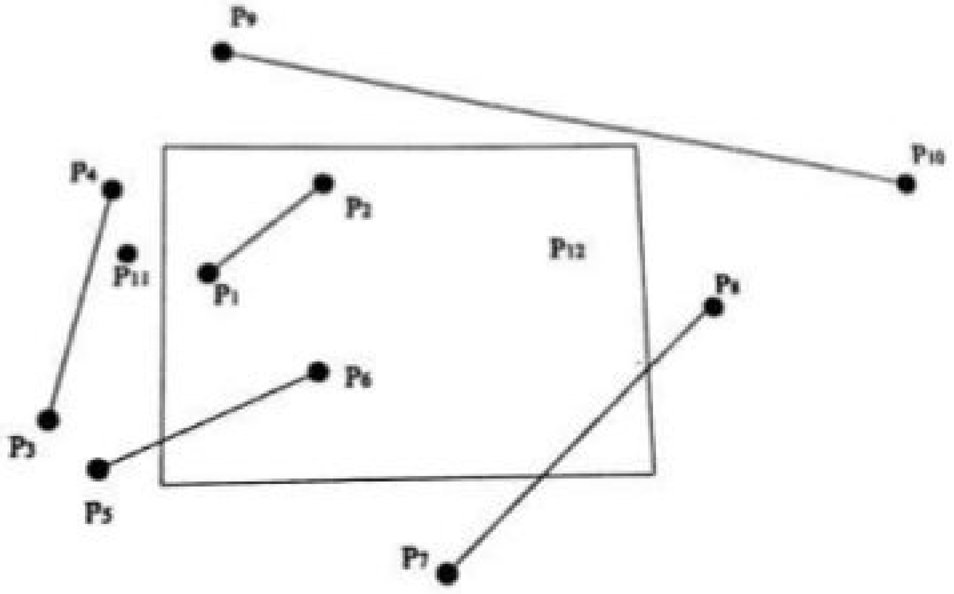


Figure 1.69 (a) Before Clipping

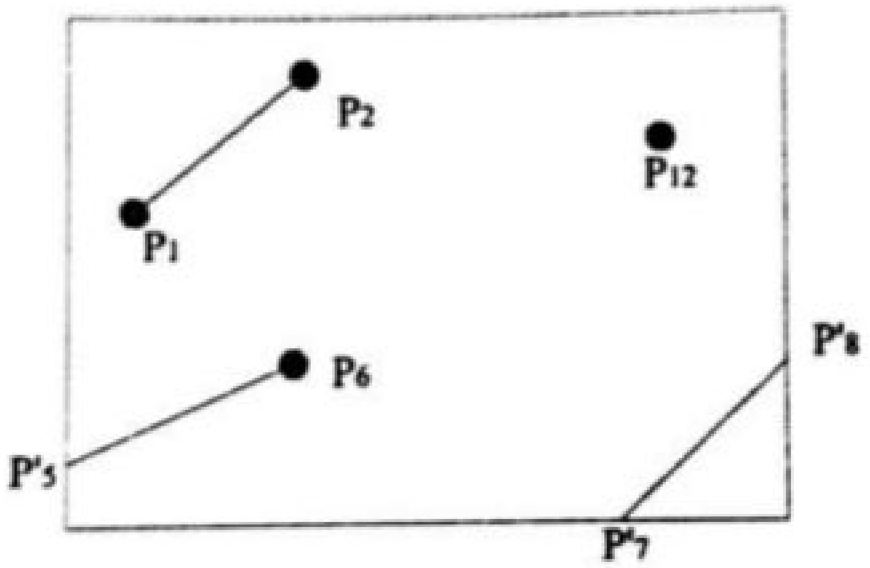


Figure 1.69 (b) After Clipping

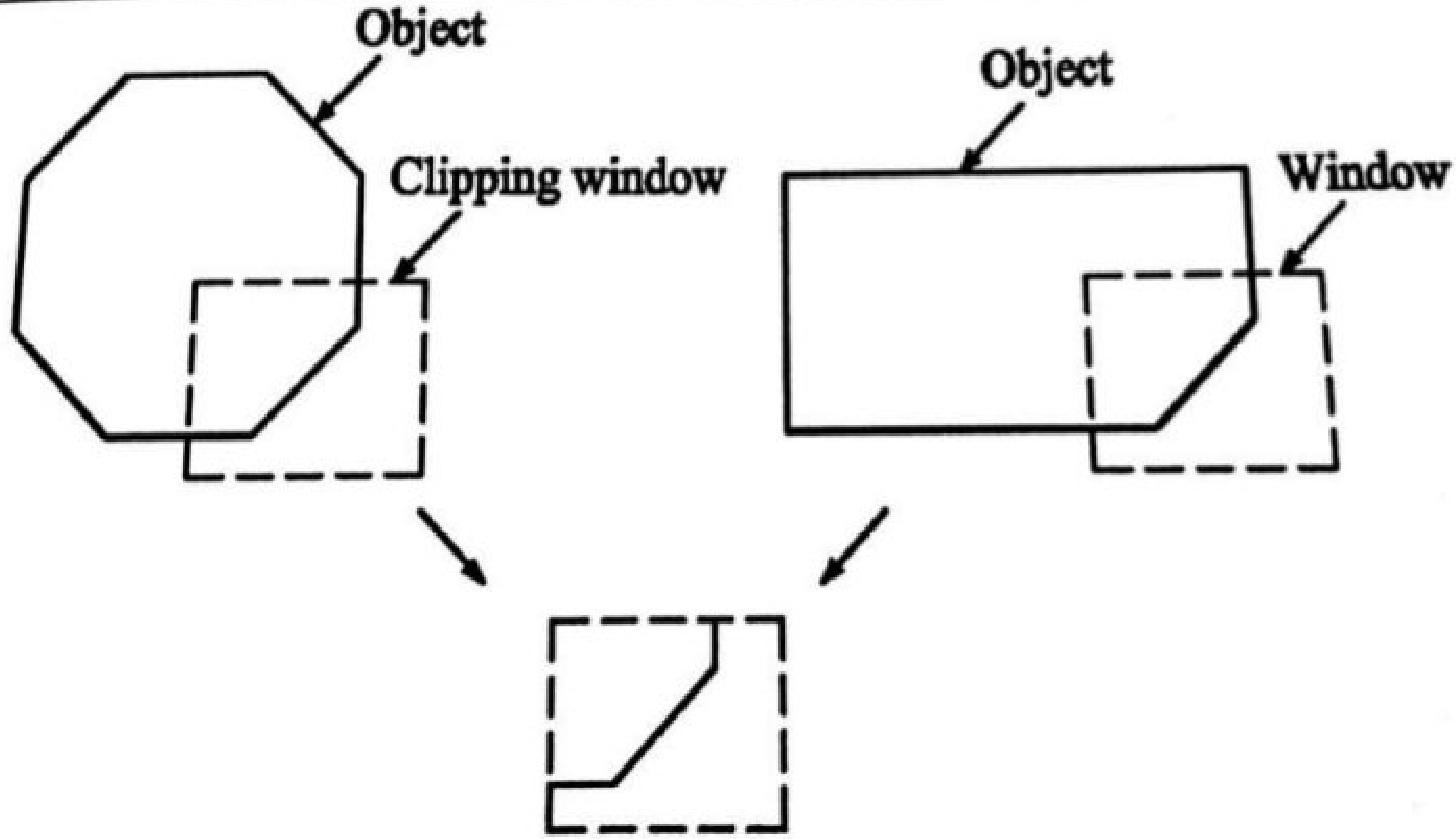
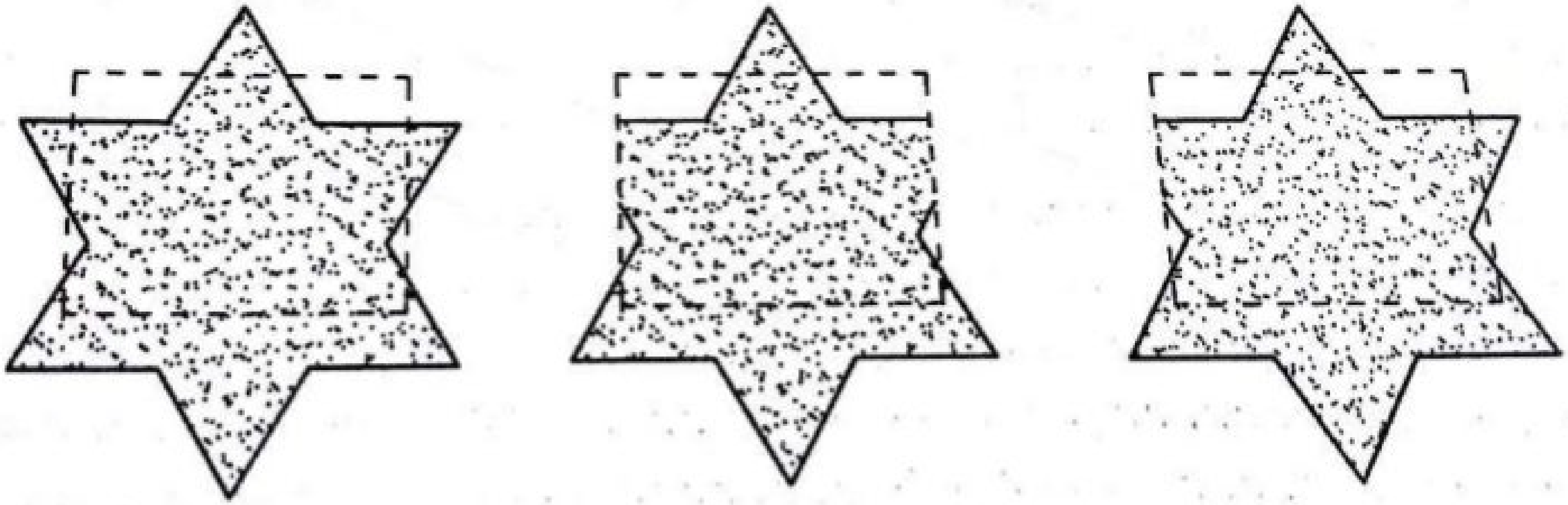


Figure 1.71



(a) Before clipping (b) Clip left (c) Clip right

VIEWING TRANSFORMATIONS

- Displaying an image of a picture involves in mapping the co-ordinates of the picture into the appropriate coordinates on the device where the image is to be displayed.

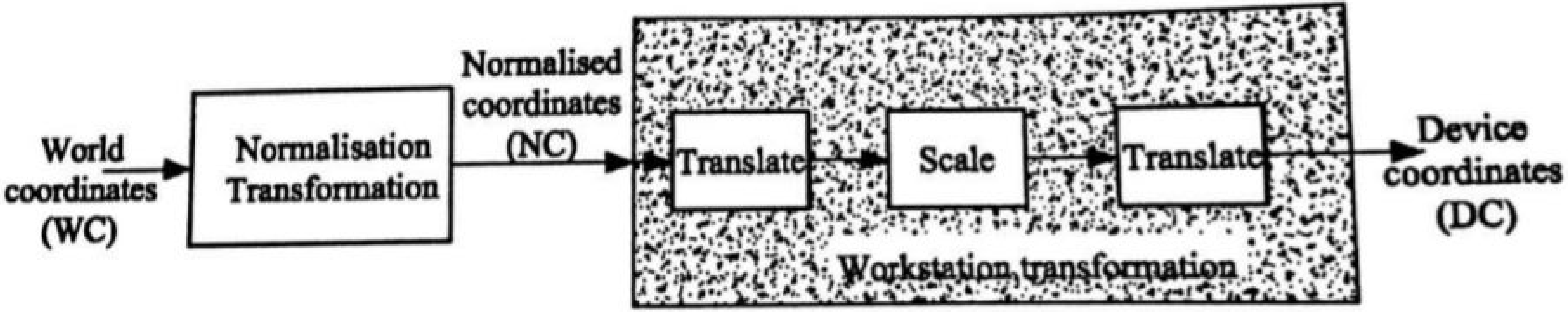


Figure 1.78 Viewing transformation process