22/1/24. POWER SYSTEM ANALYSIS

UNIT-I

REPRESENTATION OF POWER SYSTEM

System: It is a collection of many sub systems (or) elements which are interconnected in a specific manner.

Power System: Which delivers electric power

Sub systems: Generation, Transmission & distribution.

Need for system analysis in planning & operation of power system:-

* To maintain consumer voltage at satiuary levels as affected by conductor sizing & transformer.
* Fault calculations to ensure that the max. fault current can be interrupted by circuit breakers or fuses. So that large fault
* Design of protection systems to ensure faulty circuits are switched off rapidly to prevent damage.
* system design and control to maintain frequency
* To ensure sufficient generation is available to meet the expected demand - load forecasting.
* To ensure that loads are sufficient by the most efficient arrangement of generators - load Scheduling.

Per Unit Value

The per unit value of any quantity is defined as the ratio of the actual value of the quantity to the base value.

$$\text{p.u value} = \frac{\text{Actual Value}}{\text{Base Value}}$$

$$\text{\% p.u. Value} = \frac{\text{Actual Value}}{\text{Base Value}} \times 100$$

Advantages of p.u Computations

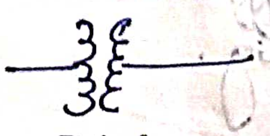
* Manufacturers usually specify the impedance of a device or machine in percent or per unit on the base of the name plate rating.
* The p.u impedances of machines of the same type & widely different rating usually lie within a narrow range.
* The p.u impedance of circuit element connected by transformers expressed on a proper base will be same.

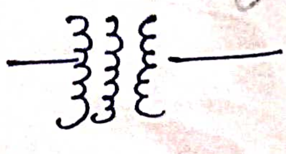
Single Line diagram

In single line representation of power system, the components of the system are represented by standard symbols and the tr. lines are represented by straight lines.

Symbols used in Single line diagram

Machine / rotating armature 

Two winding transformer 

Three winding " 

Fuse

current transformer

Potential "

Power circuit breaker

Air circuit breaker

3ϕ, 3wire delta connection Δ

3ϕ, star, neutral ungrounded Y

3ϕ, star, neutral grounded

Ammeter & voltmeter (A) (V)

MVA To KVA

1 KVA = 1000 V.

1000 KVA = 1 MVA

To convert MVA to kVA, divide the kVA by 1000, to → to MVA.

Problems.

1) A 3ϕ generator with rating 1000 KVA, 33 kV has its armature resistance and synchronous reactance as 20Ω/phase and 70Ω/phase. Calculate p.u. impedance of the generator.

To find – p.u. impedance. Z_{pu}.

Formula – $Z_{pu} = \frac{\text{Actual impedance}}{\text{Base impedance}} = \frac{Z}{Z_b}$

$Z = (R + jX)\ \Omega/\text{phase}$ $\quad Z_b = \frac{(kV_b)^2}{MVA_b}$.

Base impedance per phase $\Big]$ $Z_b = \dfrac{(kV_b)^2}{MVA_b} = \dfrac{(33)^2}{1000/1000}$

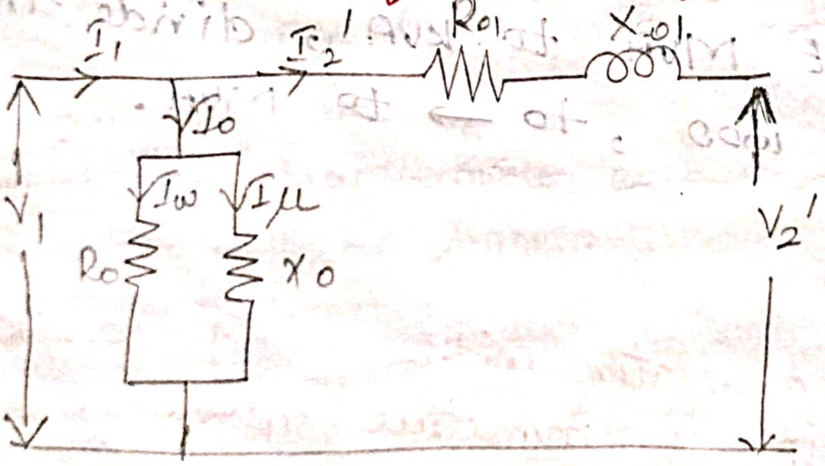
$= 1089\ \Omega$

Actual impedance per phase $\Big]$ $Z = (20 + j70)\ \Omega/\text{phase}$

$$Z_{pu} = \frac{\text{Actual impedance}}{\text{Base impedance}} = \frac{20+j70}{1089}$$

$$\boxed{Z_{pu} = 0.018 + j0.064\ \text{p.u}}$$

Equivalent Circuit of a Transformer



Transformation ratio $K = \dfrac{E_2}{E_1} = \dfrac{N_2}{N_1} = \dfrac{V_2}{V_1} = \dfrac{I_1}{I_2}$

$R_{01} = R_1 + R_2' = R_1 + R_2/K^2$

$X_{01} = X_1 + X_2' = X_1 + X_2/K^2$

$X_{02} = X_1' + X_2 = K^2 X_1 + X_2$

2. A 3ϕ Δ - Y transformer with rating 100 kVA, 11 kV / 400 V has its primary & 2° leakage reactance as 12 Ω/phase & 0.05 Ω/phase respectively. Calculate the p.u. reactance of transformer.

Solution:

case i) - 1° ratings are choosen as base values

case ii) - 2° " " " " "

case (i) To find - X_{pu}. p.u reactance of txfr.

Formula :-

$$X_{pu} = \frac{\text{Total leakage reactance}}{\text{Base impedance}}$$

Total leakage reactance $\Rightarrow X_{01} = X_1 + X_2' = X_1 + \frac{X_2}{k^2}$

Base impedance $Z_b = \frac{(kV_b)^2}{MVA_b}$

Solution

$kV_b = 11$ kV

$KVA_b = 100$ KVA

Base impedance $Z_b = \frac{(11)^2}{100/1000} = 1210\ \Omega$ //

$k = \frac{400}{11,000} = 0.0364$ //

$X_{01} = X_1 + X_2' = X_1 + \frac{X_2}{k^2} = 12 + \frac{0.05}{(0.0364)^2}$

$= 12 + 37.7 = 49.737\ \Omega/\text{phase}$

$$X_{pu} = \frac{X_{01}}{Z_b} = \frac{49.737}{1210} = 0.0411 \text{ p.u.}$$

Case ii

2^0 rating.

Base kilo volt $kV_b = 400/1000 = 0.4 \text{ kV}$

$kVA_b = 100 \text{ kVA}$

Base impedance /phase $Z_b = \frac{(0.4)^2}{100/1000} = 1.6\ \Omega$

$$k = \frac{400}{11000} = 0.0364$$

Total leakage reactance $X_{02} = X_1' + X_2 = k^2 X_1 + X_2$

$$= (0.0364)^2 \times 12 + 0.05 = 0.0659\ \Omega/\text{phase}$$

$$X_{pu} = \frac{X_{02}}{Z_b} = \frac{0.0659}{1.6} = 0.0411 \text{ p.u.}$$

③ A 3ϕ Y−Δ transformer is constructed using 3 identical single phase transformers of rating 200 kVA, 63.51 kV / 11 kV transformer. The impedances of 1^0 & 2^0 are $20 + j45\ \Omega$ & $0.1 + j0.2\ \Omega$ respectively. Calculate the p.u impedance of the transformer.

Solution

KVA rating of a 3ϕ transformer $= 3 \times 200$
$= 600$ KVA

Line voltage rating of $Y - \Delta$ transformer
$= 63.51 \times \sqrt{3}$ kV / 11 kV
$= 110$ kV / 11 kV.
1° 2°.

case i).

1° ratings are choosen

Base $KV_b = 110$ kV
$KVA_b = 600$ KVA

Formulas

$$Z_{pu} = \frac{\text{Total impedance}}{\text{Base impedance}}$$

$$Z_{01} = Z_1 + Z_2' = Z_1 + \frac{Z_2}{k^2}$$

$$Z_{02} = Z_1' + Z_2 = k^2 Z_1 + Z_2$$

$$Z_b = \frac{(kV_b)^2}{MVA_b} = \frac{(110)^2}{600/1000} = 20166.7\ \Omega$$

$$k = \frac{11}{110} = 0.1$$

$$Z_{01} = Z_1 + Z_2' = Z_1 + \frac{Z_2}{k^2} = 20+j45 + \frac{0.1+j0.2}{(0.1)^2}$$

$$= 20 + j45 + 10 + j20$$

$$= 30 + j65\ \Omega/\text{phase}$$

$$Z_{pu} = \frac{30 + j65}{20166.7} \doteq 0.0015 + j0.0032 \text{ p.u.}$$

Case ii:

$$kV_b = 400/1000 = 0.4 \text{ kV}$$

$$kVA_b = 100 \text{ kVA}$$

$$Z_b = 1.6\,\Omega$$

$$k = 0.0364.$$

Repeat

$$X_{02} = X_1' + X_2 = k^2 X_1 + X_2$$

$$= 0.0659\ \Omega/\text{phase}$$

$$X_{pu} = \frac{0.0659}{1.6} = 0.0411 \text{ p.u.}$$

(7) Equivalent Resistance:

$$\text{Total copper loss} = I_1^2 R_1 + I_2^2 R_2$$

$$= I_1^2 \left[R_1 + \frac{I_2^2}{I_1^2} R_2 \right]$$

$$= I_1^2 \left[R_1 + \frac{1}{k^2} R_2 \right]$$

$$\frac{I_2}{I_1} = \frac{1}{k} \quad //.$$

$\frac{R_2}{k^2}$ is the resistance value of R_2 shifted to 1° side.

$$R_2' = \frac{R_2}{k^2} \quad //.$$

Total resistance referred to 1°

$$R_{1e} = R_1 + R_2' = R_1 + \frac{R_2}{k^2}$$ //.

Total copper loss $= I_1^2 R_1 + I_2^2 R_2$

$$= I_2^2 \left[\frac{I_1^2}{I_2^2} R_1 + R_2 \right]$$

$$= I_2^2 \left[k^2 R_1 + R_2 \right]$$ //

ℓ.

$$R_1' = k^2 R_1$$

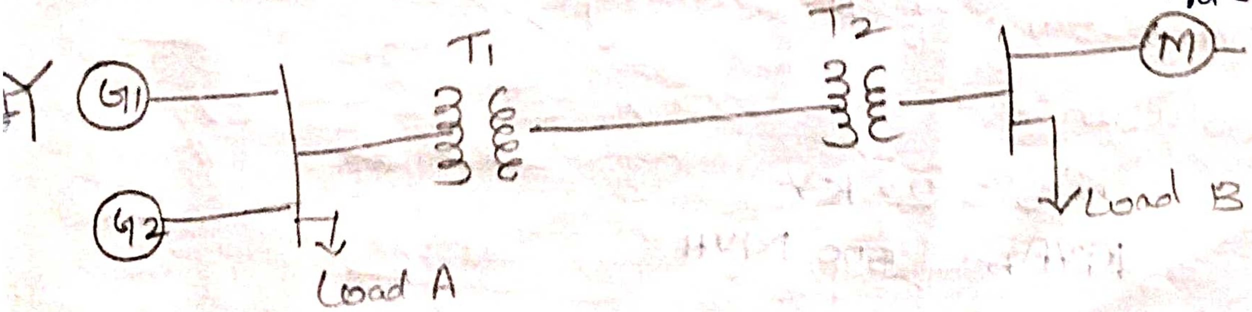
IIIly for impedance & reactance.

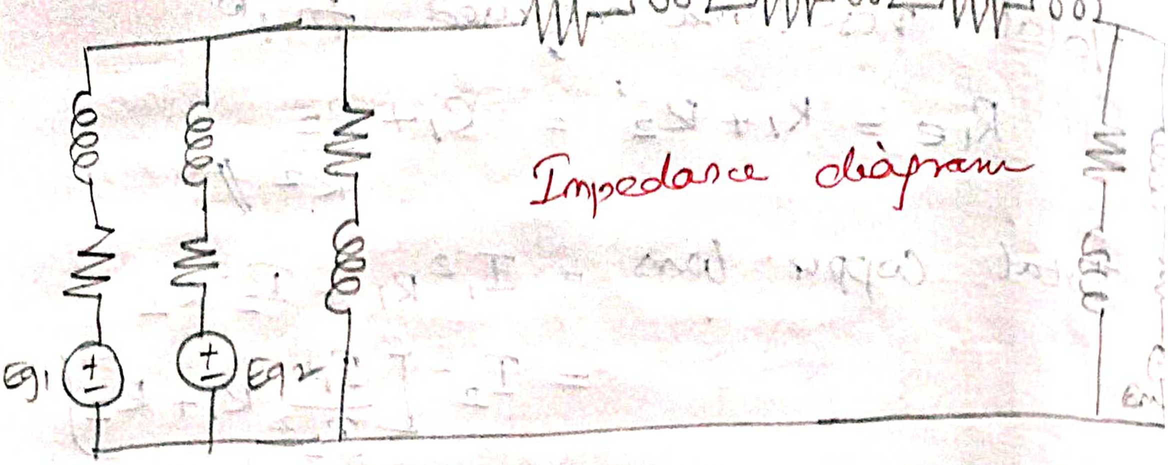
Procedure to form reactance diagram from single line diagram:

* Select a base kilovoltampere (or) megavolt-ampere. The kVA_b or MVA_b will be same for all sections of the power system.

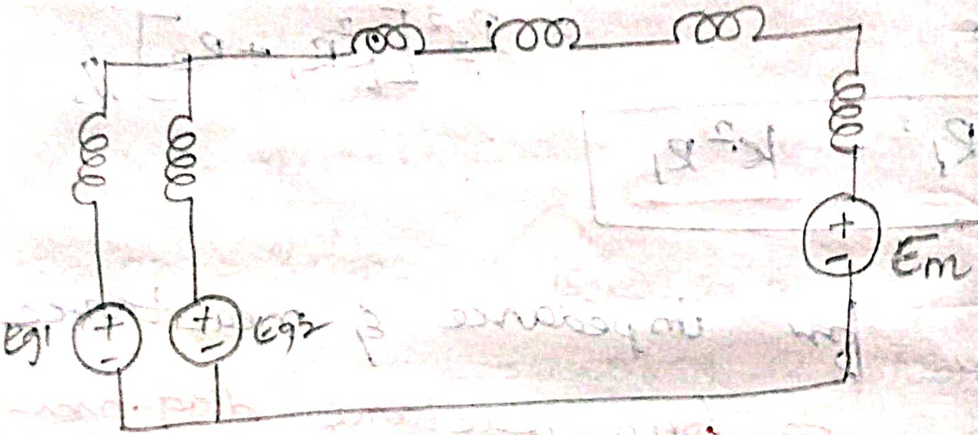
* kV_b on LT session = kV_b on HT × $\frac{\text{LT Voltage}}{\text{HT Voltage rating}}$.

kV_b on HT session = kV_b on LT × $\frac{\text{HT Voltage}}{\text{LT Voltage rating}}$.





Impedance diagram



Reactance diagram.

* p.u. reactance = $\frac{\text{Actual reactance}}{\text{Base impedance}}$

$$X_{pu,new} = X_{pu\,old} \times \left(\frac{KV_{b\,old}}{KV_{b\,new}}\right)^2 \times \left(\frac{MVA_{b\,new}}{MVA_{b\,old}}\right).$$

(4) A generator is rated 500 MVA, 22KV. Its Y connected winding has a reactance of 1.1 pu. Find the ohmic value of the reactance of the winding.

Solution:-

$KV_b = 22\,KV$

$MVA_b = 500\,MVA$

actual reactance $X = ?$

p.u. reactance $X_{pu} = \frac{\text{Actual reactance}}{\text{Base reactance}}$

$$X_{pu} = \frac{X}{Z_b}.$$

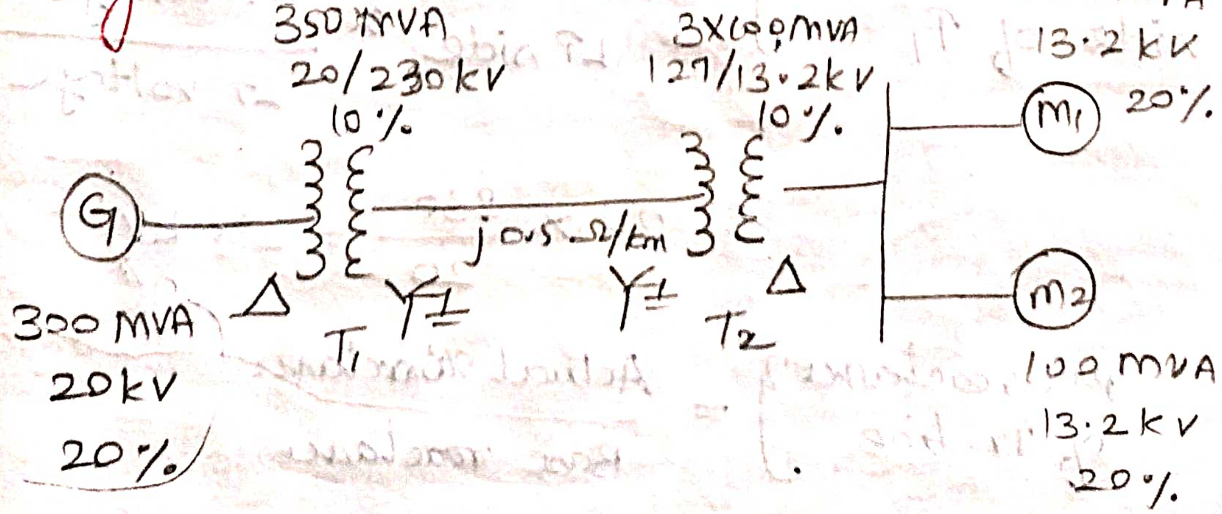
$$X = Z_b \cdot X_{pu}$$

$$Z_b = \frac{(22)^2}{500} = 0.968\ \Omega$$

$$X = 0.968 \times 1.1 = 1.0648\ \Omega/\text{phase}.$$

$$\boxed{X = 1.0648\ \Omega/\text{phase}}$$

(5) A 300 MVA, 20 kV, 3ϕ generator has a subtransient reactance of 20%. The generator supplies 2 synchronous motors through a 64 km tr. line having transformers at both ends as shown in fig. In this, T_1 is a 3ϕ transformer and T_2 is made of 3 single phase transformer of rating 100 MVA, 127/13.2 kV, 10% reactance. Series reactance of the tr. line is 0.5 Ω/km. Draw the reactance diagram with all the reactances marked in p.u. Select the generator rating as base values.



Solution:

$MVA_{b_{new}} = 300\ MVA$

$kV_{b_{new}} = 20\ kV$

Reactance of Generator G.

∵ Generator rating & base values are same, reactance will not change.

∴ pu reactance of generator $= 20\% = 0.2\ p.u.$

Reactance of Transformer T_1

The new p.u reactance of T_1 $= X_{pu\ old} \times \left(\frac{kV_{b\ old}}{kV_{b\ new}}\right)^2 \times \left(\frac{MVA_{b\ new}}{MVA_{b\ old}}\right)$

$= 0.1 \times \left(\frac{20}{20}\right)^2 \times \left(\frac{300}{350}\right)$

$= 0.0857\ p.u.$

Reactance of Tr. line.

Reactance of tr. line $= 0.5\ \Omega/km$

Total reactance $= 0.5 \times 64 = 32\ \Omega$

Base kV on HT side of T_1 $=$ Base kV on LT side $\times \frac{\text{HT voltage}}{\text{LT voltage}}$

$= 20 \times \frac{230}{20} = 230\ kV$

p.u reactance of Tr. line $= \frac{\text{Actual reactance}}{\text{Base reactance}}$

$$Z_b = \frac{(kV_b)^2}{MVA_b} = \frac{230}{300} = 176.33\ \Omega$$ //

p.u. reactance of Tr. line $= \frac{32}{176.32} = 0.1815$ p.u //

Note :-

	V_{ph}	V_L.
star	$V_{ph} = \frac{V_L}{\sqrt{3}}$	$V_L = \sqrt{3}\, V_{ph}$
Delta	$V_p = V_L$	$V_L = V_p$.

Reactance of Transformer T_2.

Voltage ratio of line voltage $= \sqrt{3} \times 127 / 13.2\,kV$

$= 220 / 13.2\,kV$ //

Base kV on LT side T_2 $=$ HT side $\times \frac{\text{LT voltage}}{\text{HT voltage}}$

$$= 230 \times \frac{13.2}{220} = 13.8\ kV$$ //.

p.u. reactance of T_2 $= X_{pu,old} \times \left(\frac{kV_{b\,old}}{kV_{b\,new}}\right)^2 \times \left(\frac{MVA_{b\,new}}{MVA_{b\,old}}\right)$

$$= 0.1 \times \left(\frac{13.2}{13.8}\right)^2 \times \left(\frac{300}{300}\right)$$

$= 0.0915$ p.u. //.

Reactance of M_1

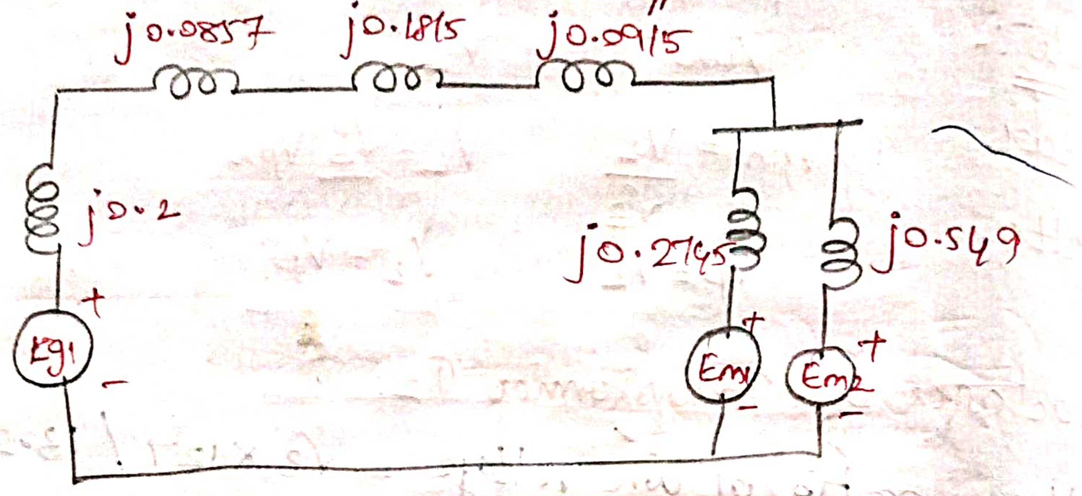
p.u reactance $= 0.2 \times \left(\frac{13.2}{13.8}\right)^2 \times \left(\frac{300}{200}\right)$

$= 0.2745$ p.u //

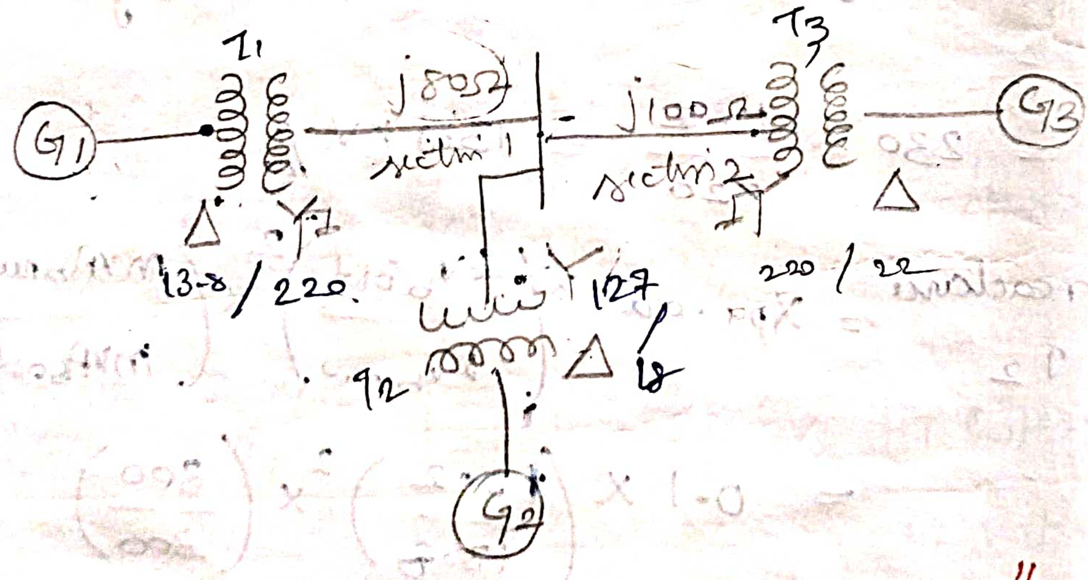
Reactance of M2

$$\text{p.u reactance} = 0.2 \times \left(\frac{13.2}{13.8}\right)^2 \times \left(\frac{300}{100}\right)$$

$$= 0.549 \text{ p.u}$$



(6) The single line diagram of an unloaded power system is shown in fig. The generators & transformers are rated as follows:



Generator G_1 = 20 MVA, 13.8 kV, $X'' = 20\%$

G_2 = 30 MVA, 18 kV, $X'' = 20\%$

G_3 = 30 MVA, 20 kV, $X'' = 20\%$

T_1 = 25 MVA, 220/13.8 kV, $X = 10\%$

T_2 = 3 1φ transformers each rated at 10 MVA, 127/18 kV, $X = 10\%$.

$T_3 = 35\,MVA,\ 220/22\,kV,\ X = 10\%.$

Draw the reactance diagram using a base of 50 MVA, 13.8 kV, on generator G_1

Solution

$MVA_{b\,new} = 50$, $kV_{b\,new} = 13.8\,kV$

Reactance of Generator G_1

$$\left.\begin{array}{l}\text{New p.u. reactance}\\ \text{of generator } G_1\end{array}\right] = X_{pu\,old} \times \left(\frac{kV_{b\,old}}{kV_{b\,new}}\right)^2 \times \left(\frac{MVA_{b\,new}}{MVA_{b\,old}}\right)$$

$$= 0.2 \times \left(\frac{13.8}{13.8}\right)^2 \times \frac{50}{20} = 0.5\ p.u.$$

Reactance of Transformer T_1

$$= 0.1 \times \left(\frac{13.8}{13.8}\right)^2 \times \left(\frac{50}{25}\right) = 0.2\ p.u.$$

Reactance of Tr. line

$$\text{Base kV on HT side} = 13.8 \times \frac{220}{13.8} = 220\,kV$$

$$kV_{b\,new} = 220\,kV$$

$$\left.\begin{array}{l}\text{p.u. reactance of}\\ \text{Tr. line}\end{array}\right\} = \frac{\text{Actual}}{\text{Base}}$$

$$\text{Base} = \frac{(220)^2}{50} = 968\ \Omega$$

$$\text{Section 1} = \frac{80}{968} = 0.0826\ p.u$$

$$\text{Section 2} = \frac{100}{968} = 0.1033\ p.u$$

Reactance of Transformer T_2

$$\left.\begin{array}{l}\text{Voltage ratio of line} \\ \text{voltage}\end{array}\right\} = \sqrt{3}\times 127 / 18$$

$$= 220 / 18 \text{ kV}$$

$$\left.\begin{array}{l}\text{Base kV on} \\ \text{LT side}\end{array}\right\} = 220 \times \frac{18}{220} = 18 \text{ kV}$$

$$\boxed{kV_{b\,new} = 18 \text{ kV}}$$

$$\text{New p.u reactance} = 0.1 \times \left(\frac{18}{18}\right)^2 \times \left(\frac{50}{3\times 10}\right)$$

$$= 0.1667 \text{ p.u}$$

Reactance of G_2

$$\text{p.u.} = 0.2 \times \left(\frac{18}{18}\right)^2 \times \left(\frac{50}{30}\right) = 0.3333 \text{ p.u}$$

Reactance of Transformer T_3

$$LT = HT \times \frac{LT}{HT}$$

$$= 220 \times \frac{22}{220} = 22 \text{ kV}$$

$$\boxed{kV_{b\,new} = 22 \text{ kV}}$$

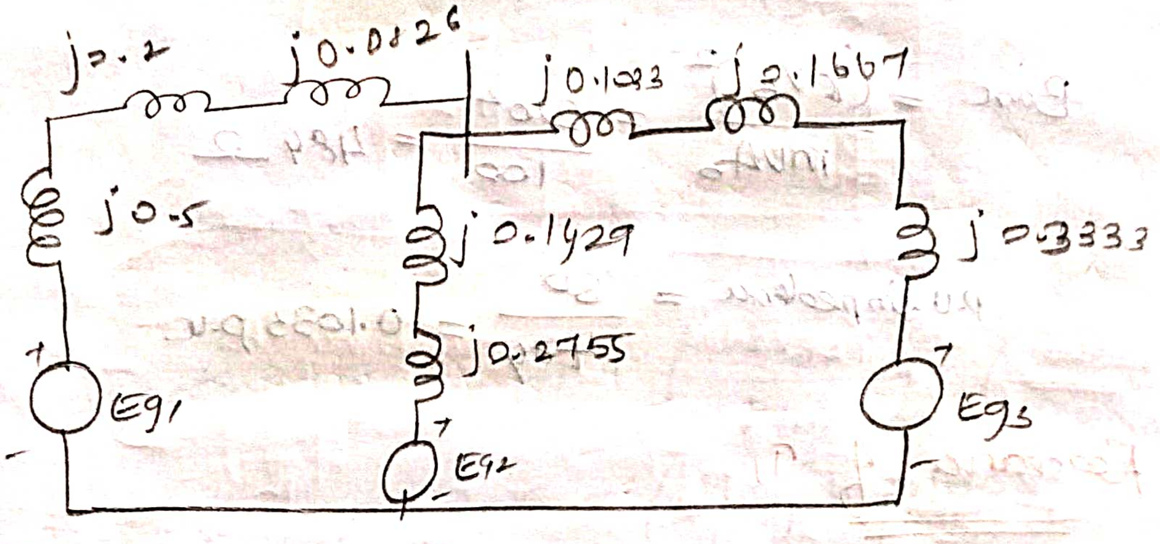
$$\text{p.u} = 0.1 \times \left(\frac{22}{22}\right)^2 \times \left(\frac{50}{35}\right) = 0.1429 \text{ p.u}$$

Reactance of G3.

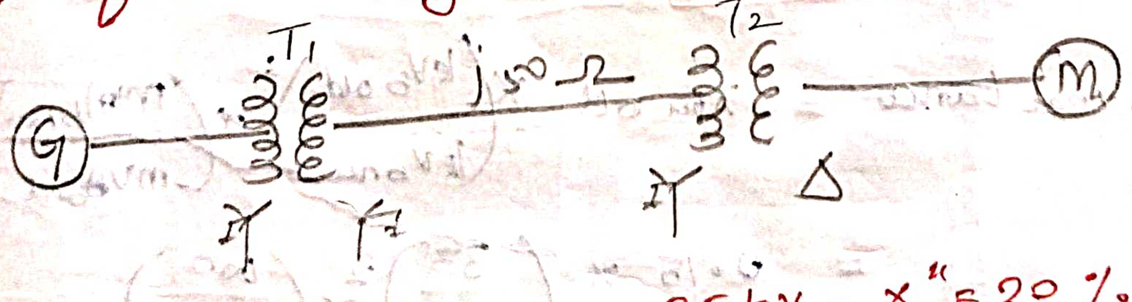
$$\text{p.u. reactance} = 0.2 \times \left(\frac{20}{22}\right)^2 \times \left(\frac{50}{30}\right)$$

$$= 0.2755 \text{ p.u.}$$

Reactance diagram



(7) Draw the reactance diagram for the power systems shown in fig. Neglect resistance and use a base of 100 MVA, 220 kV in 50 Ω line. The rating of generator, motor & transformer are given below.



Generator 40 MVA, 25 kV, $X'' = 20\%$.

Motor 50 MVA, 11 kV, $X'' = 30\%$.

Y - Y transformer : 40 MVA, 33/220 kV, 15%.

Y - Δ " : 30 MVA (11/220 kV) Δ/Y, 15%

Solution :-

$MVA_{b\,new} = 100\ MVA$

$kV_{b\,new} = 220\ kV$

Reactance of Tr. line.

$$p.u.\ impedance = \frac{Actual}{Base} =$$

$$Base = \frac{(kV_b)^2}{MVA_b} = \frac{220^2}{100} = 484\ \Omega$$

$$p.u.\ impedance = \frac{50}{484} = 0.1033\ p.u$$

Reactance of T_1

$$LT\ side = HT \times \frac{LT}{HT}$$

$$= 220 \times \frac{33}{220} = 33\ kV$$

$kV_{b\,new} = 33\ kV$

$$p.u.\ reactance = X_{pu\,old} \times \left(\frac{kV_{b\,old}}{kV_{b\,new}}\right)^2 \times \left(\frac{MVA_{b\,new}}{MVA_{b\,old}}\right)$$

$$= 0.15 \times \left(\frac{33}{33}\right)^2 \times \left(\frac{100}{40}\right)$$

$$= 0.375\ p.u$$

Reactance of G.

$$p.u.\ impedance = 0.2 \times \left(\frac{25}{33}\right)^2 \times \left(\frac{100}{40}\right)$$

$$= 0.287\ p.u.$$

Reactance of T2

$$Base\ kV\ on\ LT = HT \times \frac{LT}{HT} = 220 \times \frac{11}{220}$$

$$= 11\ kV$$

$$kV_b\ new = 11\ kV$$

$$p.u.\ impedance = X_{pu\ old} \times \left(\frac{kV_{b\ old}}{kV_{b\ new}}\right)^2 \times \left(\frac{MVA_{b\ new}}{MVA_{b\ old}}\right)$$

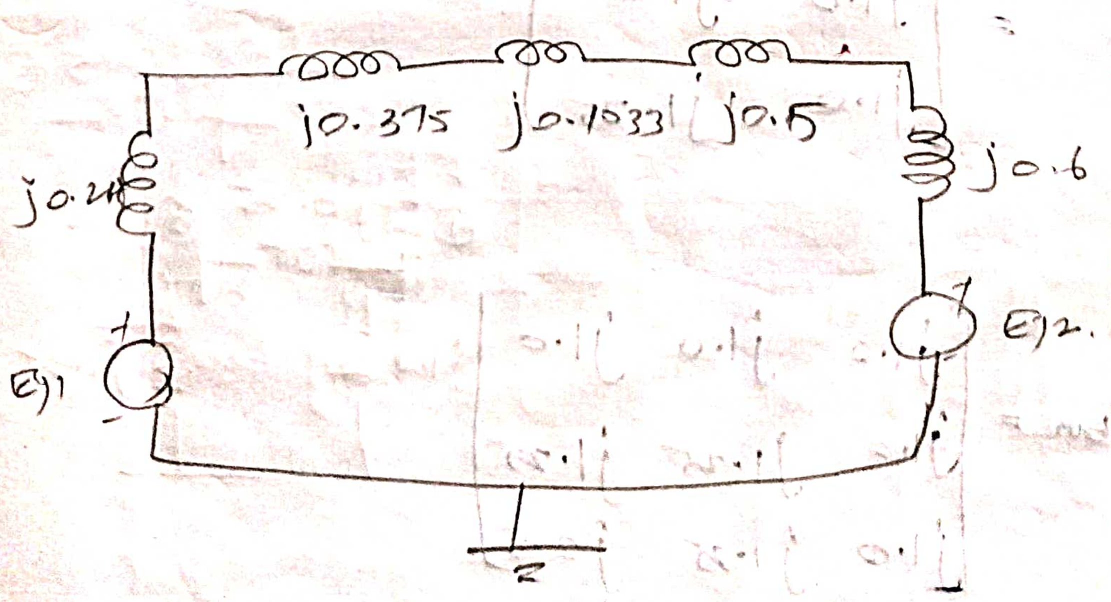
$$= 0.15 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{100}{30}\right)$$

$$= 0.5\ p.u.$$

Reactance of Synchronous motor.

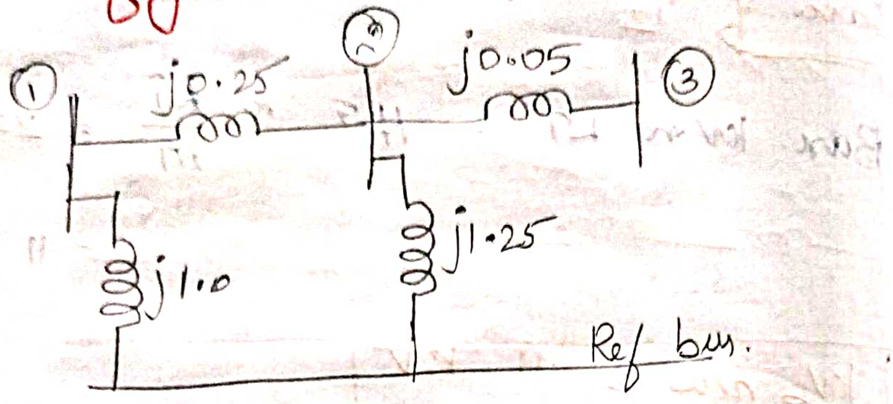
$$p.u\ impedance = 0.3 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{100}{50}\right)$$

$$= 0.6\ p.u.$$



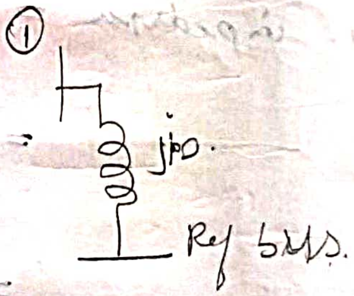
BUS IMPEDANCE MATRIX

① Find the bus impedance matrix for the system whose reactance diagram is shown in fig. All the impedance are in p.u.

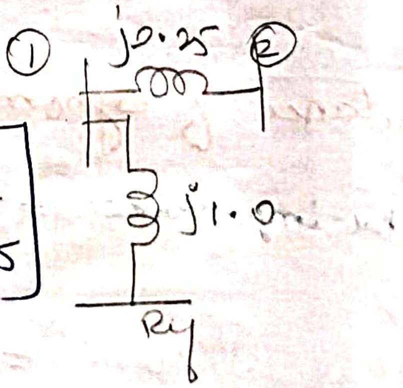


Step 1 :- Consider j1.0 p.u branch

$$Z_{bus} = [j1.0]$$



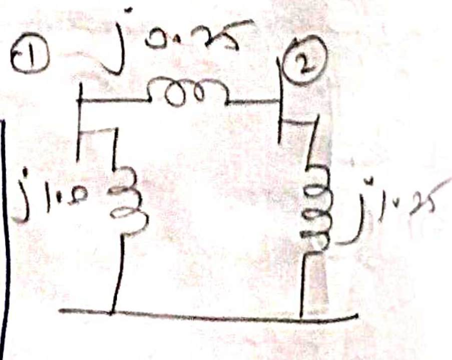
Step 2 :-



$$Z_{bus} = \begin{bmatrix} j1.0 & j1.0 \\ j1.0 & j1.0 + j0.25 \end{bmatrix}$$

$$= \begin{bmatrix} j1.0 & j1.0 \\ j1.0 & j1.25 \end{bmatrix}$$

Step 3.



$$Z_{bus} = \begin{bmatrix} j1.0 & j1.0 & j1.0 \\ j1.0 & j1.25 & j1.25 \\ j1.0 & j1.25 & j2.5 \end{bmatrix}$$

$$Z_{jk,act} = Z_{jk} - \frac{Z_{j(n+1)} Z_{(n+1)k}}{Z_{(n+1)(n+1)}}$$

where $n = 2$, $k = 1, 2$ & $j = 1, 2$.

$$Z_{11,act} = Z_{11} - \frac{Z_{13} Z_{31}}{Z_{33}} = j1.0 - \frac{j1.0 \times j1.0}{j2.5}$$

$$= j0.6$$

$$Z_{12,act} = Z_{12} - \frac{Z_{13} Z_{32}}{Z_{33}} = j1.0 - \frac{j1.0 \times j1.25}{j2.5}$$

$$= j0.5$$

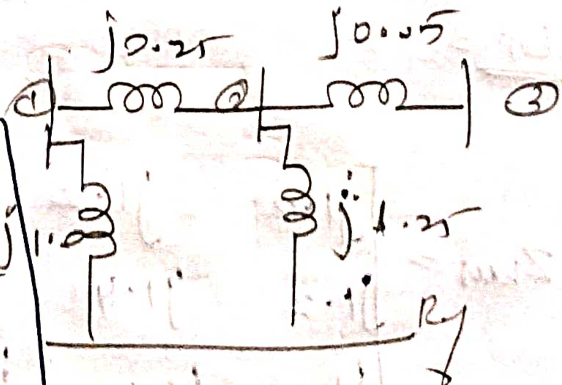
$$Z_{12} = Z_{21} = j0.5$$

$$Z_{22} = Z_{22} - \frac{Z_{23} Z_{32}}{Z_{33}} = j1.25 - \frac{j1.25 \times j1.25}{j2.5}$$

$$= j0.625$$

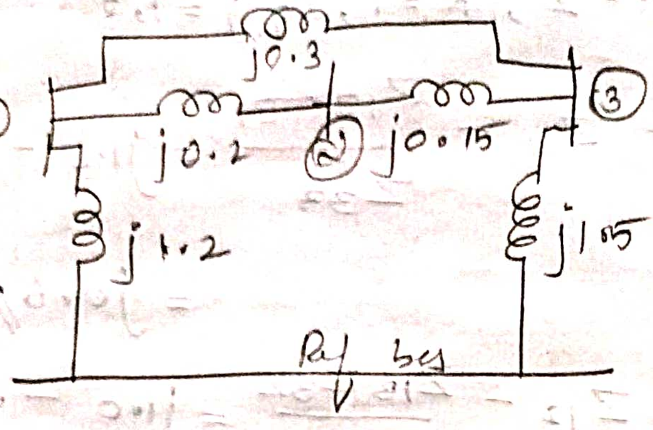
$$Z_{bus} = \begin{bmatrix} j0.6 & j0.5 \\ j0.5 & j0.625 \end{bmatrix}$$

Step 4.



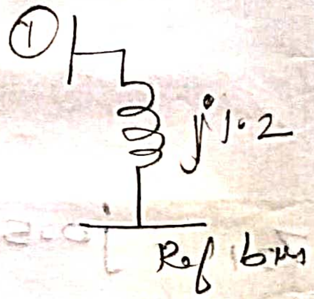
$$Z_{bus} = \begin{bmatrix} j0.6 & j0.5 & j0.5 \\ j0.5 & j0.625 & j0.625 \\ j0.5 & j0.625 & j0.675 \end{bmatrix}$$

Q) Determine Z_{bus} for system whose reactance diagram is shown in fig. where the impedance is given in p.u. Preserve all the nodes.



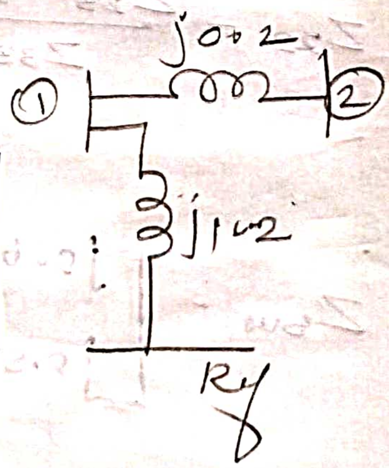
Step 1.

$$Z_{bus} = [j1.2]$$



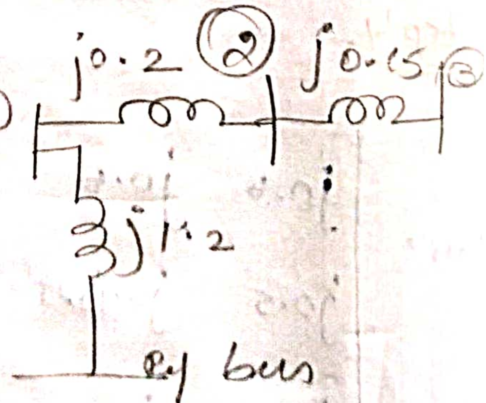
Step 2.

$$Z_{bus} = \begin{bmatrix} j1.2 & j1.2 \\ j1.2 & j1.4 \end{bmatrix}$$

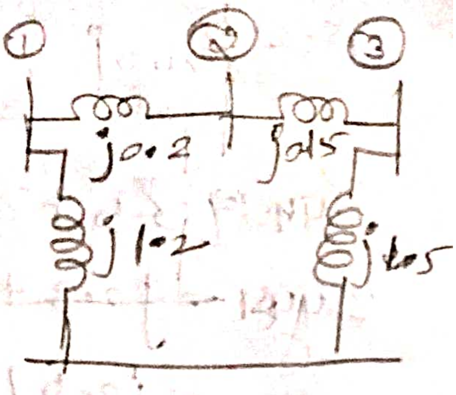


Step 3

$$Z_{bus} = \begin{bmatrix} j1.2 & j1.2 & j1.2 \\ j1.2 & j1.4 & j1.4 \\ j1.2 & j1.4 & j1.55 \end{bmatrix}$$



Step 4



$$Z_{bus} = \begin{bmatrix} j1.2 & j1.2 & j1.2 & j1.2 \\ j1.2 & j1.4 & j1.4 & j1.4 \\ j1.2 & j1.4 & j1.55 & j1.55 \\ j1.2 & j1.4 & j1.55 & j3.05 \end{bmatrix}$$

$$Z_{jk} = Z_{jk} - \frac{Z_{j(n+1)} Z_{(n+1)k}}{Z_{(n+1)(n+1)}}$$

where, $n = 3$, $j = 1, 2, 3$, $k = 1, 2, 3$

$$Z_{11} = Z_{11} - \frac{Z_{14} Z_{41}}{Z_{44}} = j0.728$$

$Z_{12} = j0.649$, $Z_{13} = j0.590$,

$Z_{21} = Z_{12} = j0.649$

$Z_{22} = j0.757$

$Z_{23} = j0.689$

$Z_{31} = Z_{13} = j0.590$

$Z_{32} = Z_{23} = j0.689$

$Z_{33} = j0.762$

$$Z_{bus} = \begin{bmatrix} j0.728 & j0.649 & j0.590 \\ j0.649 & j0.757 & j0.689 \\ j0.590 & j0.689 & j0.702 \end{bmatrix}$$

Step 5

full picture

$$Z_{44} = Z_5 + Z_{11} + Z_{33} - 2Z_{13}$$

$$Z_{44} = j0.3 + j0.728 + j0.762 - 2(j0.59)$$

$$= j0.61$$

$$Z_{bus} = \begin{bmatrix} j0.728 & j0.649 & j0.590 & j0.728 - j0.59 \\ j0.649 & j0.757 & j0.689 & j0.649 - j0.689 \\ j0.59 & j0.689 & j0.762 & j0.59 - j0.762 \\ (j0.729 - j0.59) & (j0.649 - j0.689) & (j0.59 - j0.762) & j0.61 \end{bmatrix}$$

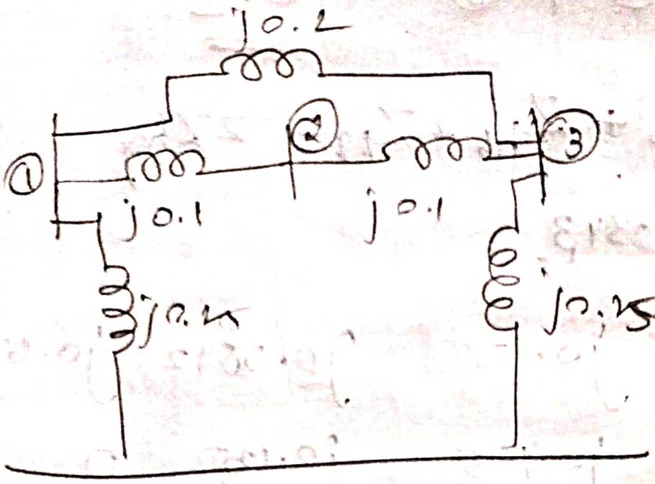
$$Z_{bus} = \begin{bmatrix} j0.728 & j0.649 & j0.590 & j0.138 \\ j0.649 & j0.757 & j0.689 & -j0.04 \\ j0.59 & j0.689 & j0.762 & -j0.172 \\ j0.138 & -j0.04 & -j0.172 & j0.61 \end{bmatrix}$$

$n = 3$, $j = 1, 2, 3$. $k = 1, 2, 3$.

$$Z_{jk} = Z_{jk} - \frac{Z_{j(n+1)} Z_{(n+1)k}}{Z_{(n+1)(n+1)}}$$

$$Z_{bus} = \begin{bmatrix} j0.697 & j0.658 & j0.629 \\ j0.658 & j0.754 & j0.678 \\ j0.629 & j0.678 & j0.714 \end{bmatrix}$$

(3)



$$Z_{bus} = [j0.25]$$

$$Z_{bus} = \begin{bmatrix} j0.25 & j0.25 \\ j0.25 & j0.35 \end{bmatrix}$$

$$Z_{bus} = \begin{bmatrix} j0.25 & j0.25 & j0.25 \\ j0.25 & j0.35 & j0.35 \\ j0.25 & j0.35 & j0.45 \end{bmatrix}$$

$$Z_{bus} = \begin{bmatrix} j0.25 & j0.25 & j0.25 & j0.25 \\ j0.25 & j0.35 & j0.35 & j0.35 \\ j0.25 & j0.35 & j0.45 & j0.45 \\ j0.25 & j0.35 & j0.45 & j0.7 \end{bmatrix}$$

4x4 to 3x3.

$$Z_{bus} = \begin{bmatrix} j0.1607 & j0.125 & j0.0892 \\ j0.125 & j0.175 & j0.125 \\ j0.0892 & j0.125 & j0.1607 \end{bmatrix}$$

node b/o (1) & (3) j0.2

$$Z_{(n+1)(n+1)} = Z_b + Z_{hh} + Z_{qq} - 2Z_{hq}$$

$$= j0.343$$

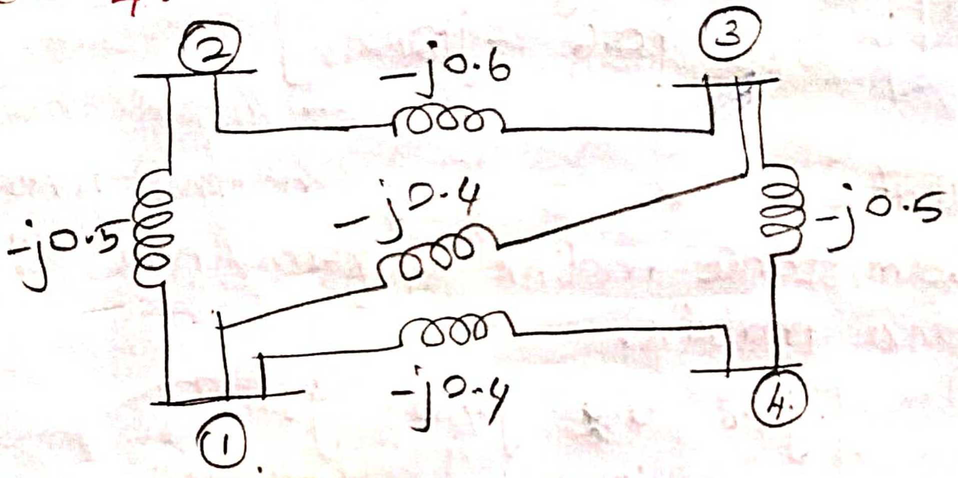
$$Z_{bus} = \begin{bmatrix} j0.1607 & j0.125 & j0.0892 & j0.072 \\ j0.125 & j0.175 & j0.125 & 0 \\ j0.0892 & j0.125 & j0.1607 & -j0.075 \\ j0.072 & 0 & -j0.0715 & j0.343 \end{bmatrix}$$

4×4 → 3×3

$$Z_{bus} = \begin{bmatrix} j0.145 & j0.125 & j0.102 \\ j0.125 & j0.175 & j0.125 \\ j0.102 & j0.125 & j0.145 \end{bmatrix}$$

Bus Admittance Matrix.

① For the network shown in fig, form the bus admittance matrix. Determine the reduced admittance matrix by eliminating node 4.



$$Y_{bus} = \begin{bmatrix} -(j0.5+j0.4+j0.4) & j0.5 & j0.4 & j0.4 \\ j0.5 & -(j0.5+j0.6) & j0.6 & 0 \\ j0.4 & j0.6 & -(j0.6+j0.5+j0.4) & j0.5 \\ j0.4 & 0 & j0.5 & -(j0.5+j0.4+j0.4) \end{bmatrix}$$

$$= \begin{bmatrix} -j1.3 & j0.5 & j0.4 & j0.4 \\ j0.5 & -j1.1 & j0.6 & 0 \\ j0.4 & j0.6 & -j1.5 & j0.5 \\ j0.4 & 0 & j0.5 & -j0.9 \end{bmatrix}$$

$$Y_{jk\,new} = Y_{jk} - \frac{Y_{jn} Y_{nk}}{Y_{nn}}$$

$n = 4$, $j = 1, 2, 3$ $k = 1, 2, 3$.

Reduced matrix

$$Y_{bus} = \begin{bmatrix} -j1.12 & j0.5 & j0.622 \\ j0.5 & -j1.1 & j0.6 \\ j0.622 & j0.6 & -j1.222 \end{bmatrix}$$

② Eliminate buses 3 & 4 in the given bus admittance matrix & form new bus admittance matrix.

$$Y_{bus} = \begin{bmatrix} -j9.8 & 0.0 & j4.0 & j5.0 \\ 0.0 & -j8.3 & j2.5 & j5.0 \\ j4.0 & j2.5 & -j14 & j8.0 \\ j5.0 & j5.0 & j8.0 & -j18.0 \end{bmatrix}$$

First eliminate bus no. 4.

$$Y_{jk,new} = Y_{jk} - \frac{Y_{jn} Y_{nk}}{Y_{nn}}, \quad n = 4, \; j = 1,2,3, \; k = 1,2,3$$

$$Y_{bus} = \begin{bmatrix} j8.411 & j1.3889 & j6.222 \\ j1.388 & -j6.911 & j4.722 \\ j6.22 & j4.722 & -j10.44 \end{bmatrix}$$

Eliminate bus no. 3.

$n = 3$, $j = 1, 2$. $k = 1, 2$.

$$Y_{bus} = \begin{bmatrix} -j4.7043 & j4.2011 \\ j4.2011 & j4.4761 \end{bmatrix}$$