UNIT – II

Short Circuit Studies

Symmetrical Components.

The analysis of unsymmetrical polyphase network by the method of symmetrical components was introduced by Dr. C. Fortesque.

Unbalanced system of n vectors can be resolved into n systems of balanced vectors called symmetrical components. | n vectors of each set of components are equal in length & phase angles b/w adjacent vectors are equal.

In a 3ϕ system, 3 unbalanced vectors V_a, V_b, V_c (or) I_a, I_b, I_c.

The vectors of the balanced system are called symmetrical components.

The symmetrical components of 3ϕ system are

- positive sequence components.
- Negative sequence components
- Zero sequence components

+ve sequence components

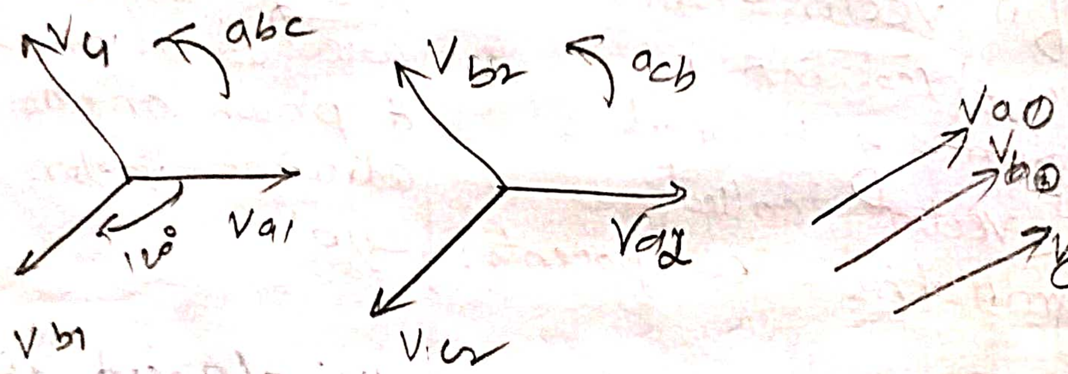
3 vectors, equal in magnitude, displaced by 120° & have same phase sequence.

-ve sequence components.

3 vectors, equal in magnitude, 120° displacement, phase sequence opposite to that of original vectors.

Zero sequence components.

3 vectors, equal in magnitude, with zero phase displacement.



on rotating V_{a1} by 120° in anticlockwise V_{c1}

" " V_{a1} by 240° " " V_{b1}

" " V_{a2} " 120° " " V_{b2}

" " V_{a2} to 240° " " V_{c2}

Computation of Unbalanced vectors from their symmetrical components.

Each of original unbalanced vector is sum of its +ve, -ve & zero sequence component.

∴ Original 3ϕ vectors can be expressed in terms of their components.

$$V_a = V_{a0} + V_{a1} + V_{a2}$$

$$V_b = V_{b0} + V_{b1} + V_{b2}$$

$$V_c = V_{c0} + V_{c1} + V_{c2}$$

from vector diagrams.

$V_{b0} = V_{a0}$	$V_{b1} = a^2 V_{a1}$	$V_{b2} = a V_{a2}$
$V_{c0} = V_{a0}$	$V_{c1} = a V_{a1}$	$V_{c2} = a^2 V_{a2}$

$$V_a = V_{a0} + V_{a1} + V_{a2}$$

$$V_b = V_{a0} + V_{a1} a^2 + a V_{a2}$$

$$V_c = V_{a0} + a V_{a1} + a^2 V_{a2}.$$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

This matrix can be used to compute the unbalanced voltage vectors from the knowledge of symmetrical components.

Formulas.

$$V_{a0} = \frac{1}{3}\left[V_a + V_b + V_c\right]$$

$$V_{a1} = \frac{1}{3}\left[V_a + aV_b + a^2 V_c\right]$$

$$V_{a2} = \frac{1}{3}\left[V_a + a^2 V_b + a V_c\right]$$

Symmetrical components of unbalanced current vectors.

$$I_{a0} = \frac{1}{3}[I_a + I_b + I_c]$$

$$I_{a1} = \frac{1}{3}[I_a + aI_b + a^2 I_c]$$

$$I_{a2} = \frac{1}{3}[I_a + a^2 I_b + aI_c]$$

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

Problem

① The voltages across a 3-ϕ unbalanced load are $V_a = 300\angle 20^\circ$ V, $V_b = 360\angle 90^\circ$ & $V_c = 500\angle -140^\circ$ V. Determine the symmetrical component of voltages. phase sequence is abc.

Solution.

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$V_{a0} = \frac{1}{3}[V_a + V_b + V_c]$$

$$V_{a1} = \frac{1}{3}[V_a + aV_b + a^2 V_c]$$

$$V_{a2} = \frac{1}{3}[V_a + a^2 V_b + aV_c]$$

Given that.

$V_a = 300\angle 20^\circ = 281.9 + j102.61\ V$

$V_b = 360\angle 90^\circ = 0 + j360\ V$

$V_c = 500\angle -140^\circ = -383.02 + j321.39\ V$

$aV_b = 1\angle 120^\circ (360^\circ \angle 90^\circ) = -311.77 - j180^\circ V.$

$= 360\angle 210^\circ$

$a^2V_b = 1\angle 240^\circ (360\angle 90^\circ) = 360\angle 330^\circ = 311.77 - j180^\circ V$

$aV_c = 1\angle 120^\circ (500^\circ \angle -140^\circ) = 500\angle -20^\circ = 469.85 - j171.01\ V.$

$a^2V_c = -86.82 + j492.40\ V$

$V_{a0} = \frac{1}{3}[V_a + V_b + V_c] = -33.70 + j47.07$

$= 57.89\angle 120^\circ\ V$

$V_{a1} = \frac{1}{3}[V_a + aV_b + a^2V_c]$

$= -38.89 + j138.34 = 143.70\angle 106^\circ\ V$

$V_{a2} = 354.51 - j82.80 = 364.05\angle -13^\circ\ V$

Zero sequence

$V_{a0} = 57.89\angle 126^\circ\ V$

$V_{b0} = 57.89\angle 126^\circ\ V$

$V_{c0} = 57.89\angle 120^\circ\ V$

+ve sequence

$\therefore V_{b1} = a^2V_{a1} \quad V_{c1} = aV_{a1}$

$V_{a1} = 143.70\angle 106^\circ\ V$

$V_{b1} = 143.70\angle 346^\circ\ V$

$V_{c1} = 143.70\angle 226^\circ\ V$

-ve sequence

$V_{b2} = aV_{a2} \quad V_{c2} = a^2V_{a2}$

$V_{a2} = 364.05\angle -13^\circ\ V$

$V_{b2} = 364.05\angle 107^\circ\ V$

$V_{c2} = 364.05\angle 227^\circ\ V$

② The symmetrical components of phase voltage in a 3ϕ unbalanced systems are $V_{a0} = 10\angle 180^\circ$, $V_{a1} = 50\angle 0^\circ V$, $V_{a2} = 20\angle 90^\circ V$. Determine phase voltage V_a, V_b, V_c.

Solution:-

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

$$V_a = V_{a0} + V_{a1} + V_{a2}$$
$$V_b = V_{a0} + a^2 V_{a1} + a V_{a2}$$
$$V_c = V_{a0} + a V_{a1} + a^2 V_{a2}$$

$$V_{a0} = 10\angle 180^\circ V = -10 + j0$$
$$V_{a1} = 50\angle 0^\circ V = 50 + j0$$
$$V_{a2} = 20\angle 90^\circ V = 0 + j20$$

$$V_a = 44.72\angle 27^\circ V = 40 + j20$$
$$V_b = 74.69\angle -134^\circ V = -52.32 - j53.33$$
$$V_c = 37.70\angle 118^\circ V = -17.68 + j33.3$$

③ The symmetrical components of phase a fault currents in a 3 phase unbalanced system are $I_{a0} = 350\angle 90°$ A $I_{a1} = 600\angle -90°$ A $I_{a2} = 250\angle 90°$ A. Determine phase currents I_a, I_b & I_c.

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \cancel{\tfrac{1}{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

$$I_{a0} = 350\angle 90° = 0 + j350$$
$$I_{a1} = 600\angle -90° = 0 - j600$$
$$I_{a2} = 250\angle 90° = 0 + j250$$

$$I_a = 0$$
$$I_b = 904.16\angle -145° \text{ A}$$
$$I_c = 904.16\angle 35° \text{ A}$$

④ Determine the symmetrical components of the unbalanced 3ϕ currents, $I_a = 10\angle 0°$, $I_b = 12\angle 230°$, $I_c = 10\angle 130°$ A.

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

Ans.

$$I_{a0} = 1.47\angle -160° = -1.38 - j0.51$$
$$I_{a1} = 10.56\angle -0.6$$
$$I_{a2} = 1.04\angle 37° = 0.83 + j0.63$$

Zero sequence $\Rightarrow I_{a0} = I_{b0} = I_{c0}$

$I_{a0} = 1.47 \angle -160^\circ$ A

$I_{b0} = 1.47 \angle -160^\circ$ A

$I_{c0} = 1.47 \angle -160^\circ$ A.

+ve sequence components

$I_{b1} = a^2 I_{a1}$; $I_{c1} = a I_{a1}$

$I_{a1} = 10.56 \angle 0^\circ$ A

$I_{b1} = a^2 I_{a1} = 10.56 \angle 240^\circ$ A

$I_{c1} = a I_{a1} = 10.56 \angle 120^\circ$ A

-ve sequence components

$I_{a2} = 1.04 \angle 37^\circ$ A

$I_{b2} = a I_{a2} = 1.04 \angle 157^\circ$ A

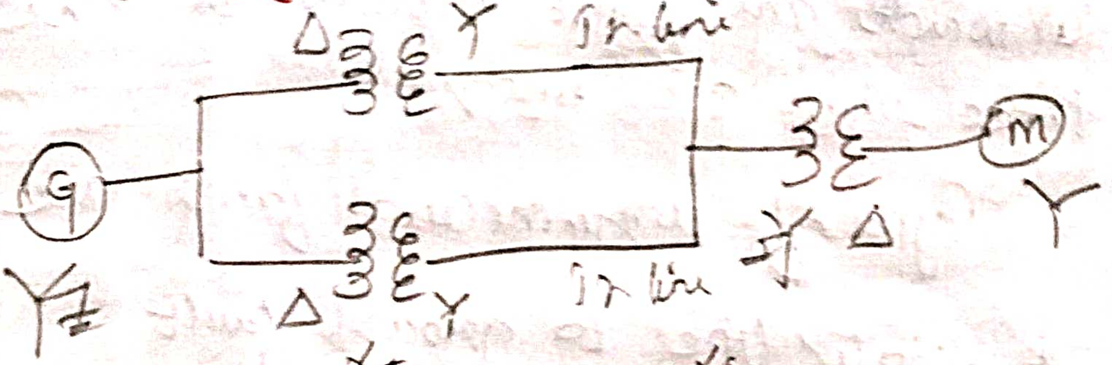
$I_{c2} = a^2 I_{a2} = 1.04 \angle 277^\circ$ A

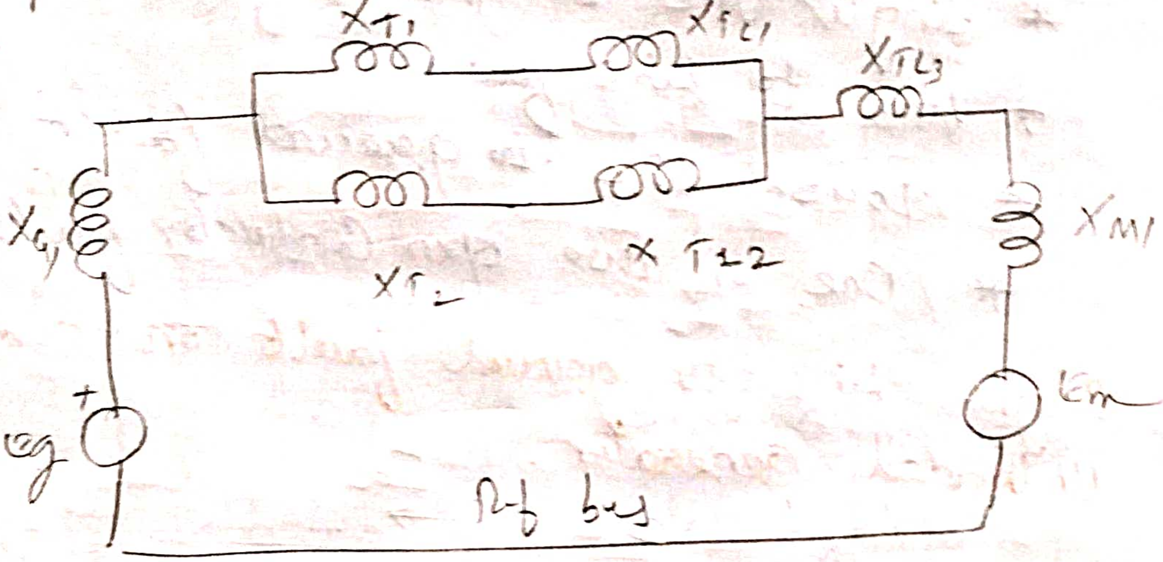
Sequence networks. & Sequence impedance.

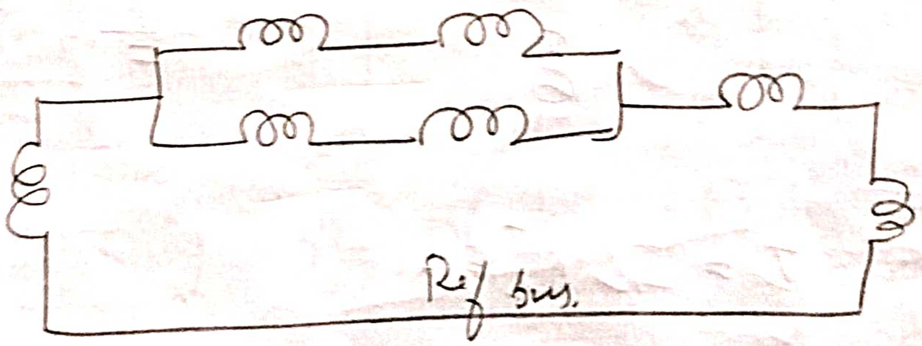
Sequence impedance are the impedances offered by the circuit element to +ve, -ve & zero sequence currents.

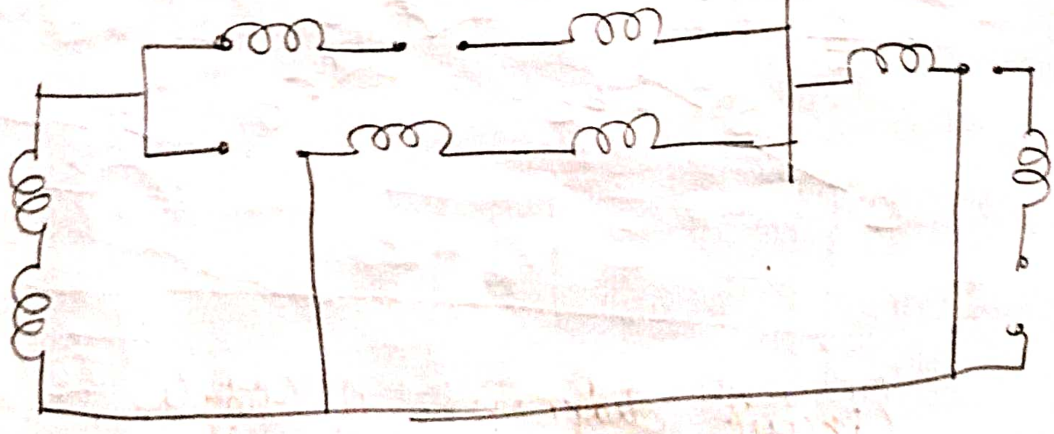
" The impedance (or) reactance diagram formed using zero sequence impedance is called zero sequence network."

Draw the +ve, -ve, zero sequence reactance diagram of the power system shown in fig









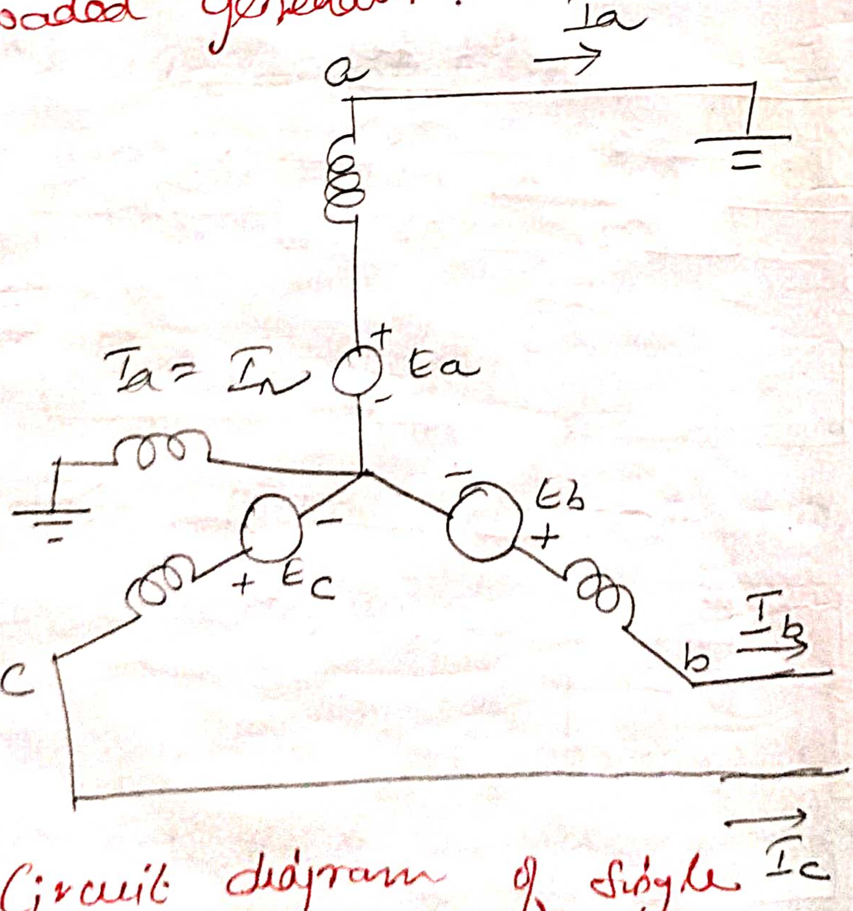
Unsymmetrical Fault Analysis.

The unsymmetrical faults are the faults in which the fault currents in the three phase are unequal.

Types of unsymmetrical faults are

* Single line to ground fault
* Line to Line fault
* Double line to ground fault
* One or two open conductor faults

Single Line to ground fault on an Unloaded generator.



Circuit diagram of single LG fault on phase a of an Unloaded generator.

phase a is shorted to ground.

fault current $I_f = I_a$

∵ generator is unloaded other phase are 0.

The conditions of fault is expressed by the following equations.

$I_b = 0, \; I_c = 0, \; V_a = 0$ —— (1)

Symmetrical Components of currents are

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \text{ —— (2)}$$

on substituting $I_b = I_c = 0$

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix} \text{ —— (3)}$$

on multiplying the above matrix.

$$I_{a0} = I_{a1} = I_{a2} = \frac{I_a}{3} \text{ —— (4)}$$

From seq network of generators.

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \text{ —— (5)}$$

On substituting $I_{a0} = I_{a1}$, $I_{a2} = I_{a1}$ in eq (5)

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a1} \\ I_{a1} \\ I_{a1} \end{bmatrix} \quad \text{—— (6)}$$

Xply eq (6)

$$V_{a0} = -Z_0 I_{a1} \quad \text{—— (7)}$$

$$V_{a1} = E_a - Z_1 I_{a1} \quad \text{—— (8)}$$

$$V_{a2} = -Z_2 I_{a1} \quad \text{—— (9)}$$

Add (7), (8) & (9)

$$V_{a0} + V_{a1} + V_{a2} = -I_{a1} Z_0 + E_a - I_{a1} Z_1 - I_{a1} Z_2 \quad \text{—— (10)}$$

W.K.T

$$V_a = V_{a0} + V_{a1} + V_{a2} = 0$$

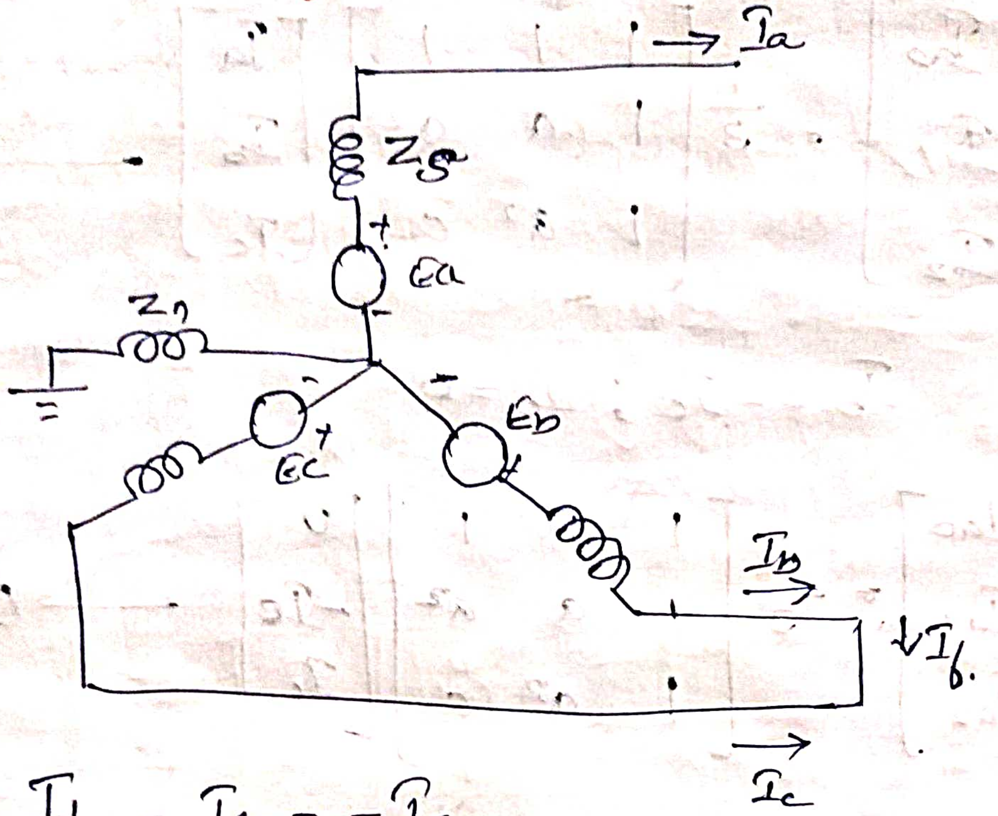
$$-I_{a1} Z_0 + E_a - I_{a1} Z_1 - I_{a1} Z_2 = 0$$

$$I_{a1} = \frac{E_a}{Z_1 + Z_2 + Z_0} \quad \text{—— (11)}$$

∴ No path exists for the flow of current in the fault unless the generator neutral is grounded.

$$I_f = I_a = 3 I_{a1} \quad \text{—— (12)}$$

Line to Line fault on an unloaded generator.



$$I_f = I_b = -I_c$$

$$V_b = V_c \; ; \; I_a = 0.$$

$$I_b + I_c = 0 \Longrightarrow I_b = -I_c \quad \text{—— (1)}$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad \text{—— (2)}$$

$$V_{a1} = \frac{1}{3}(V_a + aV_b + a^2 V_b) \quad \text{—— (3)}$$

$$V_{a2} = \frac{1}{3}(V_a + a^2 V_b + a V_b) \quad \text{—— (4)}$$

$$V_{a1} = V_{a2} \quad \text{—— (5)}$$

symmetrical components of current are

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad \text{--- (6)}$$

$$I_b = -I_c \; ; \; I_a = 0$$

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 0 \\ -I_c \\ I_c \end{bmatrix} \quad \text{--- (7)}$$

$$I_{a0} = \frac{1}{3}\left[-I_c + I_c\right] = 0$$

$$I_{a1} = \frac{1}{3}\left[-aI_c + a^2 I_c\right] \quad \text{--- (8)}$$

$$I_{a2} = \frac{1}{3}\left[-a^2 I_c + aI_c\right] \quad \text{--- (9)}$$

from eq (8) & (9)

$$I_{a1} = -I_{a2} \quad \text{--- (10)}$$

Sequence Networks

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \quad \text{--- (11)}$$

$I_{a0} = 0, \quad I_{a1} = -I_{a2}, \quad (I_{a2} = -I_{a1})$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} 0 \\ I_{a1} \\ -I_{a1} \end{bmatrix} \quad \text{--- (12)}$$

Xply (12)

$V_{a0} = 0$

$V_{a1} = E_a - Z_1 I_{a1}$ —— (13)

$V_{a2} = Z_2 I_{a1}$ —— (14)

from eq (5)

$E_a - Z_1 I_{a1} = Z_2 I_{a1}$

$$I_{a1} = \frac{E_a}{Z_1 + Z_2} \quad \text{—— (15)}$$