# Example



- A steel tube having an external diameter of 36 mm and an internal diameter of 30 mm has a brass rod of 20 mm diameter inside it, the two materials being joined rigidly at their ends when the ambient temperature is 18 $^0$C. Determine the stresses in the two materials: (a) when the temperature is raised to 68 $^0$C (b) when a compressive load of 20 kN is applied at the increased temperature.

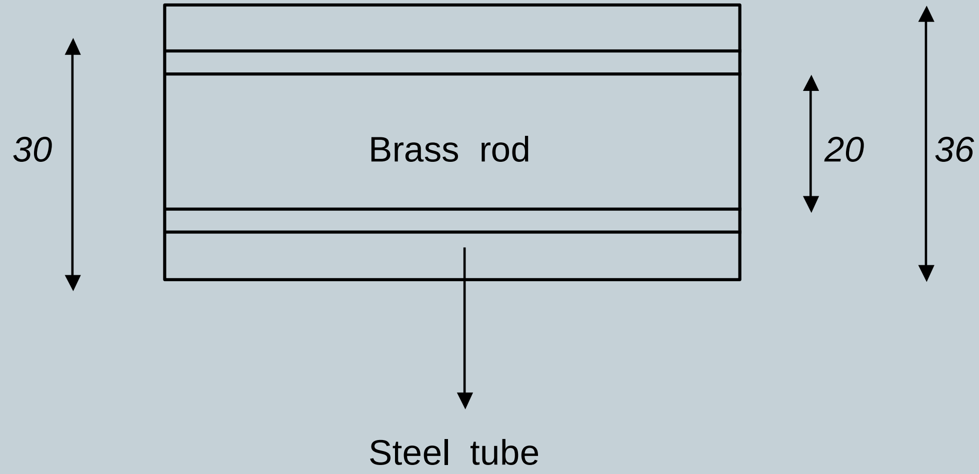
# Example Contd.



- For brass: Modulus of elasticity = 80 $GN/m^2$; Coefficient of expansion = 17 x 10 -6 $/^0C$
- For steel: Modulus of elasticity = 210 $GN/m^2$; Coefficient of expansion = 11 x 10 -6 $/^0C$

# Solution






Area of brass rod ($A_b$) = $\frac{\pi \; x \; 20^2}{4} = 314.16 \; mm^2$

Area of steel tube ($A_s$) = $\frac{\pi \; x \; (36^2 - 30^2)}{4} = 311.02 \; mm^2$

$$A_s E_s = 311.02 \; x \; 10^{-6} m^2 \; x \; 210 \; x \; 10^9 \; N / m^2 = 0.653142 \; x \; 10^8 \; N$$

$$\frac{1}{A_s E_s} = 1.53106 \; x \; 10^{-8}$$

# Solution Contd.



$$A_b E_b = 314.16\ x\ 10^{-6} m^2\ \ x\ 80\ x\ 10^9\ N/m^2 = 0.251327\ x\ 10^8\ N$$

$$\frac{1}{A_b E_b} = 3.9788736\ x\ 10^{-8}$$

$$T(\alpha_b - \alpha_s) = 50(17-11)\ x\ 10^{-6} = 3\ x\ 10^{-4}$$

With increase in temperature, brass will be in compression while steel will be in tension. This is because expands more than steel.

$$i.e.\quad F[\frac{1}{A_s E_s} + \frac{1}{A_b E_b}] = T(\alpha_b - \alpha_s)$$

i.e. F[1.53106 + 3.9788736] x 10 $^{-8}$ = 3 x 10 $^{-4}$

**F = 5444.71 N**

# Solution Concluded



Stress in steel tube = $\dfrac{5444.71N}{311.02\ mm^2} = 17.51N/mm^2 = 17.51MN/m^2 (Tension)$

Stress in brass rod = $\dfrac{5444.71N}{314.16\ mm^2} = 17.33N/mm^2 = 17.33MN/m^2 (Compression)$

(b) Stresses due to compression force, F' of 20 kN

$$\sigma_s = \frac{F' E_s}{E_s A_s + E_b A_b} = \frac{20\ x\ 10^3\ N\ x\ 210\ x\ 10^9\ N/m^2}{0.653142 + 0.251327\ x\ 10^8} = 46.44\ MN/m^2 (Compression)$$

$$\sigma_b = \frac{F' E_b}{E_s A_s + E_b A_b} = \frac{20\ x\ 10^3\ N\ x\ 80\ x\ 10^9\ N/m^2}{0.653142 + 0.251327\ x\ 10^8} = 17.69\ MN/m^2 (Compression)$$

Resultant stress in steel tube = - 46.44 + 17.51 = 28.93 MN/$m^2$ (Compression)

Resultant stress in brass rod = -17.69 - 17.33 = 35.02 MN/$m^2$ (Compression)